

# Model: radioaktivt henfald

Variable og formler:

$N$  = antal endnu ikke henfaldne kerner (enhed:  $stk$ )

$A$  = aktiviteten = antal henfald pr. tidsenhed (enhed:  $Becquerel = Bq = stk. pr. sekund = s^{-1}$ )

$k$  = sønderdelingskonstanten (enhed:  $s^{-1}$ )

$T_{\frac{1}{2}}$  = halveringstiden (enhed:  $s$ )

$$A = -\frac{dN}{dt}$$

(formlen følger direkte af definitionerne af  $A$  og  $N$ )

$$A = k \cdot N$$

(formlen siger, at aktiviteten  $A$  er proportional med antal ikke henfaldne kerner  $N$ , hvilket er meget fornuftig antagelse: dobbelt så stor radioaktiv klump giver dobbelt så stor stråling)

Sammenstilles de 2 formler, får man en differentialligning for  $N$ :  $-\frac{dN}{dt} = k \cdot N \Leftrightarrow \frac{dN}{dt} = -k \cdot N$

Det er den 1. standardtype, som vi kender løsningen til:  $y' = k \cdot y \Leftrightarrow y = c \cdot e^{kx}$

Dvs.  $N(t) = N_0 \cdot e^{-k \cdot t}$ , hvor  $N_0$  er værdien af  $N$  til start, dvs.  $t = 0$ .

Formlen for aktiviteten er så:  $A(t) = k \cdot N(t) = k \cdot N_0 \cdot e^{-k \cdot t} \Rightarrow A(t) = A_0 \cdot e^{-k \cdot t}$

**Konklusion:** både  $N(t)$  og  $A(t)$  aftager eksponentielt med samme hastighed.

**Formel for halveringstiden kan beregnes:**

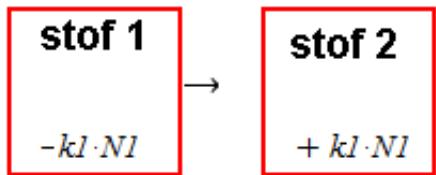
$$e^{-k \cdot T_{\frac{1}{2}}} = \frac{1}{2} \Leftrightarrow -k \cdot T_{\frac{1}{2}} = \ln\left(\frac{1}{2}\right) \Leftrightarrow T_{\frac{1}{2}} = \frac{\ln\left(\frac{1}{2}\right)}{-k} = \frac{\ln(1) - \ln(2)}{-k} = -\frac{\ln(2)}{-k} \Leftrightarrow T_{\frac{1}{2}} = \frac{\ln(2)}{k}$$

$$\Leftrightarrow k = \frac{\ln(2)}{T_{\frac{1}{2}}}$$

## Simpelt henfald

Model:

Stof1 henfalder til stof2, som er stabilt.



## Kun stof1

Differentiallignings-model:

Differentialligninger:

$$\frac{dN1}{dt} = -k1 \cdot N1$$

Betingelser:

$$N1(0) = N10$$

> restart

$$> dsolve(\{N1'(t) = -k1 \cdot N1(t), N1(0) = N10\}, N1(t))$$

$$N1(t) = N10 e^{-k1 t}$$

(1.1.1)

$$> N1 := unapply(rhs((1.1.1)), t) : N1(t)$$

(1.1.2)

$$> N10 := 10^{20}$$

$$N10 := 10000000000000000000000000$$

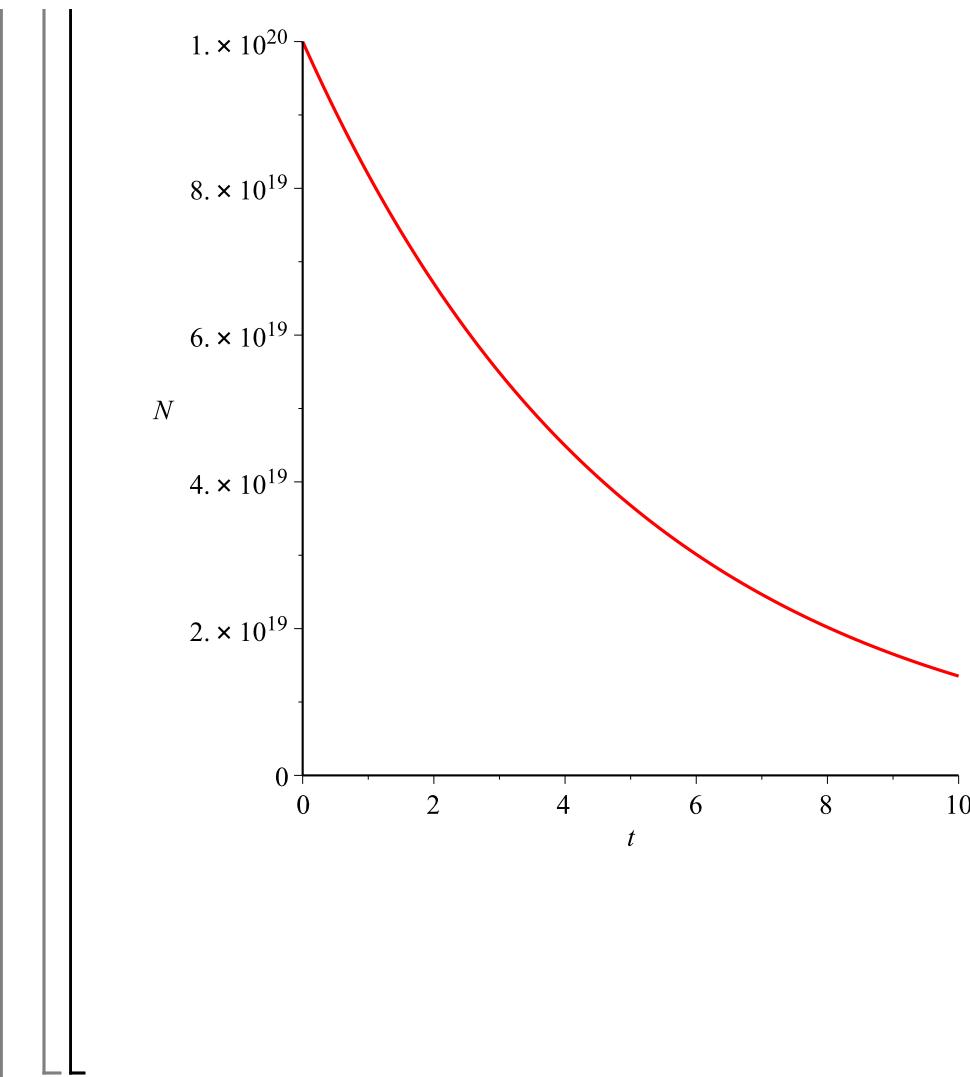
(1.1.3)

$$> k1 := 0.2$$

$$k1 := 0.2$$

(1.1.4)

$$> plot(N1(t), t=0..10, y=0..10^{20}, color=red, labels=[t, N])$$



## Kun med stof1 til start

Differentialalignings-model:

Differentialaligninger:

$$\frac{dN1}{dt} = -k1 \cdot N1$$

$$\frac{dN2}{dt} = -\frac{dN1}{dt} = k1 \cdot N1$$

Betingelser:

$$N1(0) = N10$$

$$N2(0) = 0$$

> restart

$$> dsolve(\{N1'(t) = -k1 \cdot N1(t), N1(0) = N10\}, N1(t)) \\ N1(t) = N10 e^{-k1 t} \quad (1.2.1)$$

$$> dsolve(\{N1'(t) = -k1 \cdot N1(t), N1(0) = N10, N2'(t) = k1 \cdot N1(t), N2(0) = 0\}, \{N1(t), N2(t)\}) \\ \{N1(t) = N10 e^{-k1 t}, N2(t) = -N10 e^{-k1 t} + N10\} \quad (1.2.2)$$

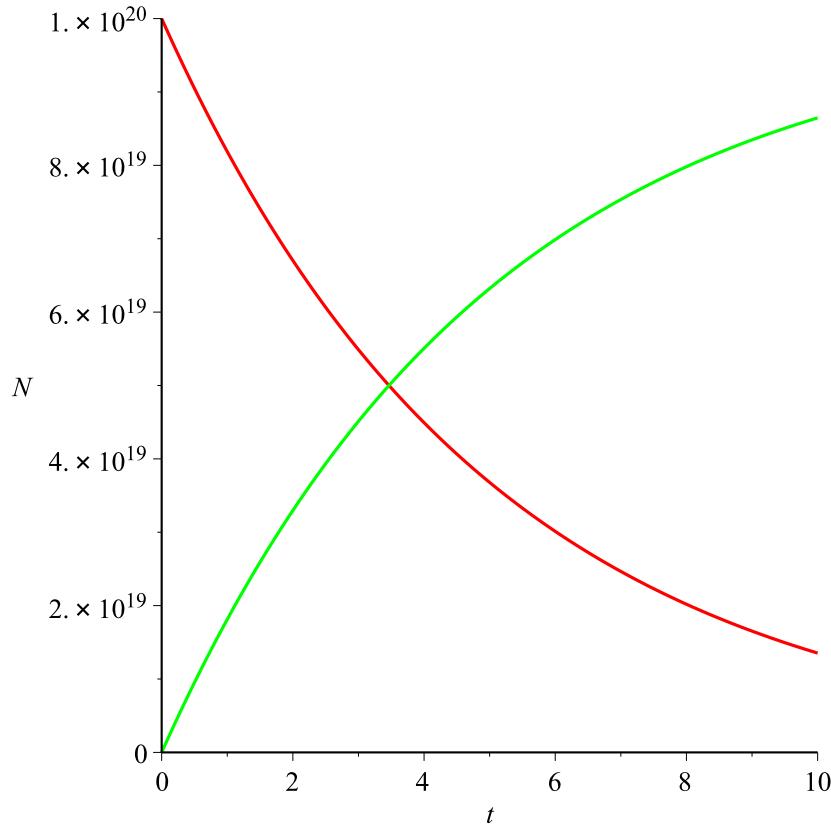
$$> N1 := \text{unapply}(\text{rhs}((\mathbf{1.2.2})[1]), t) : N1(t) \\ N10 e^{-k1 t} \quad (1.2.3)$$

$$> N2 := \text{unapply}(\text{rhs}((\mathbf{1.2.2})[2]), t) : N2(t); \text{simplify}(N2(t)) \\ -N10 e^{-k1 t} + N10 \\ -N10 (e^{-k1 t} - 1) \quad (1.2.4)$$

$$> N10 := 10^{20} \\ N10 := 10000000000000000000000000000 \quad (1.2.5)$$

$$> k1 := 0.2 \\ k1 := 0.2 \quad (1.2.6)$$

```
> plot( { N1(t), N2(t) }, t=0..10, y=0..1020, color=[red, green], labels=[t, N] )
```



## Både stof1 og stof2 til start

Differentialalignings-model:

Differentialaligninger:

$$\frac{dN1}{dt} = -k1 \cdot N1$$

$$\frac{dN2}{dt} = -\frac{dN1}{dt} = k1 \cdot N1$$

## Betingelser:

$$NI(0) = NI0$$

$$N2(0) = N20$$

=> restart

```
> dsolve( {N1'(t) =-k1·N1(t), N1(0) =N10},N1(t))
```

$$N1(t) = N10 e^{-k1 t} \quad (1.3.1)$$

```
> dsolve({NI'(t) =-k1·NI(t), NI(0) =NI0, N2'(t) =k1·NI(t), N2(0) =N20}, {NI(t), N2(t)})
```

$$\{N1(t) = N10 e^{-k1t}, N2(t) = -N10 e^{-k1t} + N10 + N20\} \quad (1.3.2)$$

**>**  $N1 := \text{unapply}(\text{rhs}((1.3.2)[1]), t) : NI(t)$

$$N10 e^{-k1 t} \quad (1.3.3)$$

**>**  $N2 := \text{unapply}(\text{rhs}((1.3.2)[2]), t) : N2(t)$

$$-N10 e^{-k1t} + N10 + N20 \quad (1.3.4)$$

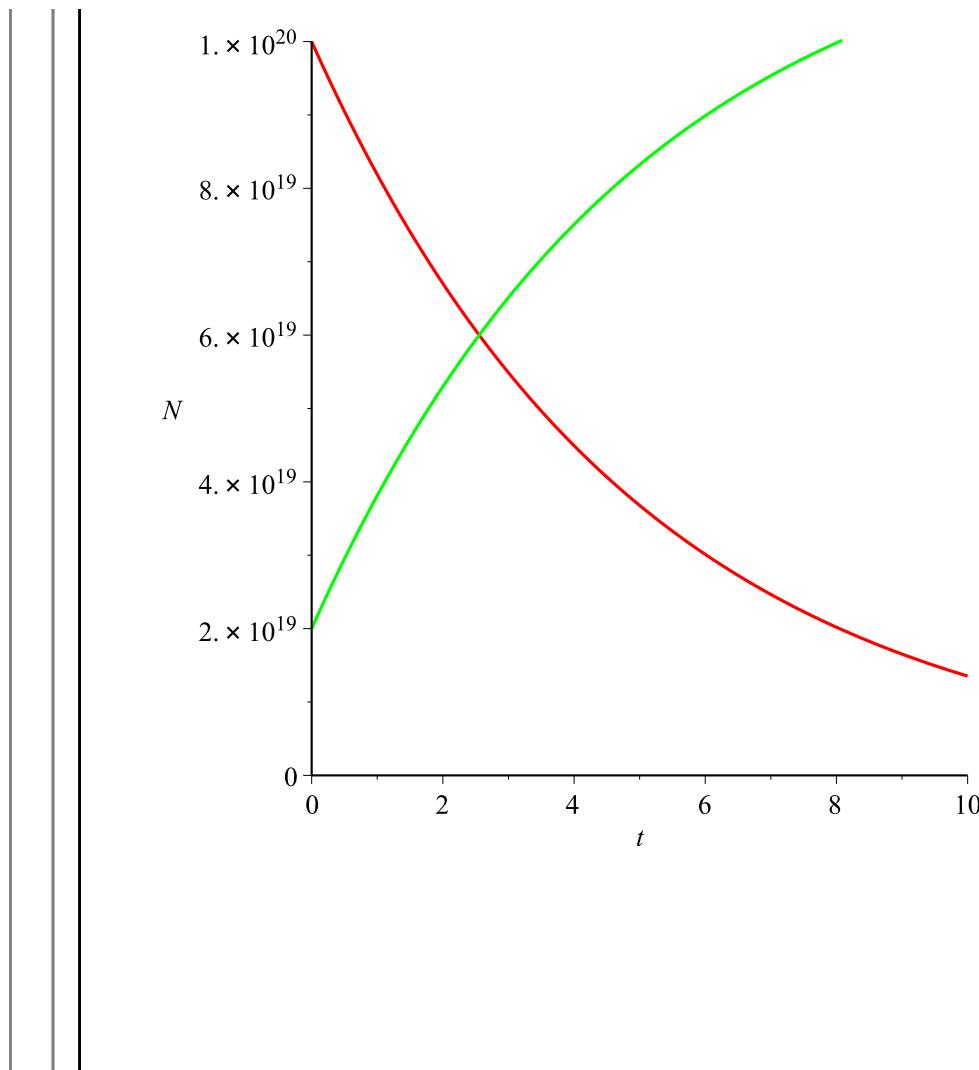
=  
N10 := 10

$N10 := 10000000000000000000000$

• 7.1

$$2 \cdot 10^{20} \text{ eV}^2 = [m^2 L_{\text{univ}}] \cdot L_{\text{univ}} = [t \cdot N]$$

```
- plot( { M(t), Nz(t) }, t=0..10, y=0..10^3, color=[red,green], labels=[t,N] )
```

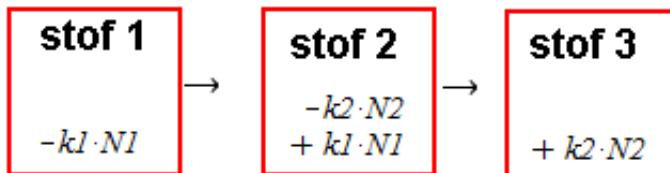


## ▼ Henfaldskæde

Model:

Stof1 er radioaktivt, og henfalder til stof2.

Stof2 er selv radioaktivt, og henfalder til stof3, som er stabilt.



## ▼ Kun stof1 til start

model

Differentialalignings-model:

Differentialaligninger:

$$\begin{aligned}\frac{dN1}{dt} &= -k1 \cdot N1 \\ \frac{dN2}{dt} &= k1 \cdot N1 - k2 \cdot N2 \\ \frac{dN3}{dt} &= k2 \cdot N2\end{aligned}$$

Betingelser:

$$N1(0) = N10$$

$$N2(0) = 0$$

$$N3(0) = 0$$

## NB: Differentialigningssystemet kan løses successivt!

Dvs. man kan løse differentialligning nr. 1 først mht.  $N1(t)$ , og indsætte den løsning i differentialligning nr. 2, og så løse nr. 2 mht.  $N2(t)$ , og igen indsætte den løsning i differentialligning nr. 3, og så endelig løse den mht.  $N3(t)$ .

Det er absolut ikke almindeligt, at differentialligningssystemer kan løses successivt.

> restart

$$> dsolve(\{N1'(t) = -k1 \cdot N1(t), N1(0) = N10\}, N1(t))$$

$$N1(t) = N10 e^{-k1 t} \quad (2.1.1.1)$$

$$> dsolve(\{N1'(t) = -k1 \cdot N1(t), N1(0) = N10, N2'(t) = k1 \cdot N1(t) - k2 \cdot N2(t), N2(0) = 0\}, \{N1(t), N2(t)\})$$

$$\left\{ N1(t) = N10 e^{-k1 t}, N2(t) = -\frac{(-k1 + k2) k1 N10 e^{-k2 t}}{(k1 - k2)^2} - \frac{k1 e^{-k1 t} N10}{k1 - k2} \right\} \quad (2.1.1.2)$$

$$> dsolve(\{N1'(t) = -k1 \cdot N1(t), N1(0) = N10, N2'(t) = k1 \cdot N1(t) - k2 \cdot N2(t), N2(0) = 0, N3'(t) = k2 \cdot N2(t), N3(0) = 0\}, \{N1(t), N2(t), N3(t)\})$$

$$\left\{ N1(t) = N10 e^{-k1 t}, N2(t) = -\frac{(-k1 + k2) k1 N10 e^{-k2 t}}{(k1 - k2)^2} - \frac{k1 e^{-k1 t} N10}{k1 - k2}, N3(t) = \right. \\ \left. -\frac{-e^{-k1 t} N10 k2 + \frac{e^{-k2 t} k1^2 N10}{k1 - k2} - \frac{e^{-k2 t} k1 N10 k2}{k1 - k2} - k1 N10 + N10 k2}{k1 - k2} \right\} \quad (2.1.1.3)$$

$$> N1 := unapply(rhs((2.1.1.3)[1]), t) : N1(t) \\ N10 e^{-k1 t} \quad (2.1.1.4)$$

$$> N2 := unapply(rhs((2.1.1.3)[2]), t) : N2(t); simplify(N2(t))$$

$$-\frac{(-k1 + k2) k1 N10 e^{-k2 t}}{(k1 - k2)^2} - \frac{k1 e^{-k1 t} N10}{k1 - k2}$$

$$\frac{k1 N10 (e^{-k2 t} - e^{-k1 t})}{k1 - k2} \quad (2.1.1.5)$$

$$> N3 := unapply(rhs((2.1.1.3)[3]), t) : N3(t); simplify(N3(t))$$

$$\begin{aligned}
 & -\frac{-e^{-k1t} N10 k2 + \frac{e^{-k2t} k1^2 N10}{k1 - k2} - \frac{e^{-k2t} k1 N10 k2}{k1 - k2} - k1 N10 + N10 k2}{k1 - k2} \\
 & - \frac{N10 (-k1 + k1 e^{-k2t} - e^{-k1t} k2 + k2)}{k1 - k2}
 \end{aligned} \tag{2.1.1.6}$$

>  $N10 := 10^{20}$

$$N10 := 1000000000000000000000000000000 \tag{2.1.1.7}$$

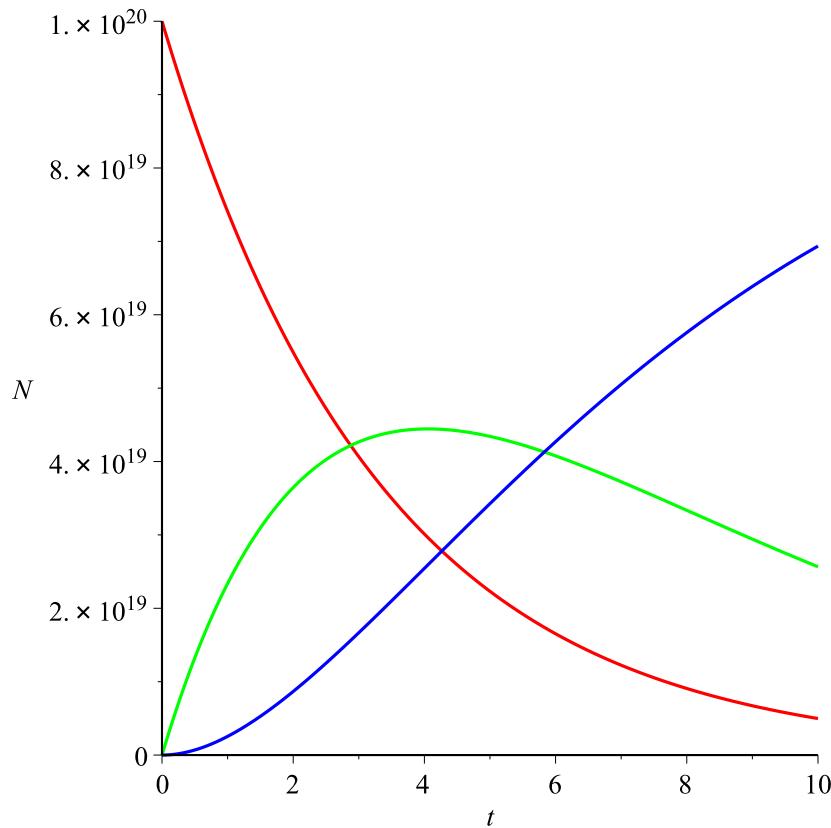
>  $k1 := 0.3; k2 := 0.2; k3 := 0.1$

$$k1 := 0.3$$

$$k2 := 0.2$$

$$k3 := 0.1 \tag{2.1.1.8}$$

>  $\text{plot}(\{N1(t), N2(t), N3(t)\}, t=0..10, y=0..10^{20}, \text{color}=[\text{red}, \text{green}, \text{blue}], \text{labels} = [t, N])$



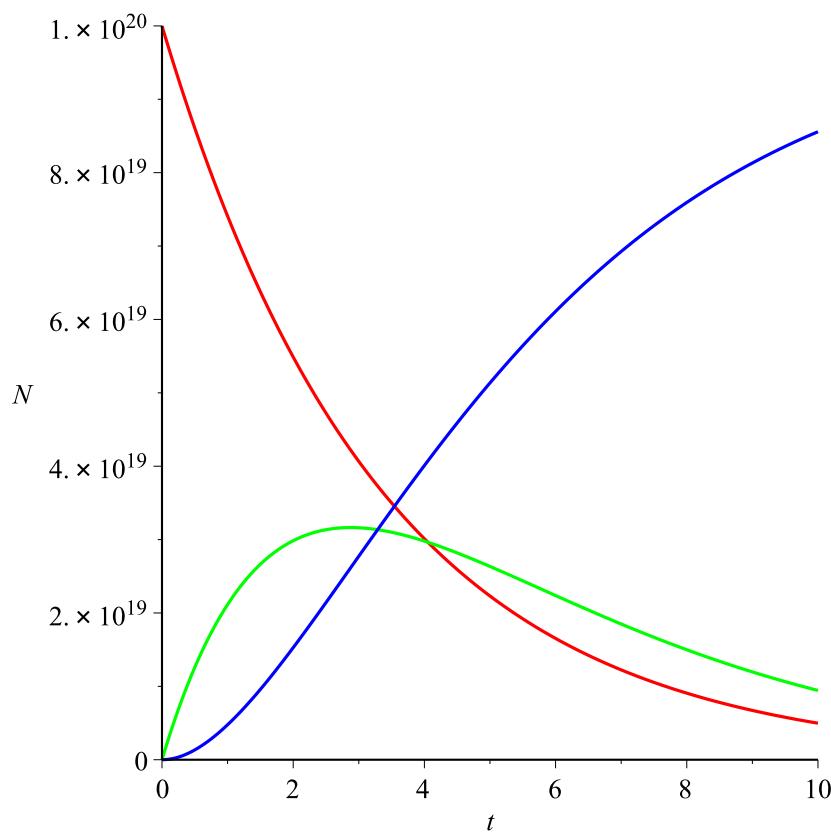
>  $k1 := 0.3; k2 := 0.4; k3 := 0.1$

$$k1 := 0.3$$

$$k2 := 0.4$$

$$k3 := 0.1 \tag{2.1.1.9}$$

```
> plot( { N1(t), N2(t), N3(t) }, t = 0 .. 10, y = 0 .. 1020, color = [red, green, blue], labels = [t, N] )
```



### Successiv løsning med håndkraft

#### Trin 1:

$$\frac{dN1}{dt} = -k1 \cdot N1$$

Er af **1. standardtype**:  $y' = k \cdot y \Leftrightarrow y = c \cdot e^{kx}$ , dvs. løsningen er  $\underline{\underline{N1(t) = c1 \cdot e^{-k1 \cdot t}}}$

Udtrykket indsættes i næste trin.

#### Trin 2:

$$\frac{dN2}{dt} = k1 \cdot N1 - k2 \cdot N2$$

lyder så:  $\frac{dN2}{dt} = k1 \cdot c1 \cdot e^{-k1 \cdot t} - k2 \cdot N2$

Denne er en **lineær type**:  $y' = -a(x) \cdot y + b(x) \Leftrightarrow y = e^{-\int a(x) dx} \cdot \int b(x) \cdot e^{\int a(x) dx} dx + c \cdot e^{-\int a(x) dx}$  hvor  $A(x)$  er en stamfunktion til  $a(x)$ .

Her er  $a(t) = k2$  og  $b(t) = k1 \cdot c1 \cdot e^{-k1 \cdot t}$  (husk at variablen  $x$  hedder  $t$  her).

$$\text{Så bliver } A(t) = \int a(t) dt = \int k2 dt = k2 \cdot t$$

og

$$\int b(t) \cdot e^{A(t)} dt = \int k1 \cdot c1 \cdot e^{-k1 \cdot t} \cdot e^{k2 \cdot t} dt = k1 \cdot c1 \cdot \int e^{(k2 - k1) \cdot t} dt = \frac{k1 \cdot c1}{k2 - k1} \cdot e^{(k2 - k1) \cdot t}$$

Hermed kan man finde  $N2(t)$ :

$$N2(t) = e^{-k2 \cdot t} \left( \frac{k1 \cdot c1}{k2 - k1} \cdot e^{(k2 - k1) \cdot t} \right) + c2 \cdot e^{-k2 \cdot t} = \frac{k1 \cdot c1}{k2 - k1} \cdot e^{-k1 \cdot t} + c2 \cdot e^{-k2 \cdot t} = \underline{\underline{c3 \cdot e^{-k1 \cdot t} + c2 \cdot e^{-k2 \cdot t}}}$$

$$\text{Så: } \underline{\underline{N2(t) = c3 \cdot e^{-k1 \cdot t} + c2 \cdot e^{-k2 \cdot t}}}$$

Udtrykket indsættes i næste trin.

### Trin 3:

$$\frac{dN3}{dt} = k2 \cdot N2$$

$$\text{lyder så: } \frac{dN3}{dt} = k2 \cdot (c3 \cdot e^{-k1 \cdot t} + c2 \cdot e^{-k2 \cdot t})$$

Denne differentialligning løses simpelt ved integration!

$$N3(t) = \int k2 \cdot (c3 \cdot e^{-k1 \cdot t} + c2 \cdot e^{-k2 \cdot t}) dt = \int (k2 \cdot c3 \cdot e^{-k1 \cdot t} + k2 \cdot c2 \cdot e^{-k2 \cdot t}) dt = \frac{k2 \cdot c3}{-k1} \cdot e^{-k1 \cdot t} - c2 \cdot e^{-k2 \cdot t} + c4$$

$$\text{Så: } \underline{\underline{N3(t) = c5 \cdot e^{-k1 \cdot t} - c2 \cdot e^{-k2 \cdot t} + c4}}$$

### Samlet:

$$N1(t) = c1 \cdot e^{-k1 \cdot t}$$

$$N2(t) = c3 \cdot e^{-k1 \cdot t} + c2 \cdot e^{-k2 \cdot t}$$

$$N3(t) = c5 \cdot e^{-k1 \cdot t} - c2 \cdot e^{-k2 \cdot t} + c4$$

Startbettingerne lyder:

$$N1(0) = N10$$

$$N2(0) = 0$$

$$N3(0) = 0$$

Det betyder, at  $c1 = N10$

$$\text{Og } c3 \text{ var givet ved } c3 = \frac{k1 \cdot c1}{k2 - k1} \Leftrightarrow c3 = \frac{k1}{k2 - k1} \cdot N10$$

$$N2(0) = 0 \text{ betyder så at } c3 \cdot e^{-k1 \cdot 0} + c2 \cdot e^{-k2 \cdot 0} = 0 \Leftrightarrow c3 + c2 = 0 \Leftrightarrow c2 = -c3$$

$$\text{Derfor er } c2 = -\frac{k1}{k2 - k1} \cdot N10$$

$$\text{Ovenfor fandt man, at } c5 = \frac{k2 \cdot c3}{-k1} \text{ . Dvs. } c5 = \frac{k2}{-k1} \cdot \frac{k1}{k2 - k1} \cdot N10 = -\frac{k2}{k2 - k1} \cdot N10$$

$$N3(0) = 0 \text{ betyder så at } c5 \cdot e^{-k1 \cdot 0} - c2 \cdot e^{-k2 \cdot 0} + c4 = 0 \Leftrightarrow c5 - c2 + c4 = 0 \Leftrightarrow$$

$$c4 = -c5 + c2 = -\left(-\frac{k2}{k2 - k1} \cdot N10\right) + \left(-\frac{k1}{k2 - k1} \cdot N10\right) \Leftrightarrow c4 = \frac{k2}{k2 - k1} \cdot N10$$

$$-\frac{k1}{k2 - k1} \cdot N10 = \frac{k2 - k1}{k2 - k1} \cdot N10 = N10$$

**Løsningerne:**

$$N1(t) = \underline{\underline{N10 \cdot e^{-k1 \cdot t}}}$$

$$N2(t) = \frac{k1}{k2 - k1} \cdot N10 \cdot e^{-k1 \cdot t} - \frac{k1}{k2 - k1} \cdot N10 \cdot e^{-k2 \cdot t} = \underline{\underline{\frac{k1}{k2 - k1} \cdot N10 \cdot (e^{-k1 \cdot t} - e^{-k2 \cdot t})}}$$

$$N3(t) = -\frac{k2}{k2 - k1} \cdot N10 \cdot e^{-k1 \cdot t} - \frac{k1}{k2 - k1} \cdot N10 \cdot e^{-k2 \cdot t} + N10 = \\ \underline{\underline{N10 \cdot \left( \frac{k1}{k2 - k1} \cdot e^{-k2 \cdot t} - \frac{k2}{k2 - k1} \cdot e^{-k1 \cdot t} + 1 \right)}}$$

**Successiv løsning i Maple**

&gt; restart

1. differentialligning løses:

$$> dsolve(\{N1'(t) = -k1 \cdot N1(t), N1(0) = N10\}, N1(t)) \\ N1(t) = N10 e^{-k1 t} \quad (2.1.3.1)$$

$$> N1 := unapply(rhs((2.1.3.1)), t) : N1(t) := N1(t) \\ N1(t) = N10 e^{-k1 t} \quad (2.1.3.2)$$

Løsningen indsættes i 2. differentialligning:

$$> N2'(t) = k1 \cdot N1(t) - k2 \cdot N2(t) \\ D(N2)(t) = k1 N10 e^{-k1 t} - k2 N2(t) \quad (2.1.3.3)$$

2. differentialligning løses:

$$> dsolve(\{N2'(t) = k1 \cdot N1(t) - k2 \cdot N2(t), N2(0) = 0\}, \{N2(t)\}) \\ N2(t) = \left( -\frac{k1 N10 e^{-t(k1 - k2)}}{k1 - k2} + \frac{k1 N10}{k1 - k2} \right) e^{-k2 t} \quad (2.1.3.4)$$

$$> N2 := unapply(rhs((2.1.3.4)), t) : N2(t) := N2(t); N2(t) := simplify(N2(t)) \\ N2(t) = \left( -\frac{k1 N10 e^{-t(k1 - k2)}}{k1 - k2} + \frac{k1 N10}{k1 - k2} \right) e^{-k2 t} \\ N2(t) = -\frac{k1 N10 (e^{-t(k1 - k2)} - 1)}{k1 - k2} e^{-k2 t} \quad (2.1.3.5)$$

Løsningen indsættes i 3. differentialligning:

$$> N3'(t) = k2 \cdot N2(t) \\ D(N3)(t) = k2 \left( -\frac{k1 N10 e^{-t(k1 - k2)}}{k1 - k2} + \frac{k1 N10}{k1 - k2} \right) e^{-k2 t} \quad (2.1.3.6)$$

3. differentialligning løses:

(NB: kan løses ved integration)

$$> dsolve(\{N3'(t) = k2 \cdot N2(t), N3(0) = 0\}, \{N3(t)\})$$

$$N3(t) = \frac{k1 N10 k2 \left( \frac{e^{-k1 t}}{k1} - \frac{e^{-k2 t}}{k2} \right)}{k1 - k2} - \frac{k1 N10 k2 \left( \frac{1}{k1} - \frac{1}{k2} \right)}{k1 - k2} \quad (2.1.3.7)$$

$$> N3 := unapply(rhs((2.1.3.7)), t) :'N3(t)'=N3(t);'N3(t)'=simplify(N3(t))$$

$$N3(t) = \frac{k1 N10 k2 \left( \frac{e^{-k1 t}}{k1} - \frac{e^{-k2 t}}{k2} \right)}{k1 - k2} - \frac{k1 N10 k2 \left( \frac{1}{k1} - \frac{1}{k2} \right)}{k1 - k2}$$

$$N3(t) = - \frac{N10 (-e^{-k1 t} k2 + e^{-k2 t} k1 - k1 + k2)}{k1 - k2} \quad (2.1.3.8)$$

## Alle 3 stoffer i spil fra starten

Differentiallignings-model:

Differentialligninger:

$$\frac{dN1}{dt} = -k1 \cdot N1$$

$$\frac{dN2}{dt} = k1 \cdot N1 - k2 \cdot N2$$

$$\frac{dN3}{dt} = k2 \cdot N2$$

Betingelser:

$$N1(0) = N10$$

$$N2(0) = N20$$

$$N3(0) = N30$$

> restart

$$> dsolve(\{N1'(t) = -k1 \cdot N1(t), N1(0) = N10\}, N1(t))$$

$$N1(t) = N10 e^{-k1 t} \quad (2.2.1)$$

$$> dsolve(\{N1'(t) = -k1 \cdot N1(t), N1(0) = N10, N2'(t) = k1 \cdot N1(t) - k2 \cdot N2(t), N2(0) = 0\}, \{N1(t), N2(t)\})$$

$$\left\{ N1(t) = N10 e^{-k1 t}, N2(t) = - \frac{(-k1 + k2) k1 N10 e^{-k2 t}}{(k1 - k2)^2} - \frac{k1 e^{-k1 t} N10}{k1 - k2} \right\} \quad (2.2.2)$$

$$> dsolve(\{N1'(t) = -k1 \cdot N1(t), N1(0) = N10, N2'(t) = k1 \cdot N1(t) - k2 \cdot N2(t), N2(0) = N20, N3'(t) = k2 \cdot N2(t), N3(0) = N30\}, \{N1(t), N2(t), N3(t)\})$$

$$\left\{ N1(t) = N10 e^{-k1 t}, N2(t) = - \frac{(-k1 + k2) (-k2 N20 + k1 N10 + N20 k1) e^{-k2 t}}{(k1 - k2)^2} \right. \\ \left. - \frac{k1 e^{-k1 t} N10}{k1 - k2}, N3(t) = - \frac{1}{k1 - k2} \left( -e^{-k1 t} N10 k2 \right. \right. \\ \left. \left. + \frac{e^{-k2 t} k1 (-k2 N20 + k1 N10 + N20 k1)}{k1 - k2} \right) \right. \\ \left. - \frac{e^{-k2 t} (-k2 N20 + k1 N10 + N20 k1) k2}{k1 - k2} - (N30 + N10 + N20) k1 + (N30 \right. \\ \left. \left. + N10 + N20\right) k2 \right\} \quad (2.2.3)$$

$$+ N10 + N20) \ k2 \Biggr\} \\ > N1 := unapply(rhs((2.2.3)[1]), t) : N1(t) \\ \qquad \qquad \qquad N10 e^{-k1t} \quad (2.2.4)$$

$$\begin{aligned} > N2 := \text{unapply}(\text{rhs}((2.2.3)[2]), t) : N2(t); \text{simplify}(N2(t)) \\ & -\frac{(-k1 + k2)(-k2 N20 + k1 N10 + N20 k1) e^{-k2 t}}{(k1 - k2)^2} - \frac{k1 e^{-k1 t} N10}{k1 - k2} \\ & \frac{-e^{-k2 t} k2 N20 + e^{-k2 t} k1 N10 + e^{-k2 t} N20 k1 - k1 e^{-k1 t} N10}{k1 - k2} \end{aligned} \quad (2.2.5)$$

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> N3 := unapply(rhs((2.2.3)[3]), t) : N3(t); simplify(N3(t))
- 
$$\frac{1}{k1 - k2} \left( -e^{-k1 t} N10 k2 + \frac{e^{-k2 t} k1 (-k2 N20 + k1 N10 + N20 k1)}{k1 - k2} \right.$$


$$- \frac{e^{-k2 t} (-k2 N20 + k1 N10 + N20 k1) k2}{k1 - k2} - (N30 + N10 + N20) k1 + (N30$$


$$+ N10 + N20) k2 \left. \right)$$

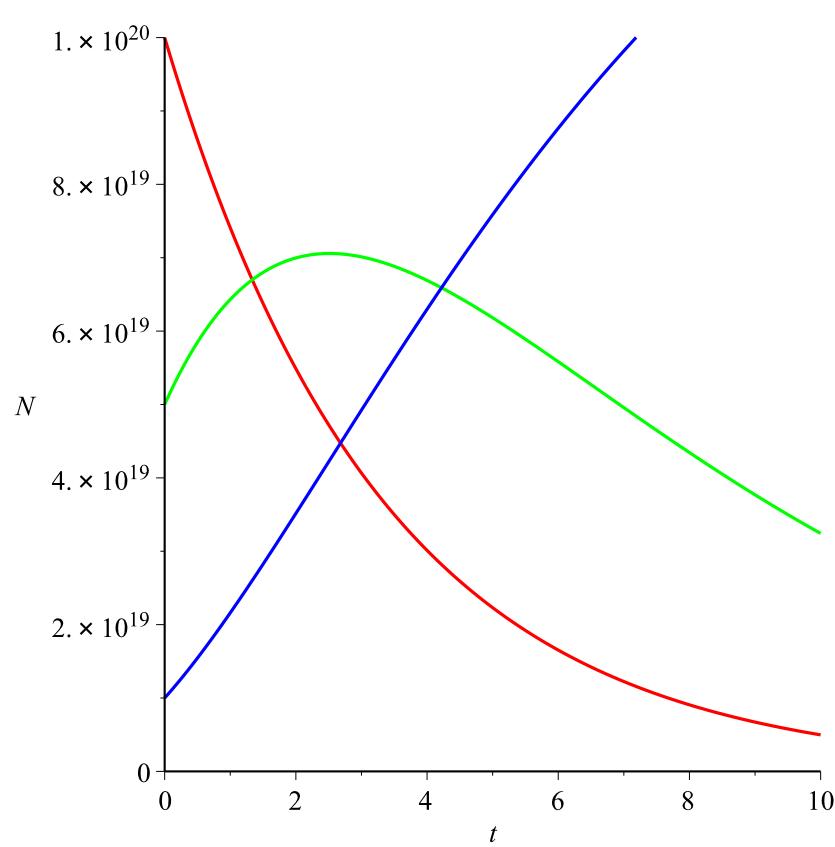
```

$$-\frac{1}{k_1 - k_2} (e^{-k_2 t} k_1 N_{10} - N_{30} k_1 + e^{-k_2 t} N_{20} k_1 - k_1 N_{10} - N_{20} k_1 - e^{-k_2 t} k_2 N_{20} + N_{30} k_2 + k_2 N_{20} - e^{-k_1 t} N_{10} k_2 + N_{10} k_2) \quad (2.2.6)$$

```
> N10 := 1020; N20 := 5 · 1019; N30 := 1019
      N10 := 1000000000000000000000000000
      N20 := 5000000000000000000000000000
      N30 := 1000000000000000000000000000
```

>  $k1 := 0.3; k2 := 0.2; k3 := 0.1$

```
> plot( { N1(t), N2(t), N3(t) }, t = 0 .. 10, y = 0 .. 1020, color = [red, green, blue], labels = [t, N])
```



>  $k1 := 0.3; k2 := 0.4; k3 := 0.1$

$k1 := 0.3$

$k2 := 0.4$

$k3 := 0.1$

(2.2.9)

>  $\text{plot}(\{N1(t), N2(t), N3(t)\}, t = 0 .. 10, y = 0 .. 10^{20}, \text{color} = [\text{red}, \text{green}, \text{blue}], \text{labels} = [t, N])$

