Differentialligninger

2.3 Bestemmelse af tangentligninger (side 36-37)

Eksemplerne løses let i hånden **uden** at løse differentialligningerne. Se lærebogen.

Her er opgaven løst i Maple ved først at finde løsningen $f(x)$, og ud fra den beregne tangentens ligning.
NB: Det er ikke altid, man kan finde løsningen $f(x)$!

Eksempel 35 side 36 i grundbog A

```
> restart
> x0 := 1
```

$$x0 := 1 \tag{1.1}$$

```
> dsolve({y'=x+y, y(x0)=2})
```

$$y(x) = -x - 1 + \frac{4\,e^{x}}{e} \tag{1.2}$$

```
> f := unapply(rhs(%), x)
```

$$f := x \mapsto -x - 1 + \frac{4\,e^{x}}{e} \tag{1.3}$$

```
> TangentLigning := y=f(x0)+f'(x0)·(x-x0)
```

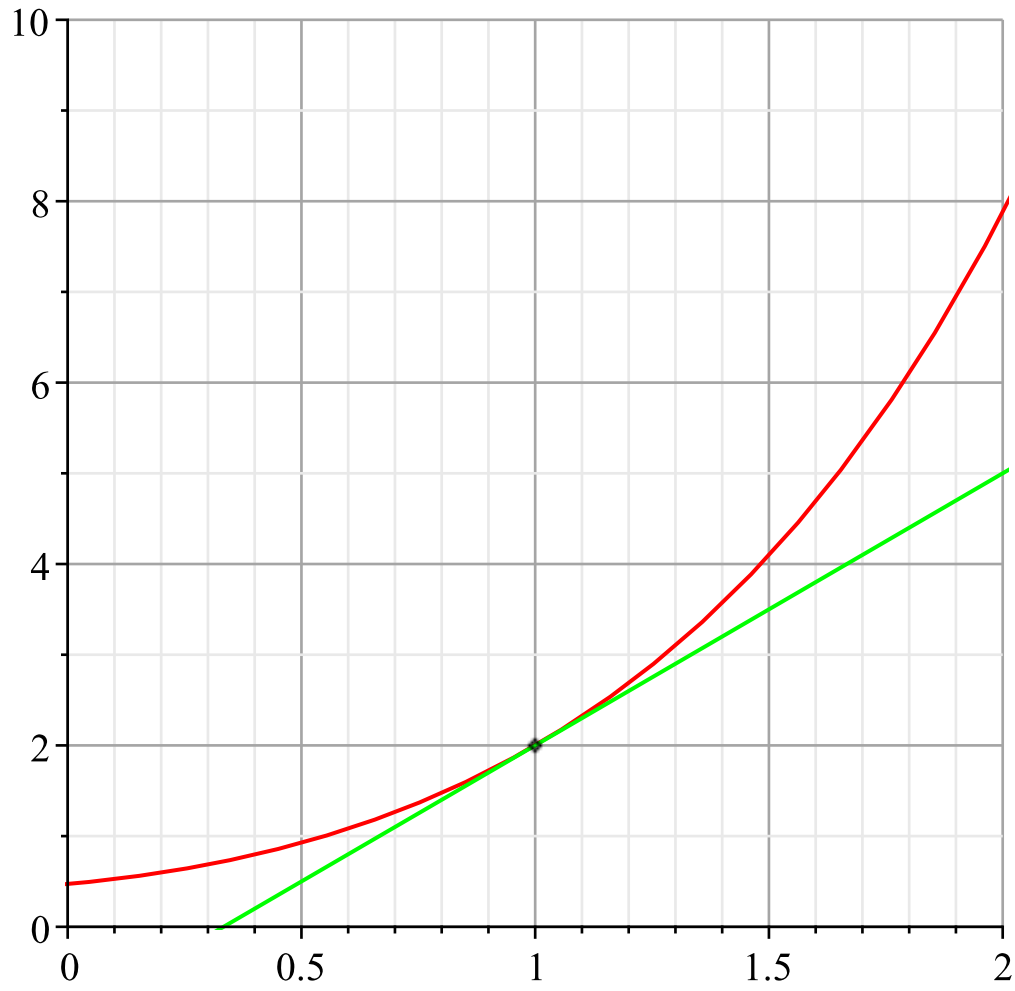
$$TangentLigning := y = -1 + 3\,x \tag{1.4}$$

```
> t := unapply(rhs(%), x)
```

$$t := x \mapsto -1 + 3\,x \tag{1.5}$$

Konklusion: Tangentligningen er $y = 3 \cdot x - 1$

```
> with(plots):
  F := plot(f, color=red):
  T := plot(t, color=green):
  punkt := pointplot([x0, f(x0)]):
  display(F, T, punkt, view=[0..2, 0..10], gridlines)
```



Eksempel 36 side 36 i grundbog A

> *restart*

> $x0 := 0$

$$x0 := 0 \tag{2.1}$$

> $dsolve\left(\left\{y' = \frac{3 \cdot y^2 + 2}{6 \cdot y}, y(x0) = 1\right\}\right)$

$$y(x) = \frac{\sqrt{-6 + 15\,e^x}}{3} \tag{2.2}$$

> $f := unapply(rhs(\%), x)$

$$f := x \mapsto \frac{\sqrt{-6 + 15\,e^x}}{3} \tag{2.3}$$

> $TangentLigning := y = f(x0) + f'(x0) \cdot (x - x0); simplify(\%)$

$$TangentLigning := y = \frac{\sqrt{9}}{3} + \frac{5\sqrt{9}\,x}{18}$$

$$y = 1 + \frac{5x}{6} \tag{2.4}$$

> $t := unapply(rhs(\%), x)$

(2.5)

$$t := x \mapsto 1 + \frac{5\,x}{6} \tag{2.5}$$

Konklusion: Tangentligningen er $y = \frac{5}{6} \cdot x + 1$

```
> with(plots) :
  F := plot(f, color = red) :
  T := plot(t, color = green) :
  punkt := pointplot([x0, f(x0)]) :
  display(F, T, punkt, view = [-2..2, 0..4], gridlines)
```

Eksempel 37 side 37 i grundbog A

```
> restart
> x0 := π/2
```

$$x0 := \frac{\pi}{2} \tag{3.1}$$

```
> dsolve({y' = e^(-y)·sin(x), y(x0) = ln(3)})
```

$$y(x) = \ln(3 - \cos(x)) \tag{3.2}$$

```
> f := unapply(rhs(%), x)
```

$$f := x \mapsto \ln(3 - \cos(x)) \tag{3.3}$$

```
> TangentLigning := y = f(x0) + f'(x0)·(x - x0)
```

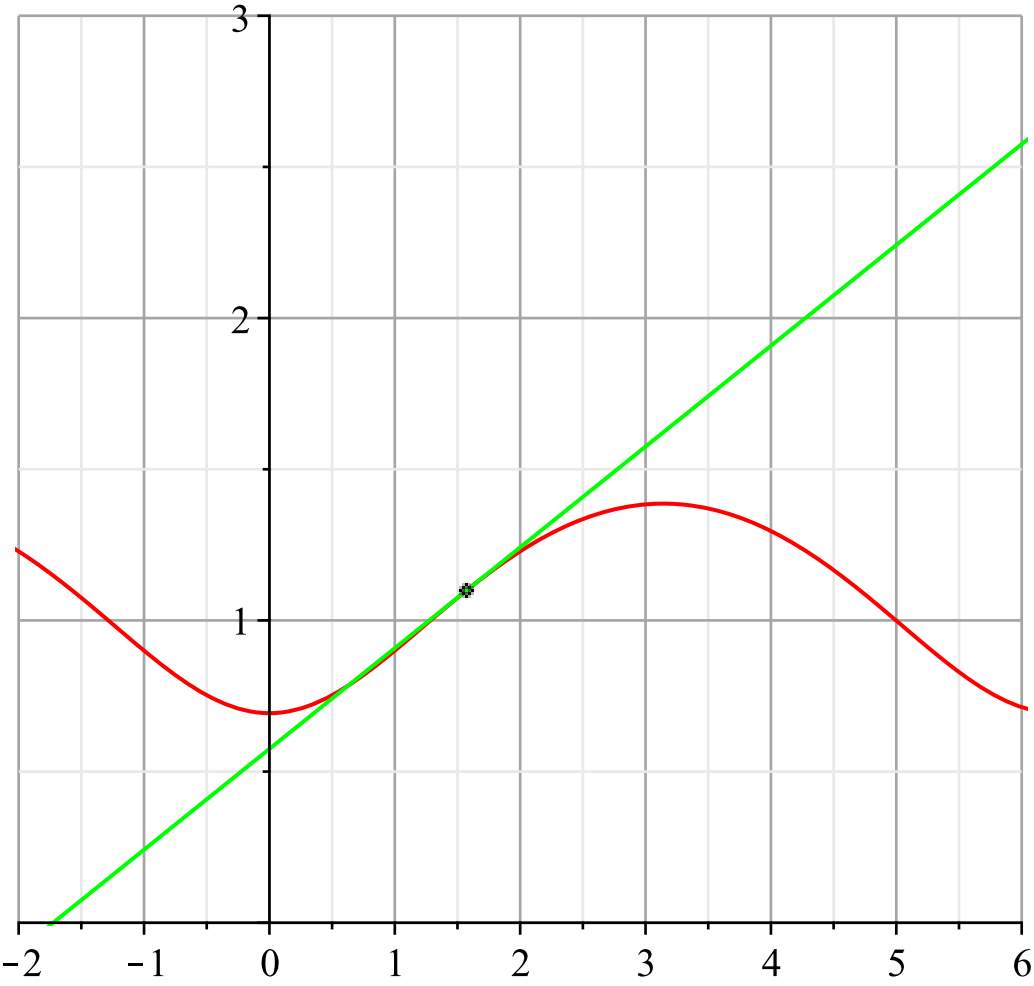
$$TangentLigning := y = \ln(3) + \frac{x}{3} - \frac{\pi}{6} \tag{3.4}$$

```
> t := unapply(rhs(%), x)
```

$$t := x \mapsto \ln(3) + \frac{x}{3} - \frac{\pi}{6} \tag{3.5}$$

```
> evalf(t(x))
```

$$0.5750135132 + 0.3333333333\,x \tag{3.6}$$

Konklusion: Tangentligningen er $y = \frac{1}{3}\cdot x - \frac{\pi}{6} + \ln(3) \approx 0.333\cdot x + 0.575$

```
> with(plots):
  F := plot(f, color = red):
  T := plot(t, color = green):
  punkt := pointplot([x0, f(x0)]):
  display(F, T, punkt, view = [-2..6, 0..3], gridlines)
```