Modellering af "Museum of the Future", Dubai

https://www.google.com/search?source=univ&tbm=isch&q=%22Museum+of+the+Future%22,+Dubai

https://www.bbc.com/future/article/20191028-museum-of-the-future-the-building-designed-by-an-algorithm

https://www.museumofthefuture.ae/

restart
with(*plots*) :

Opbygning i flere steps!

Cirkulær torus (cirkel kører i en cirkel om origo)

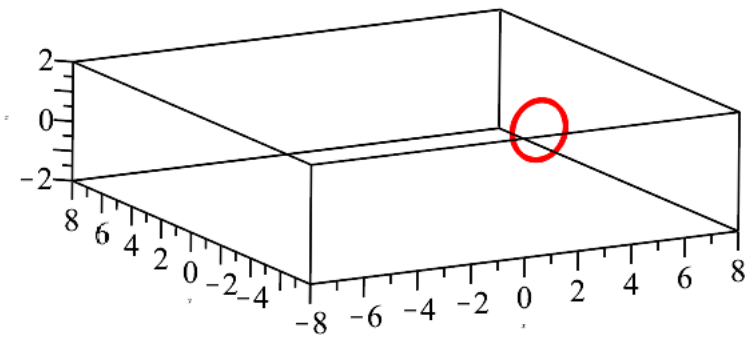
Lodret cirkel med centrum på x-aksen

Cirklens ligning i xz-planen: $(x-c)^2+z^2=r^2$

Parameterfremstilling for cirkel i xz-planen: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = r\cdot\begin{bmatrix} \cos(u) \\ 0 \\ \sin(u) \end{bmatrix} + \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix}$ hvor $u \in [0; 2\cdot\pi]$

redC(*u*) := ⟨*r*·cos(*u*) + *c*, 0, *r*·sin(*u*)⟩ :

GredC := *spacecurve*(*subs*(*r* = 1, *c* = 5, *redC*(*u*)), *u* = 0 ..2·π, *thickness* = 4, *color* = *red*, *labels* = [*x*, *y*, *z*], *scaling* = *constrained*, *view* = [−8 ..8, −8 ..8, −2 ..2])



Cirkel drejer rundt i z-aksen

Rotationsmatrix om z-aksen:
https://en.wikipedia.org/wiki/Rotation_matrix#In_three_dimensions

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

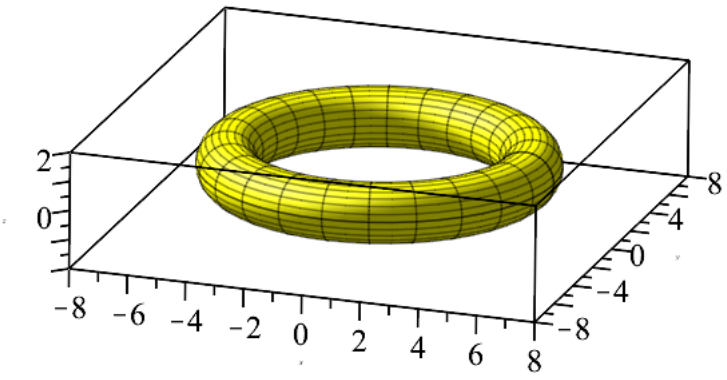
$$R(v) := \begin{bmatrix} \cos(v) & -\sin(v) & 0 \\ \sin(v) & \cos(v) & 0 \\ 0 & 0 & 1 \end{bmatrix} :$$

$$gulCT(u, v) := R(v) \cdot redC(u) :$$

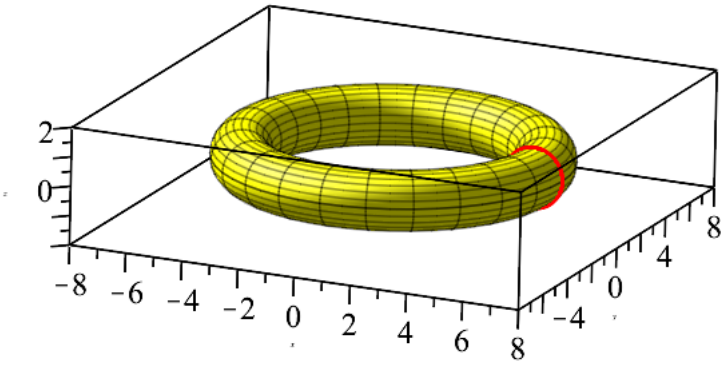
$$gulCT(u, v) = \begin{bmatrix} \cos(v)\ (r\cos(u) + c) \\ \sin(v)\ (r\cos(u) + c) \\ r\sin(u) \end{bmatrix}$$

Den gule cirkulære torus tegnes:

$GgulCT := plot3d(subs(r = 1, c = 5, gulCT(u, v)), u = 0..2\cdot\pi, v = 0..2\cdot\pi, labels = [x, y, z], scaling = constrained, color = yellow, view = [-8..8, -8..8, -2..2])$



display(*GredC*, *GgulCT*)



Elliptisk torus (cirkel kører i en ellipse med centrum i origo)

Parametrisering for ellipse:

https://en.wikipedia.org/wiki/Ellipse#Standard_parametric_representation

Using trigonometric functions, a parametric representation of the standard ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is:

$$(x,\,y)=(a\cos t,\,b\sin t),\ 0\le t<2\pi\,.$$

En grøn ellipse:

$greenE(v) := \langle a\cdot\cos(v), b\cdot\sin(v), 0\rangle$:

$$greenE(v)=\begin{bmatrix} a\cos(v)\\ b\sin(v)\\ 0\end{bmatrix}$$

En rød lodret cirkel:

$redC(u) := \langle r\cdot\cos(u), 0, r\cdot\sin(u)\rangle$:

$$redC(u)=\begin{bmatrix} r\cos(u)\\ 0\\ r\sin(u)\end{bmatrix}$$

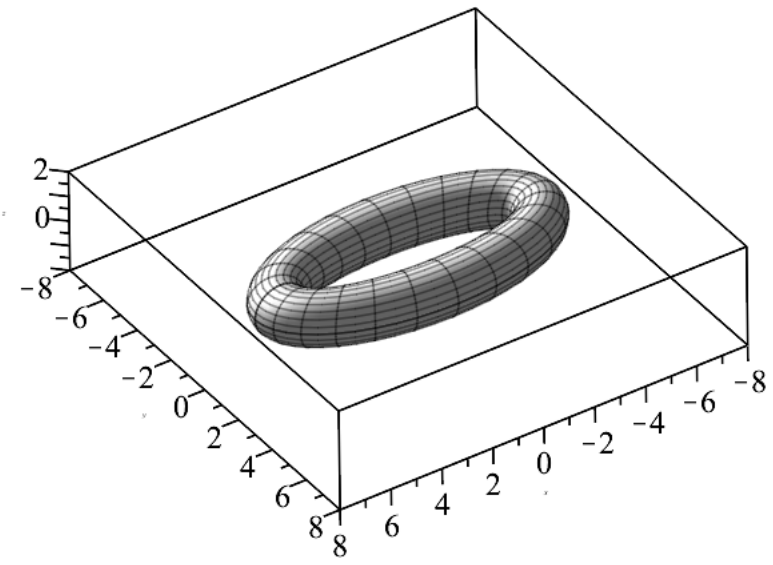
Centrum for den røde cirkel forskydes ud til den grønne ellipse:

$greyET(u,v) := R(v)\cdot redC(u)+greenE(v)$:

$$greyET(u,v)=\begin{bmatrix} \cos(v)\,r\cos(u)+a\cos(v)\\ \sin(v)\,r\cos(u)+b\sin(v)\\ r\sin(u)\end{bmatrix}$$

Den elliptiske torus kan nu tegnes:

$GgreyET := plot3d(subs(r=1, a=5, b=2, greyET(u,v)), v=0..2\cdot\pi, u=0..2\cdot\pi, labels=[x,y,z], scaling=constrained, color=grey, view=[-8..8,-8..8,-2..2])$



Elliptisk torus - hvor cirkelstørrelsen afhænger af afstanden til brændpunktet

Den elliptiske torus liggende ned

https://en.wikipedia.org/wiki/Ellipse#Standard_parametric_representation

Assuming $a \geq b$, the foci are $(\pm c, 0)$ for $c = \sqrt{a^2 - b^2}$.

Brændpunktet for ellipsen placeres i på x-aksens positive del. Mere præcist i $\left(\sqrt{a^2-b^2}, 0, 0\right)$.

$$FP := \left\langle \sqrt{a^2-b^2}, 0, 0 \right\rangle = \begin{bmatrix} \sqrt{a^2-b^2} \\ 0 \\ 0 \end{bmatrix}$$

Afstanden fra brændpunktet FP til ellipsepunktet $greenE(v)$, som er cenrtrum for den røde cirkel, er givet ved $|greenE(v) - FP|$.

$$afstand(v) := \frac{LinearAlgebra[Norm](greenE(v) - FP, 2)}{a - \sqrt{a^2-b^2}} :$$

$$afstand(v) = \frac{\sqrt{\left|a\cos(v) - \sqrt{a^2-b^2}\right|^2 + |b\sin(v)|^2}}{a - \sqrt{a^2-b^2}}$$

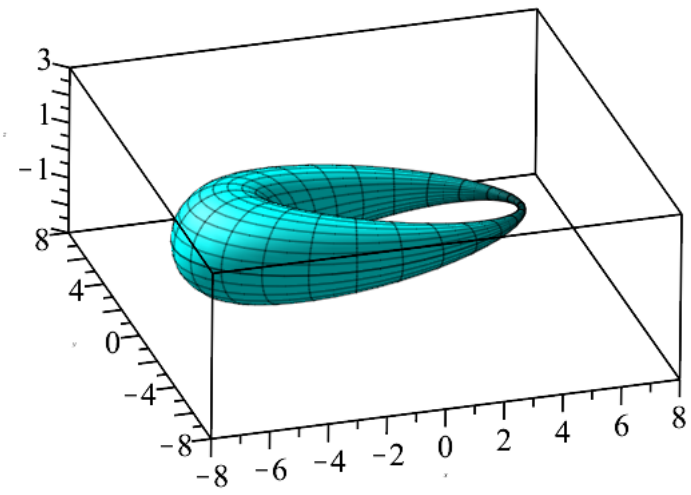
$$greyET(u, v) = \begin{bmatrix} \cos(v)\, r\cos(u) + a\cos(v) \\ \sin(v)\, r\cos(u) + b\sin(v) \\ r\sin(u) \end{bmatrix}$$

$$r := \frac{1}{10} \cdot afstand(v) :$$

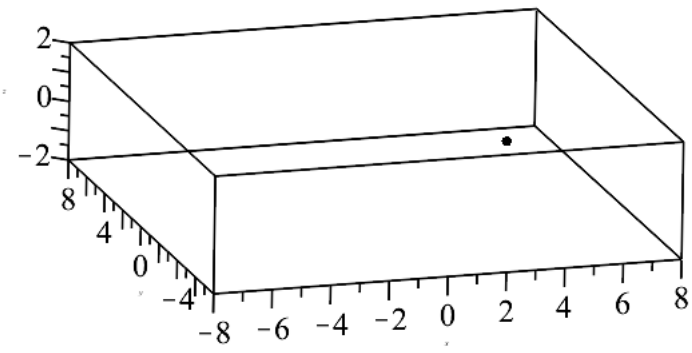
$$cyanET(u, v) := greyET(u, v) :$$

$$cyanET(u, v) = \begin{bmatrix} \frac{\cos(v)\sqrt{\left|a\cos(v) - \sqrt{a^2-b^2}\right|^2 + |b\sin(v)|^2}\cos(u)}{10\left(a - \sqrt{a^2-b^2}\right)} + a\cos(v) \\ \frac{\sin(v)\sqrt{\left|a\cos(v) - \sqrt{a^2-b^2}\right|^2 + |b\sin(v)|^2}\cos(u)}{10\left(a - \sqrt{a^2-b^2}\right)} + b\sin(v) \\ \frac{\sqrt{\left|a\cos(v) - \sqrt{a^2-b^2}\right|^2 + |b\sin(v)|^2}\sin(u)}{10\left(a - \sqrt{a^2-b^2}\right)} \end{bmatrix}$$

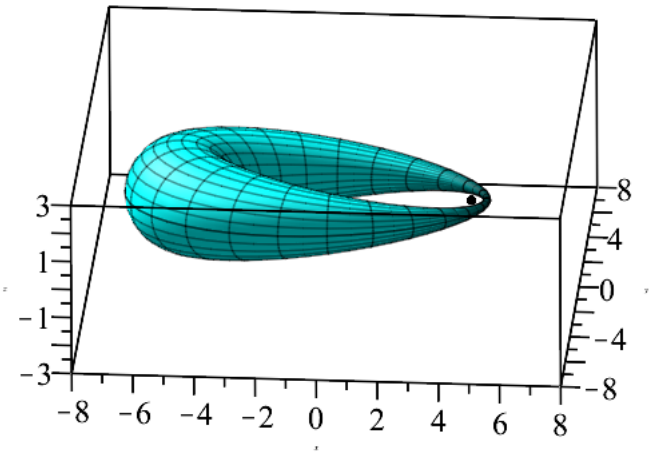
$$GcyanET := plot3d\left(subs(a = 5, b = 2.2, cyanET(u, v)), v = 0..2\cdot\pi, u = 0..2\cdot\pi, labels = [x, y, z], scaling = constrained, color = cyan, view = [-8..8, -8..8, -3..3]\right)$$



$focus := pointplot3d(\{subs(a=5, b=2.2, FP)\}, symbol=solidsphere, symbolsize=10, color=black, labels=[x, y, z], scaling=constrained, view=[-8..8, -8..8, -2..2])$



$display(focus, GcyanET)$



Den elliptiske torus orienteret lodret som museet i Dubai

Rotation om x-aksen:
https://en.wikipedia.org/wiki/Rotation_matrix#In_three_dimensions

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Hertil drejes figuren 90° om x-aksen:

$$R_x(\theta) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} :$$

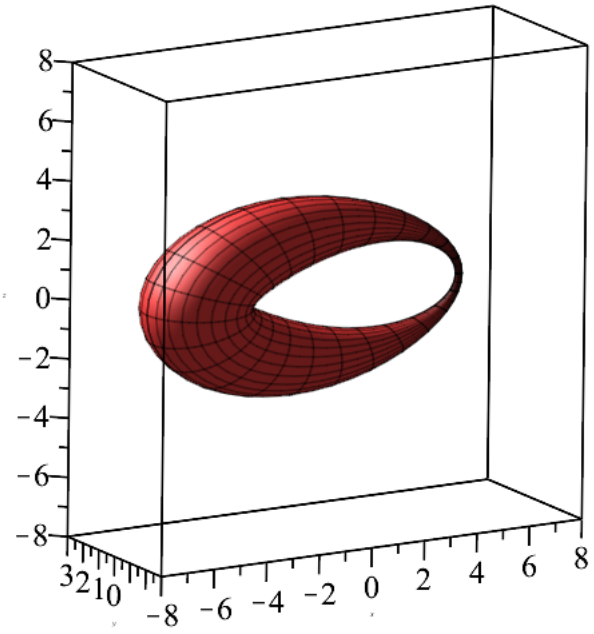
$$R_x\left(\frac{\pi}{2}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$orangeET(u, v) := R_x\left(\frac{\pi}{2}\right) \cdot cyanET(u, v) :$$

$$orangeET(u, v) =$$

$$\left[\begin{array}{c} \dfrac{\cos(v)\sqrt{\left|a\cos(v)-\sqrt{a^2-b^2}\right|^2+\left|b\sin(v)\right|^2}\cos(u)}{10\left(a-\sqrt{a^2-b^2}\right)}+a\cos(v) \\ -\dfrac{\sqrt{\left|a\cos(v)-\sqrt{a^2-b^2}\right|^2+\left|b\sin(v)\right|^2}\sin(u)}{10\left(a-\sqrt{a^2-b^2}\right)} \\ \dfrac{\sin(v)\sqrt{\left|a\cos(v)-\sqrt{a^2-b^2}\right|^2+\left|b\sin(v)\right|^2}\cos(u)}{10\left(a-\sqrt{a^2-b^2}\right)}+b\sin(v) \end{array}\right]$$

$GorangeET := plot3d(subs(a=5, b=2.2, orangeET(u, v)), v=0..2\cdot\pi, u=0..2\cdot\pi, labels=[x, y, z], scaling=constrained, color=orange, view=[-8..8, -3..3, -8..8])$



Fremstilling af STL-filen "dubai.stl":

$Export(\text{"dubai.stl"}, GorangeET, base=homedir) = 230484$

NB: Filen ligger i brugerens mappe i Windows (f.eks. "C:\Users\Steen").