Herons formel for areal af trekant

Sætning 37 side 192 i bog B1, 2. udgave

Areal af ΔABC er givet ved *Herons formel* $\sqrt{s\cdot(s-a)\cdot(s-b)\cdot(s-c)}$, hvor $s=\frac{a+b+c}{2}$

> restart

s er den halve omkreds :

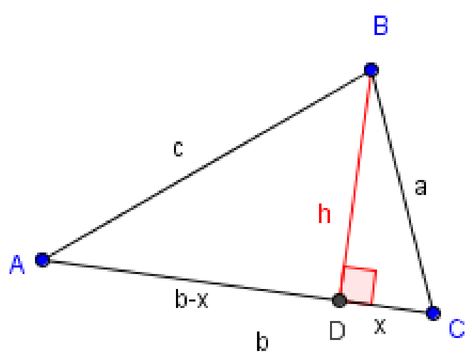
> $s := \frac{a+b+c}{2}$

$$s := \frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}c \quad (1.1)$$

> $Heron := \sqrt{s\cdot(s-a)\cdot(s-b)\cdot(s-c)}$

$$Heron := \left(\left(\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}c\right)\left(-\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}c\right)\left(\frac{1}{2}a-\frac{1}{2}b+\frac{1}{2}c\right)\left(\frac{1}{2}a+\frac{1}{2}b-\frac{1}{2}c\right)\right)^{1/2} \quad (1.2)$$

Pythagoras' sætning anvendt på de 2 trekanter, som opstår når højden h tegnes fra vinkel B til siden b :



> $solve(\{h^2+x^2=a^2, (b-x)^2+h^2=c^2\}, \{x,h\})$

$$\left\{h=\frac{1}{2}\,\frac{RootOf(a^4-2a^2b^2-2a^2c^2+b^4-2b^2c^2+c^4+\_Z^2)}{b}, x=\frac{1}{2}\,\frac{a^2+b^2-c^2}{b}\right\} \quad (1.3)$$

> $allvalues(\%)$

$$\left\{h=\frac{1}{2}\,\frac{\sqrt{-a^4+2a^2b^2+2a^2c^2-b^4+2b^2c^2-c^4}}{b}, x=\frac{1}{2}\,\frac{a^2+b^2-c^2}{b}\right\}, \left\{h=-\frac{1}{2}\,\frac{\sqrt{-a^4+2a^2b^2+2a^2c^2-b^4+2b^2c^2-c^4}}{b}, x=\frac{1}{2}\,\frac{a^2+b^2-c^2}{b}\right\} \quad (1.4)$$

> $h := rhs(\%[1,1])$

$$h := \frac{1}{2}\,\frac{\sqrt{-a^4+2\,a^2\,b^2+2\,a^2\,c^2-b^4+2\,b^2\,c^2-c^4}}{b} \quad \textbf{(1.5)}$$

> $Areal := \frac{1}{2}\cdot b\cdot h$

$$Areal := \frac{1}{4}\,\sqrt{-a^4+2\,a^2\,b^2+2\,a^2\,c^2-b^4+2\,b^2\,c^2-c^4} \quad \textbf{(1.6)}$$

For at undersøge formlerne nærmere, kvadrerer vi udtrykkene:

> $Areal^2$

$$-\frac{1}{16}\,a^4+\frac{1}{8}\,a^2\,b^2+\frac{1}{8}\,a^2\,c^2-\frac{1}{16}\,b^4+\frac{1}{8}\,b^2\,c^2-\frac{1}{16}\,c^4 \quad \textbf{(1.7)}$$

> $Heron^2$

$$\left(\frac{1}{2}\,a+\frac{1}{2}\,b+\frac{1}{2}\,c\right)\left(-\frac{1}{2}\,a+\frac{1}{2}\,b+\frac{1}{2}\,c\right)\left(\frac{1}{2}\,a-\frac{1}{2}\,b+\frac{1}{2}\,c\right)\left(\frac{1}{2}\,a+\frac{1}{2}\,b-\frac{1}{2}\,c\right) \quad \textbf{(1.8)}$$

> $expand(\%)$

$$-\frac{1}{16}\,a^4+\frac{1}{8}\,a^2\,b^2+\frac{1}{8}\,a^2\,c^2-\frac{1}{16}\,b^4+\frac{1}{8}\,b^2\,c^2-\frac{1}{16}\,c^4 \quad \textbf{(1.9)}$$

> $Areal^2 - Heron^2$

$$-\frac{1}{16}\,a^4+\frac{1}{8}\,a^2\,b^2+\frac{1}{8}\,a^2\,c^2-\frac{1}{16}\,b^4+\frac{1}{8}\,b^2\,c^2-\frac{1}{16}\,c^4-\left(\frac{1}{2}\,a+\frac{1}{2}\,b+\frac{1}{2}\,c\right)\left(-\frac{1}{2}\,a+\frac{1}{2}\,b+\frac{1}{2}\,c\right)\left(\frac{1}{2}\,a-\frac{1}{2}\,b+\frac{1}{2}\,c\right)\left(\frac{1}{2}\,a+\frac{1}{2}\,b-\frac{1}{2}\,c\right) \quad \textbf{(1.10)}$$

> $simplify(\%)$

$$0 \quad \textbf{(1.11)}$$

Q.E.D., dvs. Herons formel er bevist med et **CAS-bevis**.