

Differentiation af specielle funktioner: \ln , \log , e^x , a^x , x^a , x^x

> restart

Naturlige logaritmefunktion \ln :

> $\ln(x)$

$$\ln(x) \quad (1)$$

Differentiation med Maple:

> $(\ln(x))'$

$$\frac{1}{x} \quad (2)$$

Definition af differentiation:

> $\lim_{h \rightarrow 0} \frac{\ln(x+h) - \ln(x)}{h}$

$$\frac{1}{x} \quad (3)$$

Konklusion: $(\ln(x))' = \frac{1}{x}$

Titalslogaritmen $\log(x)$:

Husk at \log indtastet som $\log10$:

> $\log10(x)$

$$\frac{\ln(x)}{\ln(10)} \quad (4)$$

Differentiation med Maple:

> $(\log10(x))'$

$$\frac{1}{x \ln(10)} \quad (5)$$

Definition af differentiation:

> $\lim_{h \rightarrow 0} \frac{\log10(x+h) - \log10(x)}{h}$

$$\frac{1}{x \ln(2) + x \ln(5)} \quad (6)$$

> $is(5) = (6)$

$$true \quad (7)$$

NB: $\ln(2) + \ln(5) = \ln(2 \cdot 5) = \ln(10)$

Konklusion: $(\log(x))' = \frac{1}{x \cdot \ln(10)}$

Den naturlige eksponentialfunktion e^x :

Husk at e^x indtastet som $\exp(x)$. Eller via skabelonen e^a .

$$> \exp(x) \quad e^x \quad (8)$$

$$> e^x \quad e^x \quad (9)$$

Differentiation med Maple:

$$> (\exp(x))' \quad e^x \quad (10)$$

$$> (e^x)' \quad e^x \quad (11)$$

Definition af differentiation:

$$> \lim_{h \rightarrow 0} \frac{\exp(x+h) - \exp(x)}{h} \quad e^x \quad (12)$$

$$> \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} \quad e^x \quad (13)$$

Konklusion: $(e^x)' = e^x$

Den generelle eksponentialfunktion a^x :

$$> a^x \quad a^x \quad (14)$$

Differentiation med Maple:

$$> (a^x)' \quad a^x \ln(a) \quad (15)$$

Definition af differentiation:

$$> \lim_{h \rightarrow 0} \frac{a^{x+h} - a^x}{h} \quad a^x \ln(a) \quad (16)$$

Konklusion: $(a^x)' = a^x \cdot \ln(a)$

Potensfunktion x^a :

$$> x^a \quad x^a \quad (17)$$

Differentiation med Maple:

$$\begin{aligned} > (x^a)' & \frac{x^a a}{x} \end{aligned} \tag{18}$$

$$\begin{aligned} > \text{simplify}(\%) & x^{a-1} a \end{aligned} \tag{19}$$

Definition af differentiation:

$$\begin{aligned} > \lim_{h \rightarrow 0} \frac{(x+h)^a - x^a}{h} & \frac{x^a a}{x} \end{aligned} \tag{20}$$

$$\begin{aligned} > \text{simplify}(\%) & x^{a-1} a \end{aligned} \tag{21}$$

Konklusion: $(x^a)' = a \cdot x^{a-1}$

Hverken-eller-funktion x^x :

$$\begin{aligned} > x^x & x^x \end{aligned} \tag{22}$$

Differentiation med Maple:

$$\begin{aligned} > (x^x)' & x^x (\ln(x) + 1) \end{aligned} \tag{23}$$

Definition af differentiation:

$$\begin{aligned} > \lim_{h \rightarrow 0} \frac{(x+h)^{x+h} - x^x}{h} & x^x \ln(x) + x^x \end{aligned} \tag{24}$$

$$\begin{aligned} > \text{factor}(\%) & x^x (\ln(x) + 1) \end{aligned} \tag{25}$$

Konklusion: $(x^x)' = x^x \cdot (\ln(x) + 1)$