

## Trigonometriske funktioner med radiantal

Maple regner i radiantal.

Skal man regne i gradtal, er det lettest at anvende en Maple-pakke.  
"sin" med lille s regner i radiantal, "Sin" med stort S regner i gradtal.

**Radianer** bruges ved **funktioner** (grafer, differentiation, integraler).

Grader bruges i forbindelse med geometri (trekantsberegninger).

$$\text{Omregningsformel: } \frac{\text{Radiantal}}{\pi} = \frac{\text{Gradtal}}{180}$$

> restart

## Radianer, grader

> with(Gym)

[*ChiKvadratGOFtest, ChiKvadratUtest, Cos, ExpReg, LinReg, LogistReg, PolyReg, PowReg, Sin, Tan, antalstabel, arealP, arealT, binomialTest, boksplot, cart2pol, det, dotP, ev, forventet, fraktil, frekvens, frekvensTabel, gennemsnit, grupperData, hat, hyppighed, invCos, invSin, invTan, kumuleretFrekvens, kvartiler, len, median, middel, pindediagramBIN, plotHistogram, plotPindediagram, plotSumkurve, plotTrappekurve, pol2cart, proj, spredning, sumkurve, trappekurve, trappekurveBIN, typeinterval, typetal, varians, vinkel, visMatrix*]

>  $\sin(0) = \text{Sin}(0)$

$$0 = 0. \quad (1.2)$$

>  $\sin(\pi) = \text{Sin}(180)$

$$0 = 0. \quad (1.3)$$

>  $\sin\left(\frac{\pi}{2}\right) = \text{Sin}(90)$

$$1 = 1. \quad (1.4)$$

>  $\cos(0) = \text{Cos}(0)$

$$1 = 1. \quad (1.5)$$

>  $\cos(\pi) = \text{Cos}(180)$

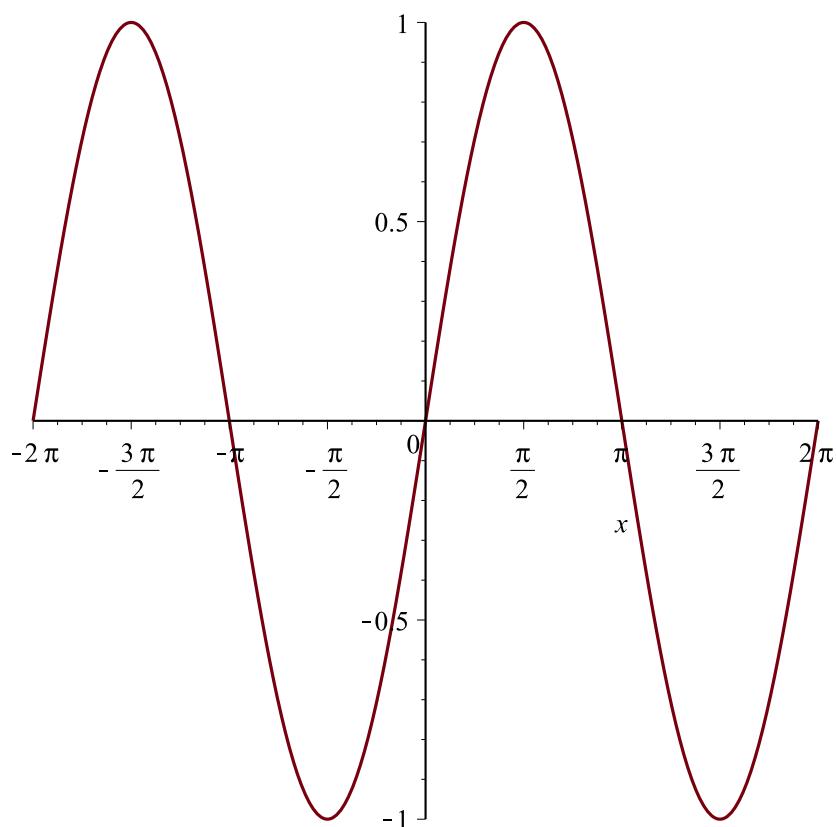
$$-1 = -1. \quad (1.6)$$

>  $\cos\left(\frac{\pi}{2}\right) = \text{Cos}(90)$

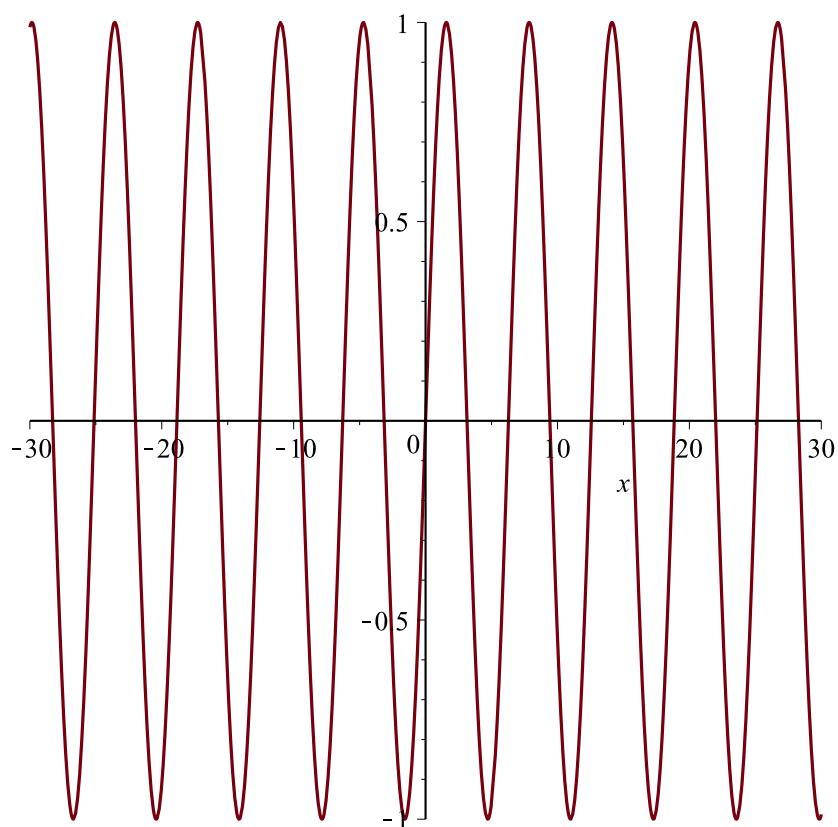
$$0 = 0. \quad (1.7)$$

## Grafer

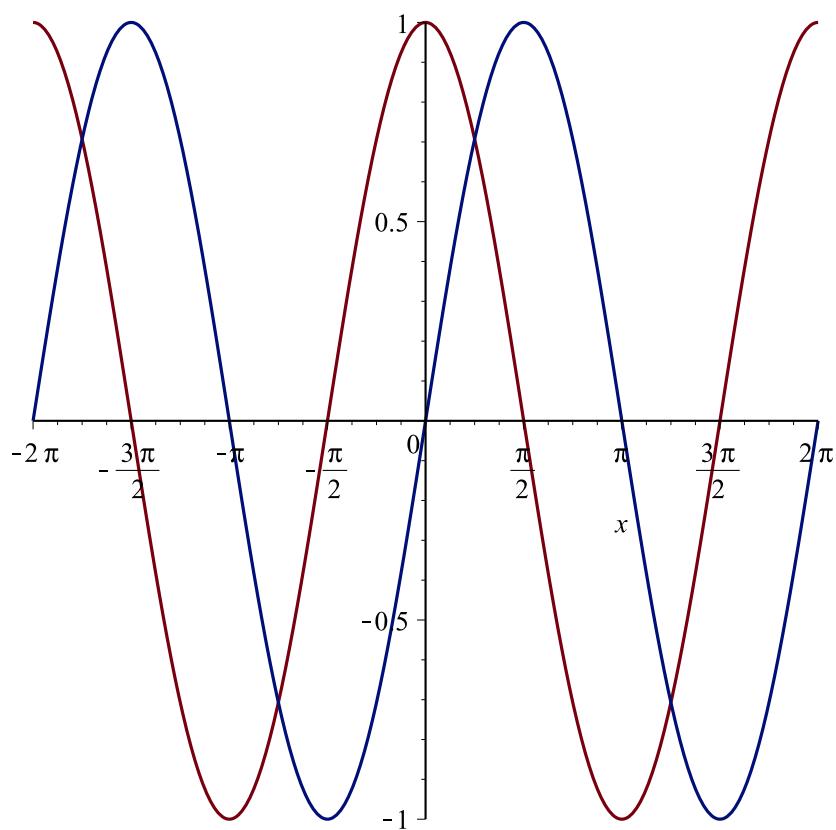
>  $\text{plot}(\sin(x))$



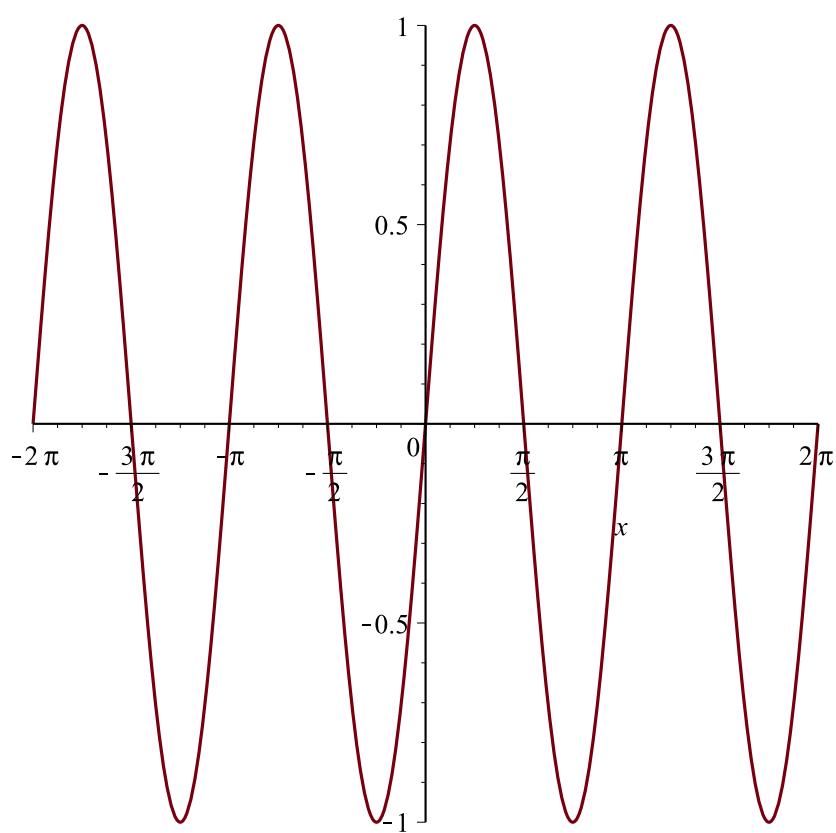
```
> plot(sin(x), x=-30..30)
```



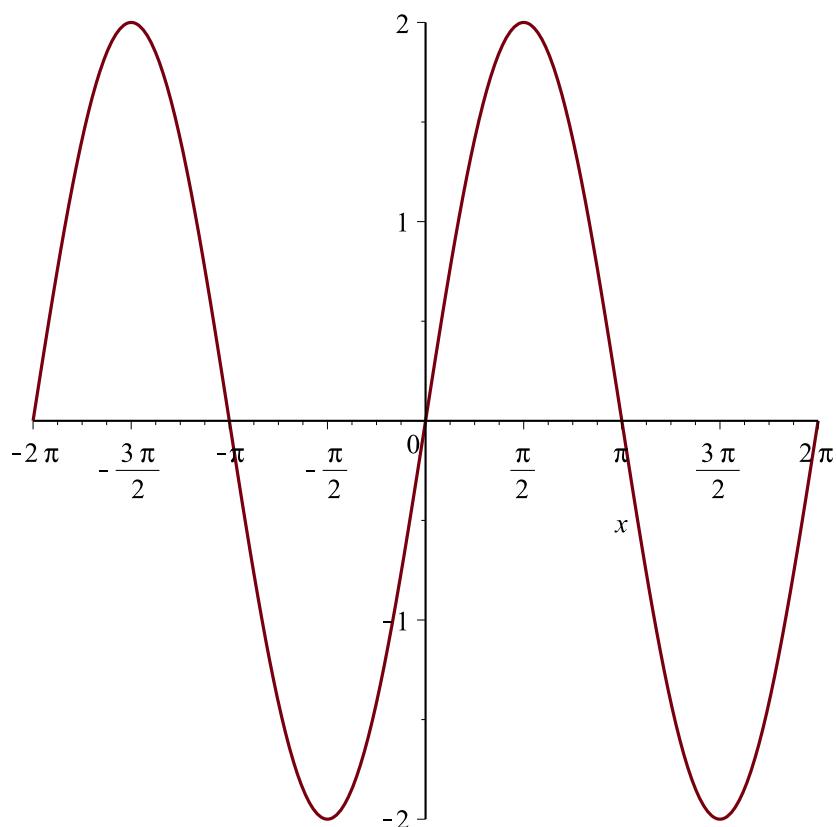
```
> plot( {sin(x), cos(x)} )
```



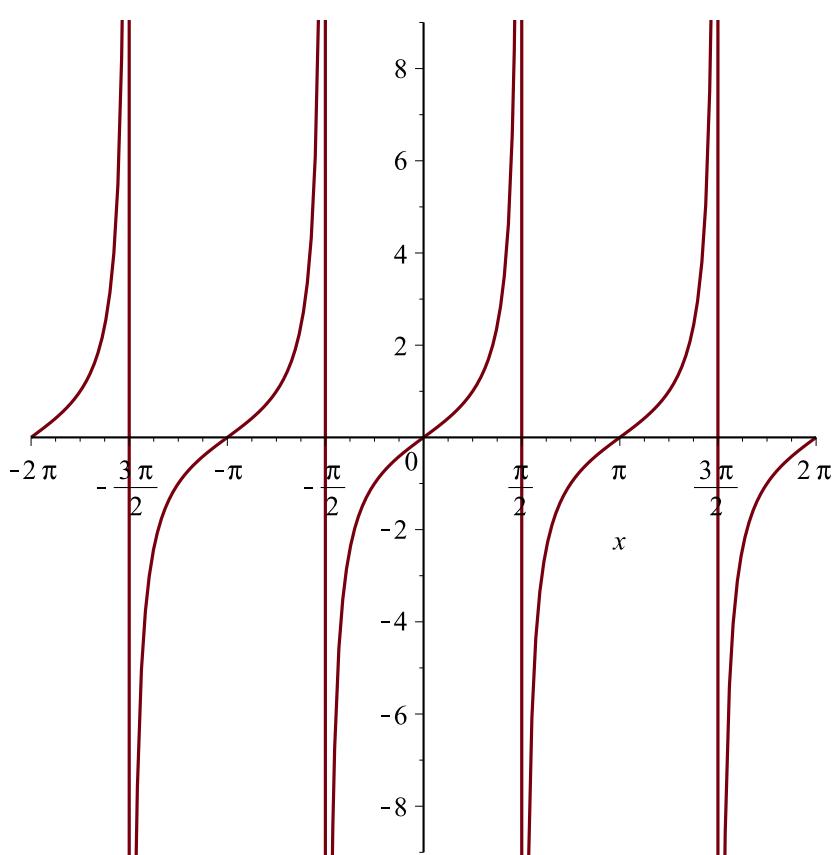
```
> plot(sin(2·x))
```



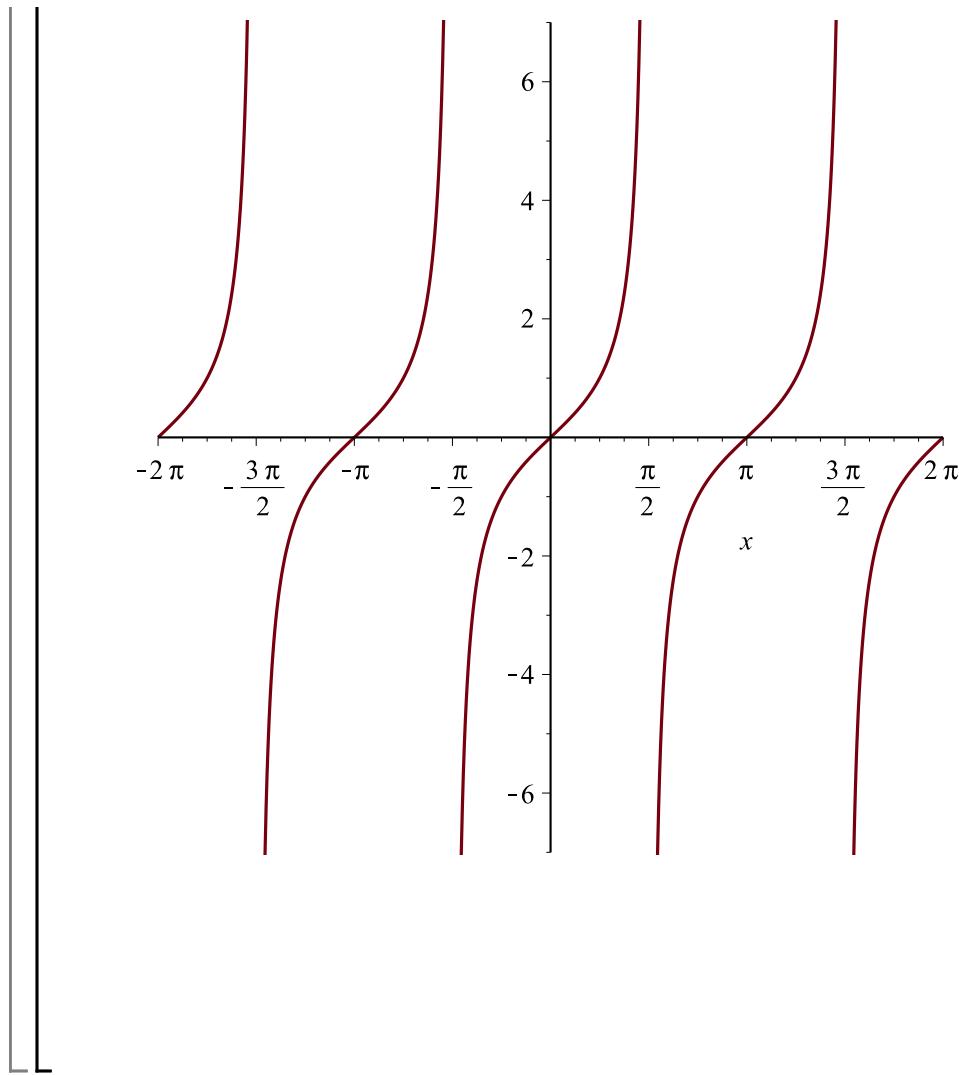
```
> plot(2·sin(x))
```



```
> plot(tan(x))
```



```
> plot(tan(x), discontinuous = true)
```



## Differentiation

$$\begin{aligned} &> (\sin(x))' \\ &= \cos(x) \end{aligned} \tag{3.1}$$

$$\begin{aligned} &> \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} \\ &= \cos(x) \end{aligned} \tag{3.2}$$

$$\begin{aligned} &> (\cos(x))' \\ &= -\sin(x) \end{aligned} \tag{3.3}$$

$$\begin{aligned} &> \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos(x)}{h} \\ &= -\sin(x) \end{aligned} \tag{3.4}$$

$$\begin{aligned} &> (\tan(x))' \\ &= 1 + \tan(x)^2 \end{aligned} \tag{3.5}$$

$$\begin{aligned} &> \lim_{h \rightarrow 0} \frac{\tan(x+h) - \tan(x)}{h} \\ &= 1 + \tan(x)^2 \end{aligned} \tag{3.6}$$

Håndcheck af  $\tan(x)$  vha. brøkreglen:

$$(\tan(x))' = \left( \frac{\sin(x)}{\cos(x)} \right)' = \frac{\cos(x) \cdot \cos(x) - \sin(x) \cdot (-\sin(x))}{(\cos(x))^2} = \frac{(\cos(x))^2 + (\sin(x))^2}{(\cos(x))^2} = 1$$

$$+ \left( \frac{\sin(x)}{\cos(x)} \right)^2 = \underline{\underline{1 + (\tan(x))^2}}$$

## Integration

$$> \int \sin(x) \, dx \quad -\cos(x) \quad (4.1)$$

$$> \int \cos(x) \, dx \quad \sin(x) \quad (4.2)$$

$$> \int \tan(x) \, dx \quad -\ln(\cos(x)) \quad (4.3)$$

Håndcheck (integrationsprøven) vha. sammensat differentiation:

$$(-\ln(\cos(x)))' = -\frac{1}{\cos(x)} \cdot (\cos(x))' = -\frac{1}{\cos(x)} \cdot (-\sin(x)) = \frac{\sin(x)}{\cos(x)} = \underline{\underline{\tan(x)}}$$

## Ligningsløsning

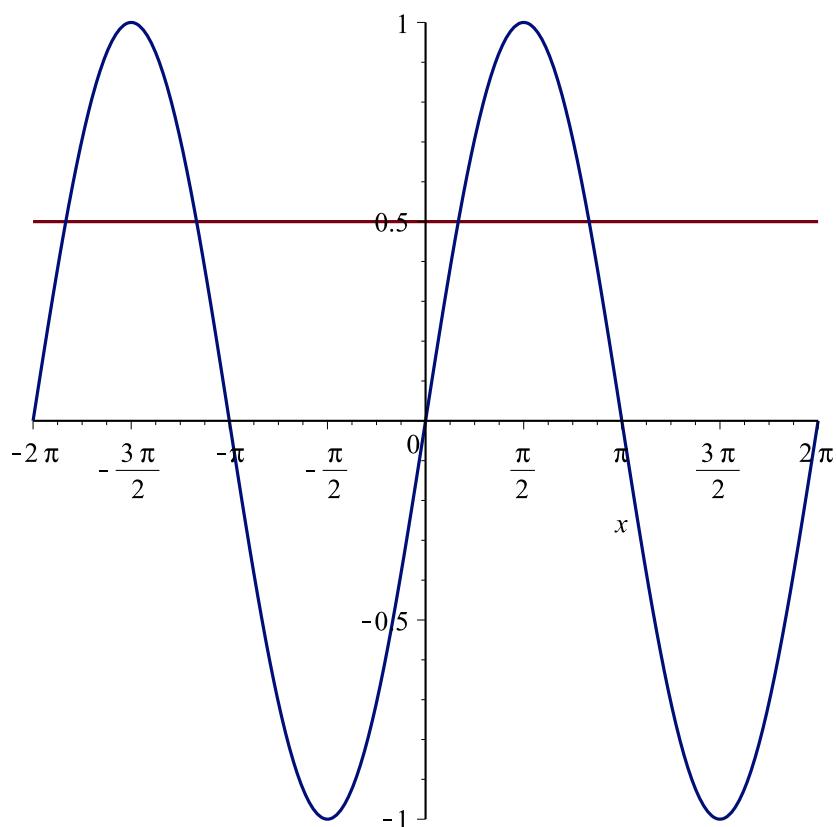
$$> solve\left(\sin(x) = \frac{1}{2}, x\right) \quad \frac{1}{6}\pi \quad (5.1)$$

$$> solve\left(\sin(x) = \frac{1}{2}, x, AllSolutions\right) \quad \frac{1}{6}\pi + \frac{2}{3}\pi_B I \sim + 2\pi_Z 2 \quad (5.2)$$

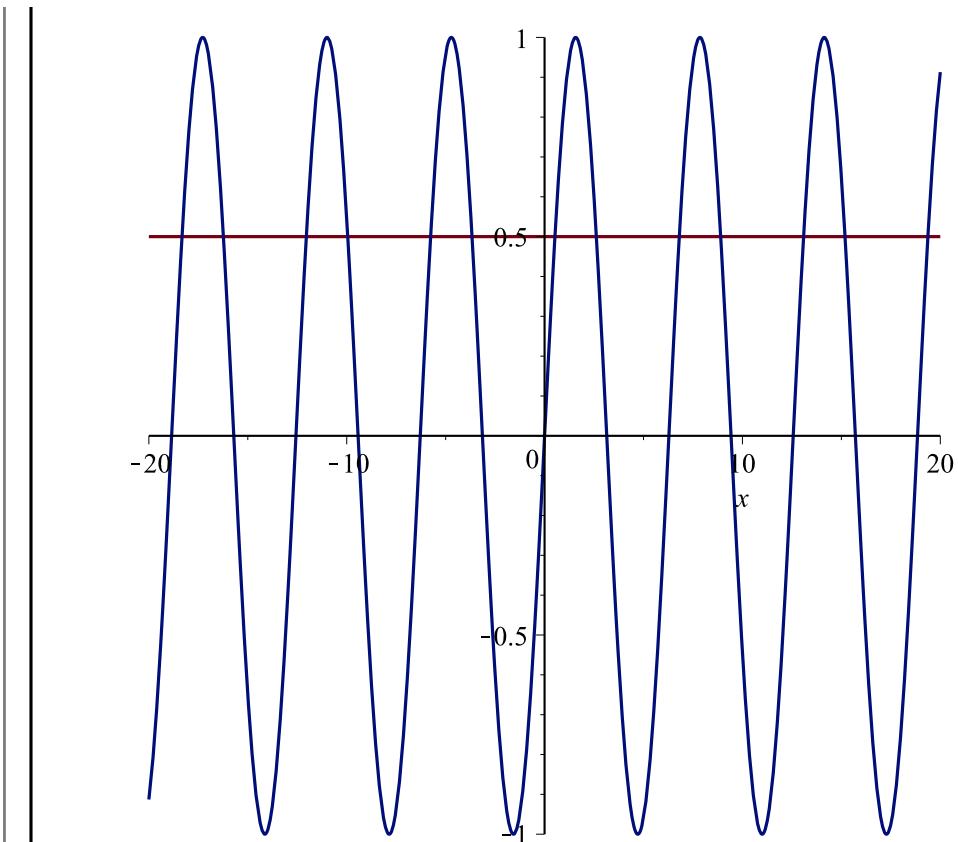
NB:  $_B$  betyder, at faktoren kan være 0 eller 1, dvs. binær.  $_Z$  betyder, at faktoren kan være et helt tal, dvs.  $\in \mathbb{Z}$ .

På den måde får man opskrevet alle løsninger. Det binære giver de 2 løsninger fra 0 til  $2\pi$ , og heltalsfaktoren angiver at man kan køre flere gange rundt i enhedscirklen.

$$> plot\left(\left\{\sin(x), \frac{1}{2}\right\}\right)$$



➤  $\text{plot}\left(\{\sin(x), \frac{1}{2}\}, x = -20..20\right)$



$$> \sin\left(\frac{\pi}{6}\right) \quad \frac{1}{2} \quad (5.3)$$

$$> \sin\left(\pi - \frac{\pi}{6}\right) \quad \frac{1}{2} \quad (5.4)$$

$$> \sin\left(\frac{\pi}{6} + \frac{2}{3} \cdot \pi\right) \quad \frac{1}{2} \quad (5.5)$$

$$> \sin\left(2 \cdot \pi + \frac{\pi}{6}\right) \quad \frac{1}{2} \quad (5.6)$$