

Kleins flaske

Med parametrisering 1

https://en.wikipedia.org/wiki/Klein_bottle#Bottle_shape

restart
with(plots) :

Parametrisering (fra Wikipedia):

$$\begin{aligned}
 x(u, v) &= -\frac{2}{15} \cos u (3 \cos v - 30 \sin u + 90 \cos^4 u \sin u \\
 &\quad - 60 \cos^6 u \sin u + 5 \cos u \cos v \sin u) \\
 y(u, v) &= -\frac{1}{15} \sin u (3 \cos v - 3 \cos^2 u \cos v - 48 \cos^4 u \cos v + 48 \cos^6 u \\
 &\quad \cos v - 60 \sin u + 5 \cos u \cos v \sin u - 5 \cos^3 u \cos v \sin u - 80 \\
 &\quad \cos^5 u \cos v \sin u + 80 \cos^7 u \cos v \sin u) \\
 z(u, v) &= \frac{2}{15} (3 + 5 \cos u \sin u) \sin v
 \end{aligned}$$

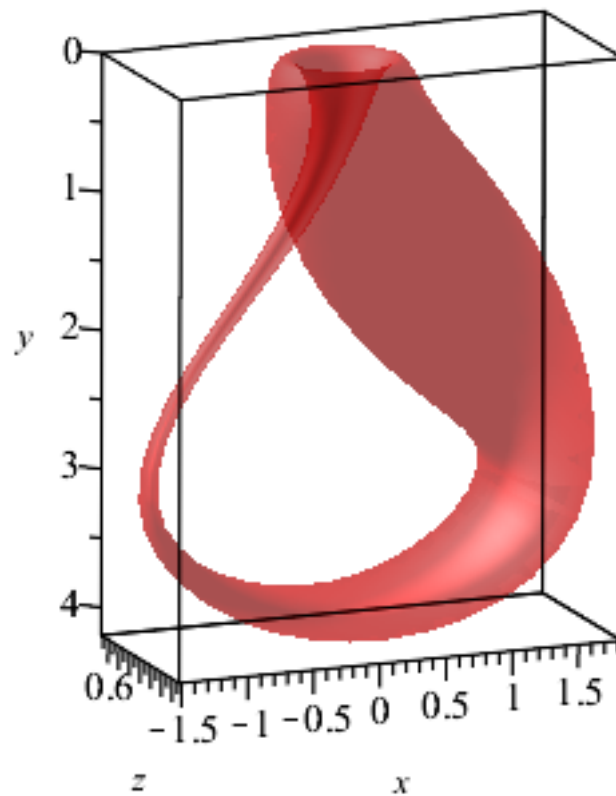
for $0 \leq u < \pi$ and $0 \leq v < 2\pi$.

Implementering af formlerne:

$$\begin{aligned}
 x(u, v) &:= -\frac{2}{15} \cdot \cos(u) \cdot (3 \cdot \cos(v) - 30 \cdot \sin(u) + 90 \cdot (\cos(u))^4 \cdot \sin(u) - 60 \cdot (\cos(u))^6 \cdot \sin(u) + 5 \\
 &\quad \cdot \cos(u) \cdot \cos(v) \cdot \sin(u)) : \\
 y(u, v) &:= -\frac{1}{15} \cdot \sin(u) \cdot (3 \cdot \cos(v) - 3 \cdot (\cos(u))^2 \cdot \cos(v) - 48 \cdot (\cos(u))^4 \cdot \cos(v) + 48 \cdot (\cos(u))^6 \\
 &\quad \cdot \cos(v) - 60 \cdot \sin(u) + 5 \cdot \cos(u) \cdot \cos(v) \cdot \sin(u) - 5 \cdot (\cos(u))^3 \cdot \cos(v) \cdot \sin(u) - 80 \cdot (\cos(u))^5 \\
 &\quad \cdot \cos(v) \cdot \sin(u) + 80 \cdot (\cos(u))^7 \cdot \cos(v) \cdot \sin(u)) : \\
 z(u, v) &:= \frac{2}{15} \cdot (3 + 5 \cdot \cos(u) \cdot \sin(u)) \cdot \sin(v) :
 \end{aligned}$$

$$p1(u, v) := \langle x(u, v), y(u, v), z(u, v) \rangle :$$

$$\begin{aligned}
 \text{klein1} &:= \text{plot3d}(p1(u, v), u=0..2 \cdot \pi, v=0..2 \cdot \pi, \text{color}=\text{red}, \text{labels}=[x, y, z], \text{numpoints}=10000, \\
 &\quad \text{scaling}=\text{constrained}, \text{transparency}=0.75, \text{style}=\text{patchnograd})
 \end{aligned}$$

**Fremstilling af STL-fil:**

```
Export("klein1.stl", klein1, base = homedir) = 1000084
```

▼ Med parametrisering 2

<http://paulbourke.net/geometry/klein/>

```
restart  
with(plots) :
```

Parametrisering fra kilden:

$$v = 0 \rightarrow 2\pi$$

$$u = 0 \rightarrow 2\pi$$

$$r = 4(1 - \cos(u) / 2)$$

$$x = \begin{cases} 6 \cos(u) (1 + \sin(u)) + r \cos(u) \cos(v) & 0 \leq u < \pi \\ 6 \cos(u) (1 + \sin(u)) + r \cos(v + \pi) & \pi < u \leq 2\pi \end{cases}$$

$$y = \begin{cases} 16 \sin(u) + r \sin(u) \cos(v) & 0 \leq u < \pi \\ 16 \sin(u) & \pi < u \leq 2\pi \end{cases}$$

$$z = r \sin(v)$$

Implementering af formlerne:

$$r := 4 \cdot \left(1 - \frac{\cos(u)}{2}\right) :$$

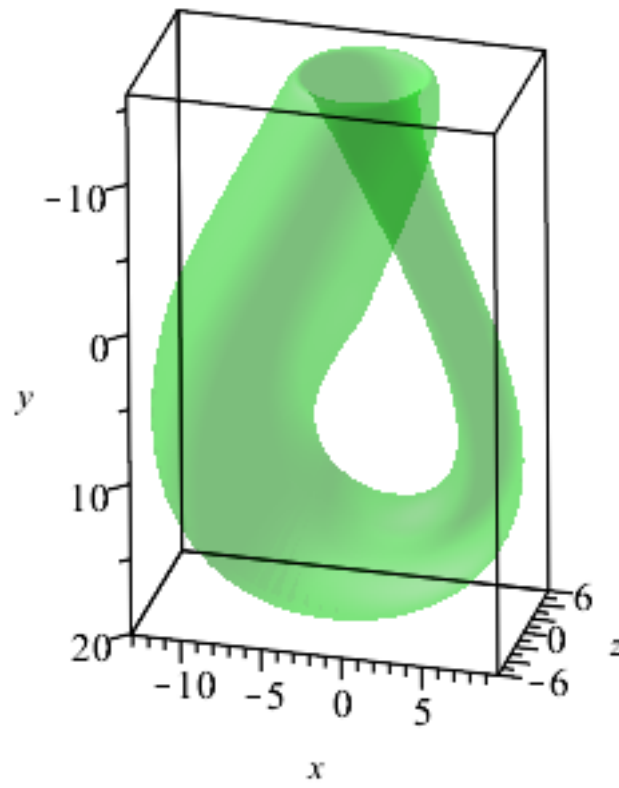
$$x(u, v) := \begin{cases} 6 \cdot \cos(u) \cdot (1 + \sin(u)) + r \cdot \cos(u) \cdot \cos(v) & 0 \leq u \leq \pi \\ 6 \cdot \cos(u) \cdot (1 + \sin(u)) + r \cdot \cos(v + \pi) & \pi < u \leq 2 \cdot \pi \end{cases} :$$

$$y(u, v) := \begin{cases} 16 \cdot \sin(u) + r \cdot \sin(u) \cdot \cos(v) & 0 \leq u \leq \pi \\ 16 \cdot \sin(u) & \pi < u \leq 2 \cdot \pi \end{cases} :$$

$$z(u, v) := r \cdot \sin(v) :$$

$$p2(u, v) := \langle x(u, v), y(u, v), z(u, v) \rangle :$$

$$\text{klein2} := \text{plot3d}(p2(u, v), u = 0 .. 2 \cdot \pi, v = 0 .. 2 \cdot \pi, \text{color} = \text{green}, \text{labels} = [x, y, z], \text{numpoints} = 10000, \text{scaling} = \text{constrained}, \text{transparency} = 0.70, \text{style} = \text{patchnograd})$$

**Fremstilling af STL-fil:**

`Export("klein2.stl", klein2, base = homedir)` [1000084](#)