

Faktorisering af polynomium

Spørgsmål:

Faktorisering af polynomiet $x^n - 1$ har tilsyneladende altid koefficienterne **-1, 0 eller 1**. Gælder det altid?

> restart

> factor($x^3 - 1$)

$$(x - 1) (x^2 + x + 1) \quad (1)$$

> factor($x^{30} - 1$)

$$(x - 1) (x^4 + x^3 + x^2 + x + 1) (x^2 + x + 1) (x^8 - x^7 + x^5 - x^4 + x^3 - x + 1) (x + 1) (x^4 - x^3 + x^2 - x + 1) (x^2 - x + 1) (x^8 + x^7 - x^5 - x^4 - x^3 + x + 1) \quad (2)$$

> factor($x^{47} - 1$)

$$(x - 1) (x^{46} + x^{45} + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) \quad (3)$$

> factor($x^{100} - 1$)

$$(x - 1) (x^4 + x^3 + x^2 + x + 1) (x^{20} + x^{15} + x^{10} + x^5 + 1) (x + 1) (x^4 - x^3 + x^2 - x + 1) (x^{20} - x^{15} + x^{10} - x^5 + 1) (x^2 + 1) (x^8 - x^6 + x^4 - x^2 + 1) (x^{40} - x^{30} + x^{20} - x^{10} + 1) \quad (4)$$

Tilsyneladende er det sandt.

MEN det svigter for $n = 105$!

> factor($x^{105} - 1$)

$$(x - 1) (x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) (x^4 + x^3 + x^2 + x + 1) (x^{24} - x^{23} + x^{19} - x^{18} + x^{17} - x^{16} + x^{14} - x^{13} + x^{12} - x^{11} + x^{10} - x^8 + x^7 - x^6 + x^5 - x + 1) (x^2 + x + 1) (x^{12} - x^{11} + x^9 - x^8 + x^6 - x^4 + x^3 - x + 1) (x^8 - x^7 + x^5 - x^4 + x^3 - x + 1) (x^{48} + x^{47} + x^{46} - x^{43} - x^{42} - 2x^{41} - x^{40} - x^{39} + x^{36} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} - x^{28} - x^{26} - x^{24} - x^{22} - x^{20} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} - x^9 - x^8 - 2x^7 - x^6 - x^5 + x^2 + x + 1) \quad (5)$$