Opgave 6005 side 104 i Arbejdsbog A

Opstart

```
> restart
> with(Gym) :
> unprotect('O') :
  unprotect('D')
> A := ⟨10, 0, 0⟩; B := ⟨10, 4, 0⟩; C := ⟨0, 4, 0⟩; O := ⟨0, 0, 0⟩
```

$$A := \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$B := \begin{bmatrix} 10 \\ 4 \\ 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$$

$$O := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(1.1.1)}$$

Punkterne D, E, F og G ligger i samme højde (nemlig z = 3):

```
> D := ⟨10, 0, 3⟩; E := ⟨10, 4, 3⟩; F := ⟨0, 4, 3⟩; G := ⟨0, 0, 3⟩
```

$$D := \begin{bmatrix} 10 \\ 0 \\ 3 \end{bmatrix}$$

$$E := \begin{bmatrix} 10 \\ 4 \\ 3 \end{bmatrix}$$

$$F := \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}$$

$$G := \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \quad \textbf{(1.1.2)}$$

Punkterne H og J ligger i samme højde, nemlig z = 8:

> $H := \langle 7, 2, 8 \rangle ; J := \langle 3, 2, 8 \rangle$

$$H := \begin{bmatrix} 7 \\ 2 \\ 8 \end{bmatrix}$$

$$J := \begin{bmatrix} 3 \\ 2 \\ 8 \end{bmatrix} \quad \textbf{(1.1.3)}$$

Ligninger for de 4 tagflader

P

> $P := \langle x, y, z \rangle$

$$P := \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \textbf{(1.2.1.1)}$$

DEH

> $\overrightarrow{DE} := E - \mathrm{D}; \overrightarrow{DH} := H - \mathrm{D}; \overrightarrow{nDEHud} := \overrightarrow{DE} \times \overrightarrow{DH}$

$$\overrightarrow{DE} := \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$$

$$\overrightarrow{DH} := \begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix}$$

$$\overrightarrow{nDEHud} := \begin{bmatrix} 20 \\ 0 \\ 12 \end{bmatrix} \quad \textbf{(1.2.2.1)}$$

Ligning for tagfladens plan:

> $LigningDEH := \overrightarrow{nDEHud} \bullet (P - \mathrm{D}) = 0$

$$LigningDEH := 20\,x - 236 + 12\,z = 0 \quad \textbf{(1.2.2.2)}$$

> $LigningDEH := \frac{\%}{4}$

$$LigningDEH := 5\,x - 59 + 3\,z = 0 \quad \textbf{(1.2.2.3)}$$

Konklusion: Ligningen for planen gennem DEH er $5 \cdot x + 3 \cdot z - 59 = 0$

EFJH

> $\overrightarrow{EF} := F - E; \overrightarrow{EH} := H - E; \overrightarrow{nEFJHud} := \overrightarrow{EF} \times \overrightarrow{EH}$

$$\overrightarrow{EF} := \begin{bmatrix} -10 \\ 0 \\ 0 \end{bmatrix}$$

$$\overrightarrow{EH} := \begin{bmatrix} -3 \\ -2 \\ 5 \end{bmatrix}$$

$$\overrightarrow{nEFJHud} := \begin{bmatrix} 0 \\ 50 \\ 20 \end{bmatrix} \quad \textbf{(1.2.3.1)}$$

Ligning for tagfladens plan:

> $LigningEFJH := \overrightarrow{nEFJHud} \bullet (P - E) = 0$

$$LigningEFJH := 50\,y - 260 + 20\,z = 0 \quad \textbf{(1.2.3.2)}$$

> $LigningEFJH := \frac{\%}{10}$

$$LigningEFJH := 5\,y - 26 + 2\,z = 0 \quad \textbf{(1.2.3.3)}$$

Konklusion: Ligningen for planen gennem EFJH er $5 \cdot y + 2 \cdot z - 26 = 0$

FGJ

> $\overrightarrow{FJ} := J - F; \overrightarrow{FG} := G - F; \overrightarrow{nFGJud} := \overrightarrow{FG} \times \overrightarrow{FJ}$

$$\overrightarrow{FJ} := \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$$

$$\overrightarrow{FG} := \begin{bmatrix} 0 \\ -4 \\ 0 \end{bmatrix}$$

$$\overrightarrow{nFGJud} := \begin{bmatrix} -20 \\ 0 \\ 12 \end{bmatrix} \quad \textbf{(1.2.4.1)}$$

Ligning for tagfladens plan:

> $LigningFGJ := \overrightarrow{nFGJud} \bullet (P - F) = 0$

$$LigningFGJ := -20\,x + 12\,z - 36 = 0 \quad \textbf{(1.2.4.2)}$$

> $LigningFGJ := \frac{\%}{4}$

$$LigningFGJ := -5\,x + 3\,z - 9 = 0 \quad \textbf{(1.2.4.3)}$$

Konklusion: Ligningen for planen gennem FGJ er $-5 \cdot x + 3 \cdot z - 9 = 0$

DHJG

> $\overrightarrow{DG} := G - \mathrm{D};\ \overrightarrow{DH};\ \overrightarrow{nDHJGud} := \overrightarrow{DH} \times \overrightarrow{DG}$

$$\overrightarrow{DG} := \begin{bmatrix} -10 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix}$$

$$\overrightarrow{nDHJGud} := \begin{bmatrix} 0 \\ -50 \\ 20 \end{bmatrix} \quad \textbf{(1.2.5.1)}$$

Ligning for tagfladens plan:

> $LigningDHJG := \overrightarrow{nDHJGud} \bullet (P - \mathrm{D}) = 0$

$$LigningDHJG := -50\,y + 20\,z - 60 = 0 \quad \textbf{(1.2.5.2)}$$

> $LigningDHJG := \frac{\%}{10}$

$$LigningDHJG := -5\,y + 2\,z - 6 = 0 \quad \textbf{(1.2.5.3)}$$

Konklusion: Ligningen for planen gennem DHJG er $-5 \cdot y + 2 \cdot z - 6 = 0$

Vinklen mellem tagfladerne

NB: Den ene normalvektor skal gå **udad**, den anden **indad**!
Det klares ved at sætte minus foran normalvektor, så skifter den til at gå indad.

> $VinkelDEHogEFJH := vinkel\left(\overrightarrow{nDEHud}, -\overrightarrow{nEFJHud}\right)$

$$VinkelDEHogEFJH := 101.0157558 \quad \textbf{(1.3.1)}$$

> $VinkelEFJHogFGJ := vinkel\left(\overrightarrow{nEFJHud}, -\overrightarrow{nFGJud}\right)$

$$VinkelEFJHogFGJ := 101.0157558 \quad \textbf{(1.3.2)}$$

> $VinkelFGJogDHJG := vinkel\left(\overrightarrow{nFGJud}, -\overrightarrow{nDHJGud}\right)$

$$VinkelFGJogDHJG := 101.0157558 \quad \textbf{(1.3.3)}$$

> $VinkelDHJGogDEH := vinkel\left(\overrightarrow{nDHJGud}, -\overrightarrow{nDEHud}\right)$

$$VinkelDHJGogDEH := 101.0157558 \quad \textbf{(1.3.4)}$$

Ikke så overraskende bliver alle 4 vinkler ens (symmetri!).

Konklusion: Skrå vinklen mellem 2 tagflader er ca. 101.02°

> $VinkelDHJGogEFJH := vinkel\left(\overrightarrow{nDHJGud}, -\overrightarrow{nEFJHud}\right)$

$$VinkelDHJGogEFJH := 43.60281898 \quad \textbf{(1.3.5)}$$

Konklusion: Vinklen mellem tagfladerne *langs tagrykken* er ca. 43.60°

Meteor nedslag på tagflade

Start

Metoritnedslag. Givet $t > -200$.

> $meteor := \langle 6, 5, 7\rangle + t\cdot\langle -1, -1, -2\rangle$

$$meteor := \begin{bmatrix} 6-t \\ 5-t \\ 7-2\,t \end{bmatrix} \quad \textbf{(1.4.1.1)}$$

DEH

> $solve(subs(x = meteor[1], y = meteor[2], z = meteor[3], LigningDEH), t)$

$$-\frac{8}{11} \quad \textbf{(1.4.2.1)}$$

> $subs(t = \%, meteor)$

$$\begin{bmatrix} \frac{74}{11} \\ \frac{63}{11} \\ \frac{93}{11} \end{bmatrix} \quad \textbf{(1.4.2.2)}$$

> $evalf(\%)$

$$\begin{bmatrix} 6.727272727 \\ 5.727272727 \\ 8.454545455 \end{bmatrix} \quad \textbf{(1.4.2.3)}$$

Ikke inde på tagfladen, for der skal x være mellem 7 og 10.

EFJH

> $solve(subs(x = meteor[1], y = meteor[2], z = meteor[3], LigningEFJH), t)$

$$\frac{13}{9} \quad \textbf{(1.4.3.1)}$$

> $subs(t = \%, meteor)$

$$\begin{bmatrix} \frac{41}{9} \\ \frac{32}{9} \\ \frac{37}{9} \end{bmatrix} \quad \textbf{(1.4.3.2)}$$

> $M := \%$

$$M := \begin{bmatrix} \frac{41}{9} \\ \frac{32}{9} \\ \frac{37}{9} \end{bmatrix} \quad \textbf{(1.4.3.3)}$$

> $evalf(\%)$

$$\begin{bmatrix} 4.555555556 \\ 3.555555556 \\ 4.111111111 \end{bmatrix} \quad \textbf{(1.4.3.4)}$$

Dette punkt ligger i tagfladen EFJH.

Konklusion: Meteoren rammer tagfladen EFJH i punkter (4.56, 3.56, 4.11)

FGJ

```
> solve(subs(x = meteor[1], y = meteor[2], z = meteor[3], LigningFGJ), t)
```

$$-18 \quad \textbf{(1.4.4.1)}$$

```
> subs(t = %, meteor)
```

$$\begin{bmatrix} 24 \\ 23 \\ 43 \end{bmatrix} \quad \textbf{(1.4.4.2)}$$

Klart LANGT væk fra huset!

DHJG

```
> solve(subs(x = meteor[1], y = meteor[2], z = meteor[3], LigningDHJG), t)
```

$$17 \quad \textbf{(1.4.5.1)}$$

```
> subs(t = %, meteor)
```

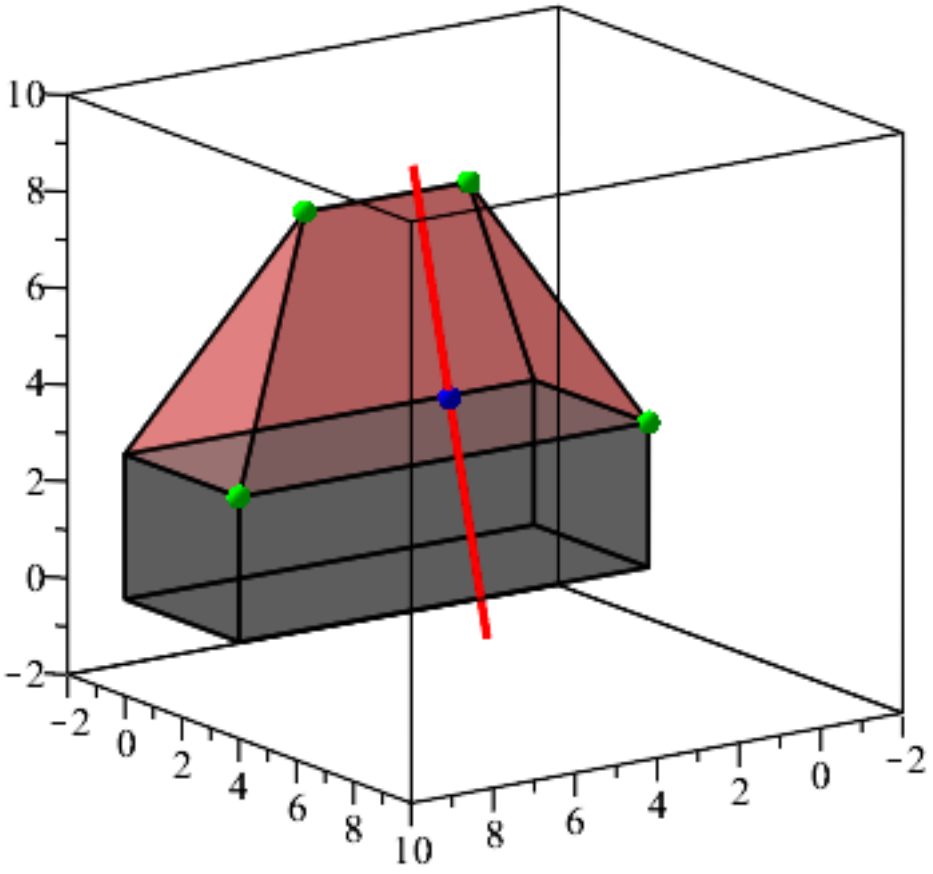
$$\begin{bmatrix} -11 \\ -12 \\ -27 \end{bmatrix} \quad \textbf{(1.4.5.2)}$$

Klart LANGT væk fra huset!

Plot af huset og meteoren

```
> with(plots) :
  with(plottools) :

  #meteorbanen og nedslagspunktet
  L := spacecurve(meteor, t = -10..10, thickness = 3, color = red) :
  punktM := point(M, color = blue, symbol = solidsphere, symbolsize = 20) :

  #punkter på ramt tagflade
  RamtFlade := pointplot3d([E, F, J, H], color = green, symbol = solidsphere,
      symbolsize = 20) :

  #taget
  EFJH := polygonplot3d([convert(E, list), convert(F, list), convert(J, list),
      convert(H, list)], transparency = 0.60, color = red) :
  DHJG := polygonplot3d([convert(D, list), convert(H, list), convert(J, list),
      convert(G, list)], transparency = 0.60, color = red) :
  DEH := polygonplot3d([convert(D, list), convert(E, list), convert(H, list)],
      transparency = 0.60, color = red) :
  FGJ := polygonplot3d([convert(F, list), convert(G, list), convert(J, list)],
      transparency = 0.60, color = red) :

  #siderne
```

```
ABED := polygonplot3d( [convert(A, list), convert(B, list), convert(E, list),
    convert(D, list) ], transparency = 0.60, color = black) :
BCFE := polygonplot3d( [convert(B, list), convert(C, list), convert(F, list),
    convert(E, list) ], transparency = 0.60, color = black) :
OCFG := polygonplot3d( [convert(O, list), convert(C, list), convert(F, list),
    convert(G, list) ], transparency = 0.60, color = black) :
OADG := polygonplot3d( [convert(O, list), convert(A, list), convert(D, list),
    convert(G, list) ], transparency = 0.60, color = black) :

#gulv
OABC := polygonplot3d( [convert(O, list), convert(A, list), convert(B, list),
    convert(C, list) ], transparency = 0.60, color = black) :

#plot af det hele
display(L, punktM, EFJH, DHJG, DEH, FGJ, ABED, BCFE, OCFG, OADG,
    OABC, RamtFlade, axes = box, view = [ -2 ..10, -2 ..10, -2 ..10 ])
```



Huset har sorte sider, og rødt tag!