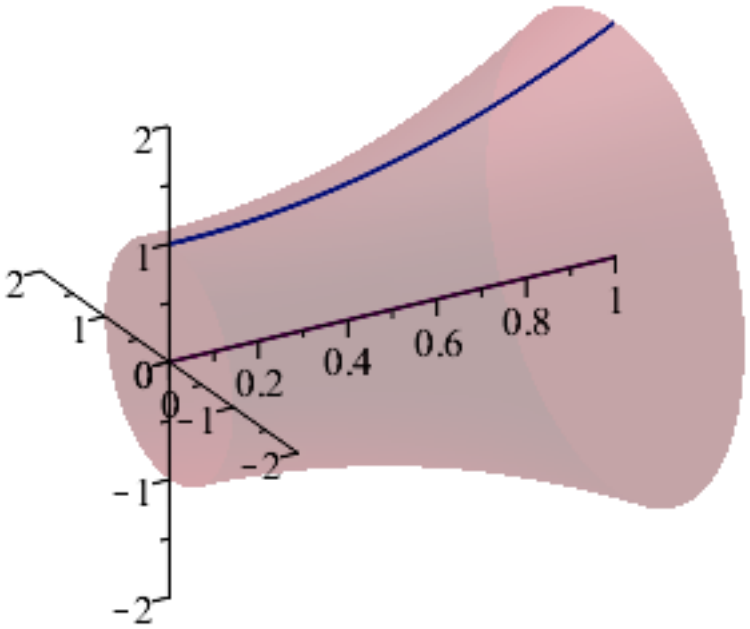
Rumfang af omdrejningslegeme

Brug af indbygget værktøj i Maple

Menu: Tools, Tutors, Calculus - Single Variable, Volume of Revolution

```
restart
with(Student[Calculus1]):
VolumeOfRevolution(x^2 + 1, 0 .. 1, 'axis'= horizontal, 'distancefromaxis' = 0, 'showvolume'= true,
    'showsum'= false, 'showregion'= false, 'method'= midpoint, 'partition'= 6, 'output'= plot)
```



The solid of revolution created on $0 \le x \le 1$ by rotation of $f(x) = x^2 + 1$ about the axis $y = 0$.

```
VolumeOfRevolution(x^2 + 1, x = 0..1)
```

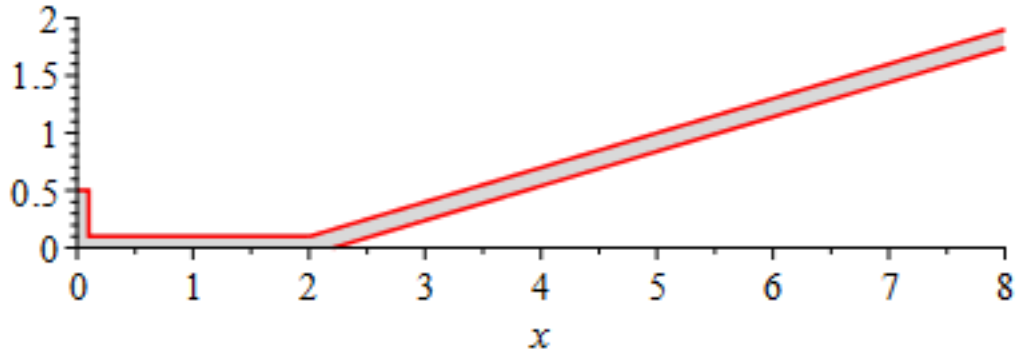
$$\frac{28}{15}\,\pi \tag{1.1}$$

```
π·int((x^2 + 1)^2, x = 0 .. 1)
```

$$\frac{28}{15}\,\pi \tag{1.2}$$

Eksempel: et vinglas

Givet et vinglas, hvor man skal bestemme rumfanget af glasset samt rumfanget af vinen, som der er plads til i glasset.



Funktionen er 0.5 fra 0 til 0.1 ("foden"), og 0.1 fra 0.1 til 2 ("stilken").
Glasset ligger imellem 2 lineære funktioner:
$f(x) = 0.3 \cdot (x - 2) + 0.1$ for x fra 2 ti 8
$g(x) = 0.3 \cdot (x - 2) - 0.06$ for x fra 2.2 til 8

Målene er i cm.

Definerer de 3 funktioner:

restart

$$h(x) := \begin{cases} 0.5 & 0 \le x \le 0.1 \\ 0.1 & 0.1 < x \le 2 \end{cases}$$

$$x \to piecewise(0 \le x \text{ and } x \le 0.1, 0.5, 0.1 < x \text{ and } x \le 2, 0.1) \quad \textbf{(2.1)}$$

$$f(x) := 0.3 \cdot (x - 2) + 0.1$$

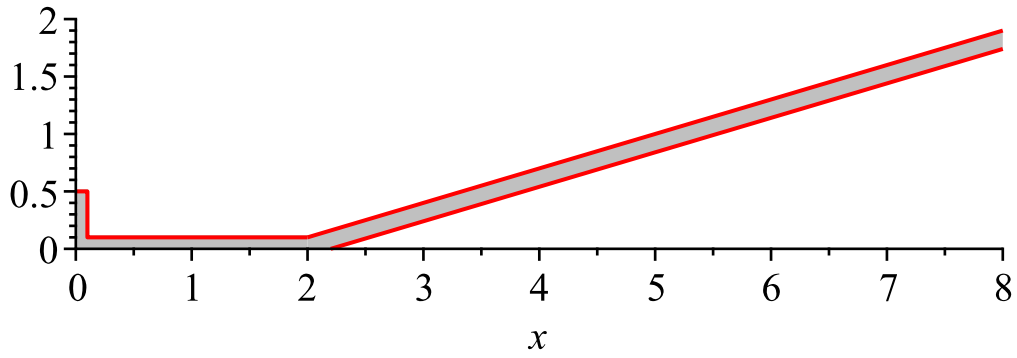
$$x \to 0.3\ (x - 2) + 0.1 \quad \textbf{(2.2)}$$

$$g(x) := 0.3 \cdot (x - 2) - 0.06$$

$$x \to 0.3\ (x - 2) - 0.06 \quad \textbf{(2.3)}$$

Plot af vinglasset

```
with(plots):
H := plot(h(x), x = 0..2, color = red):
Hf := plot(h(x), x = 0..2, filled, color = grey):
F := plot(f(x), x = 2..8, color = red):
G := plot(g(x), x = 2.2..8, color = red):
S := shadebetween(f(x), g(x), x = 2..8, color = grey, showboundary = false):
display(Hf, H, F, G, S, view = [0..8, 0..2], scaling = constrained)
```



Rumfanget af vinen, når glasset er helt fyldt op

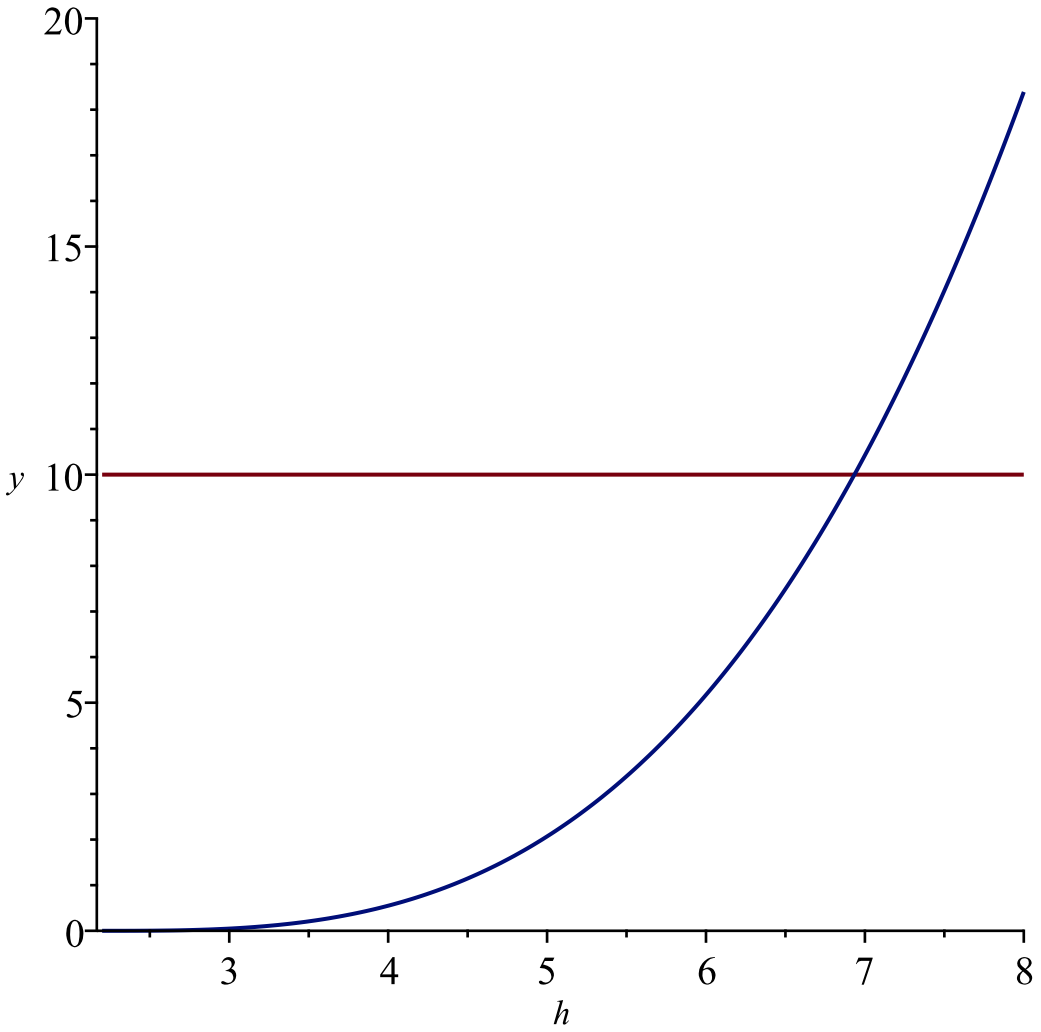
$$\pi\cdot\int_{2.2}^{8}(g(x))^2\,dx$$

18.38887278 **(2.2.1)**

Dvs. glasset indeholder ca. $18.4\ cm^3$ vin

Højden af vinen i glasset, når der skal være 10 cm^3 vin i glasset

$$plot\left(\left\{\pi\cdot\int_{2.2}^{h}(g(x))^2\,dx,\ 10\right\},\ h=2.2..8,\ y=0..20\right)$$



Der er tydelig netop én løsning.

$$solve\left(\pi\cdot\int_{2.2}^{h}(g(x))^2\,dx=10,\,h\right)$$

$$6.934160282,\ -0.1670801410+4.099903070\,I,\ -0.1670801410-4.099903070\,I \qquad \textbf{(2.3.1)}$$

Ønsker ikke de komplekse løsninger!

$$with(RealDomain):$$

$$solve\left(\pi\cdot\int_{2.2}^{h}(g(x))^2\,dx=10,\,h\right)$$

$$6.934160282 \qquad \textbf{(2.3.2)}$$

$$unwith(RealDomain):$$

Dvs. vinen når op til ca. 6.9 *cm*, når der er 10 cm^3 i glasset.

Rumfanget af glasset, som indgår i vinglasset

$$\pi\cdot\int_{0}^{2}(h(x))^2\,dx+\pi\cdot\int_{2}^{8}(f(x))^2\,dx-\pi\cdot\int_{2.2}^{8}(g(x))^2\,dx$$

$$5.68829332 \qquad \textbf{(2.4.1)}$$

eller:

$$\pi\cdot\int_{0}^{0.1}(h(x))^2\,dx+\pi\cdot\int_{0.1}^{2}(h(x))^2\,dx+\pi\cdot\int_{2}^{8}(f(x))^2\,dx-\pi\cdot\int_{2.2}^{8}(g(x))^2\,dx$$

5.68829332 **(2.4.2)**

Dvs. der er anvendt ca. $5.7\ cm^3$ glas