Skæring mellem linjer og planer i $\mathbb{R}^3$

Eksempel 143 side 169 - skæring mellem 2 linjer (ét punkt)

Givet parameterfremstilling for 2 linjer:

$$l: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} + t \cdot \begin{bmatrix} 5 \\ 1 \\ -4 \end{bmatrix} \text{ og } m: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ -2 \end{bmatrix} + s \cdot \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

NB: Vigtigt, at parametreren ikke hedder det samme i de 2 parameterfremstillinger! Kald den ene for *t*, den anden for *s*.

```
> restart
> l := <3,-2, 1> + t·<5, 1,-4>
```

$$l := \begin{bmatrix} 3+5t \\ -2+t \\ 1-4t \end{bmatrix} \quad \textbf{(1.1)}$$

```
> m := <3, 5,-2> + s·<-2, 1, 1>
```

$$m := \begin{bmatrix} 3-2s \\ 5+s \\ -2+s \end{bmatrix} \quad \textbf{(1.2)}$$

```
> solve({l[1]=m[1], l[2]=m[2], l[3]=m[3]}, {t, s})
```

$$\{s=-5, t=2\} \quad \textbf{(1.3)}$$

Altså netop ét skæringspunkt!

```
> subs(t=2, l)
```

$$\begin{bmatrix} 13 \\ 0 \\ -7 \end{bmatrix} \quad \textbf{(1.4)}$$

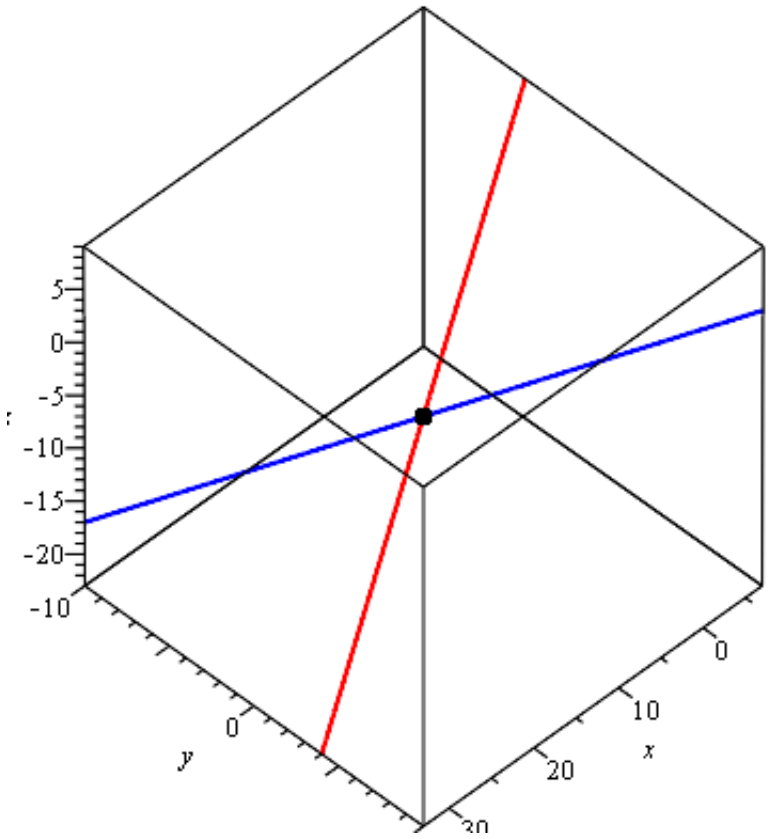
```
> subs(s=-5, m)
```

$$\begin{bmatrix} 13 \\ 0 \\ -7 \end{bmatrix} \quad \textbf{(1.5)}$$

Konklusion: De 2 rette linjer skæring hinanden i netop ét punkt, som hedder $(13, 0, -7)$

```
> with(plots) : with(plottools) :
> linjeL := spacecurve([l[1], l[2], l[3]], t=-2..6, thickness=2, color=red) :
  linjeM := spacecurve([m[1], m[2], m[3]], s=-15..5, thickness=2, color=blue) :
```

```
punkt := pointplot3d([13, 0, -7], symbolsize = 20, symbol = solidcircle), color = black:
display(linjeL, linjeM, punkt, axes = boxed, labels = [x, y, z]);
```



Eksempel 144 side 170 - skæring mellem 2 linjer (ingen)

Givet parameterfremstilling for 2 linjer:

$$l: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -11 \\ 12 \end{bmatrix} + t \cdot \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} \quad \text{og} \quad m: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} + s \cdot \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

NB: Vigtigt, at parametreren ikke hedder det samme i de 2 parameterfremstillinger! Kald den ene for *t*, den anden for *s*.

```
> restart
> l := ⟨10, -11, 12⟩ + t·⟨1, -1, 5⟩
```

$$l := \begin{bmatrix} 10 + t \\ -11 - t \\ 12 + 5\,t \end{bmatrix} \tag{2.1}$$

> $m := \langle -1, -3, 1\rangle + s\cdot\langle 4, -1, 1\rangle$

$$m := \begin{bmatrix} -1+4s \\ -3-s \\ 1+s \end{bmatrix} \quad \textbf{(2.2)}$$

Skæring i 3 dimensioner?

> $solve(\{l_1 = m_1, l_2 = m_2, l_3 = m_3\}, \{t, s\})$

Altså intet skæringspunkt!

Checker skæring i 2 dimensioner:

> $solve(\{l_1 = m_1, l_2 = m_2\}, \{t, s\})$

$$\{s = 1, t = -7\} \quad \textbf{(2.3)}$$

> $subs(t = -7, l)$

$$\begin{bmatrix} 3 \\ -4 \\ -23 \end{bmatrix} \quad \textbf{(2.4)}$$

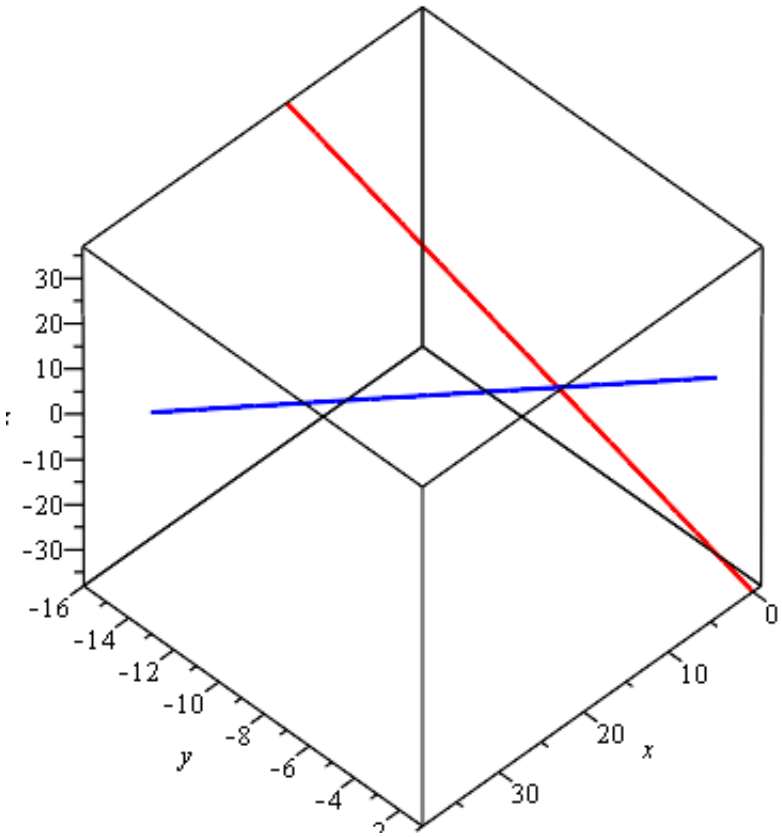
> $subs(s = 1, m)$

$$\begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix} \quad \textbf{(2.5)}$$

Konklusion: De 2 rette linjer skæring ikke hinanden i netop ét punkt. Linjerne er faktisk *vindskæve*.
NB: Passer kun i *x* og *y*, ikke i *z*.

> $with(plots) : with(plottools) :$

> $linjeL := spacecurve([l_1, l_2, l_3], t = -10..5, thickness = 2, color = red) :$
$linjeM := spacecurve([m_1, m_2, m_3], s = 0..10, thickness = 2, color = blue) :$
$display(linjeL, linjeM, axes = boxed, labels = [x, y, z]);$



Eksempel 145 side 171 - skæring mellem linje og plan (ét punkt)

Givet plan $\alpha: 4\cdot x - 6\cdot y + 10\cdot z - 26 = 0.$

Givet linje m: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -6 \end{bmatrix} + t\cdot \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$

```
> restart
> α := 4·x − 6·y + 10·z − 26 = 0
```

$$\alpha := 4x - 6y + 10z - 26 = 0 \quad \textbf{(3.1)}$$

```
> m := ⟨1, -1, -6⟩ + t·⟨2, -2, 4⟩
```

$$m := \begin{bmatrix} 1 + 2t \\ -1 - 2t \\ -6 + 4t \end{bmatrix} \quad \textbf{(3.2)}$$

```
> subs(x = m1, y = m2, z = m3, α)
```

$$-76 + 60\,t = 0 \quad \textbf{(3.3)}$$

```
> tSKÆR := solve((3.3), t)
```

$$tSKÆR := \frac{19}{15} \quad \textbf{(3.4)}$$

```
> S := subs(t = tSKÆR, m)
```

$$S := \begin{bmatrix} \frac{53}{15} \\ -\frac{53}{15} \\ -\frac{14}{15} \end{bmatrix} \quad \textbf{(3.5)}$$

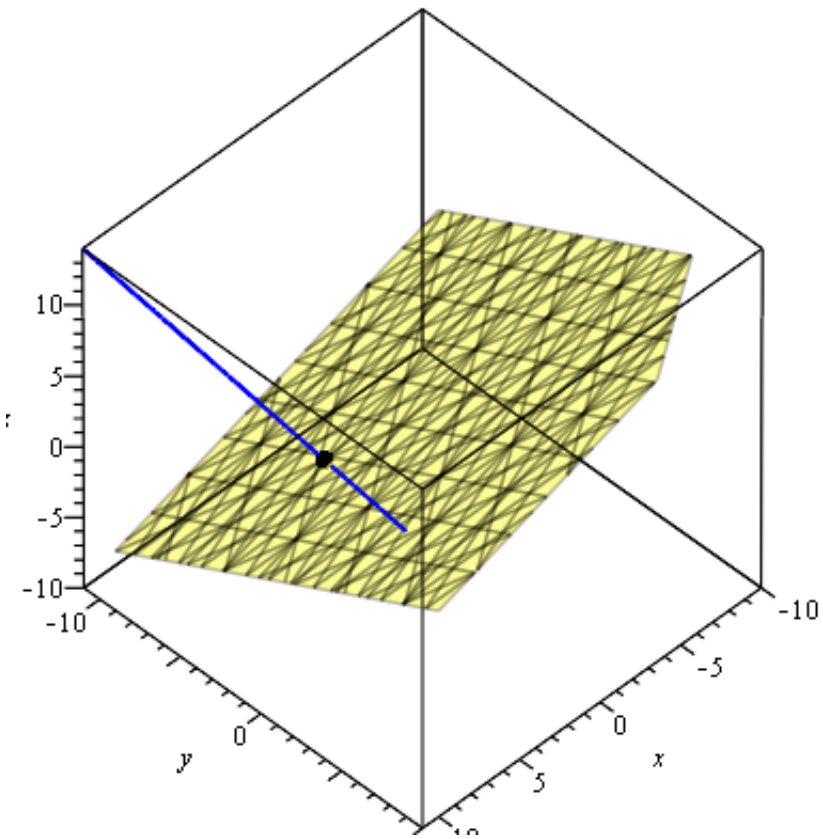
```
> evalf(S)
```

$$\begin{bmatrix} 3.533333333 \\ -3.533333333 \\ -0.9333333333 \end{bmatrix} \quad \textbf{(3.6)}$$

Konklusion: Der er netop ét skæringspunkt. Det har koordinaterne

$$\left(\frac{53}{15}, -\frac{53}{15}, -\frac{14}{15}\right) \approx (3.53, -3.53, -0.93)$$

```
> with(plots) : with(plottools) :
> linjeM := spacecurve([m1, m2, m3], t = 0..5, thickness = 2, color = blue) :
  planα := implicitplot3d(α, x = -10..10, y = -10..10, z = -10..10, color = yellow, transparency
      = 0.60) :
  punkt := pointplot3d(convert(S, list), symbolsize = 20, symbol = solidcircle, color = black) :
  display(linjeM, planα, punkt, axes = boxed, labels = [x, y, z]);
```


10
5
0
-5
-10
-10
0
y
-10
-5
0
5
x