

Möbius bånd med tykkelse

https://en.wikipedia.org/wiki/M%C3%B6bius_strip

Ønsker at lave et Möbius bånd med en vis tykkelse, så det kan 3D-printes.

Rotation om z-aksen:

https://en.wikipedia.org/wiki/Rotation_matrix#In_three_dimensions

restart

with(plots) :

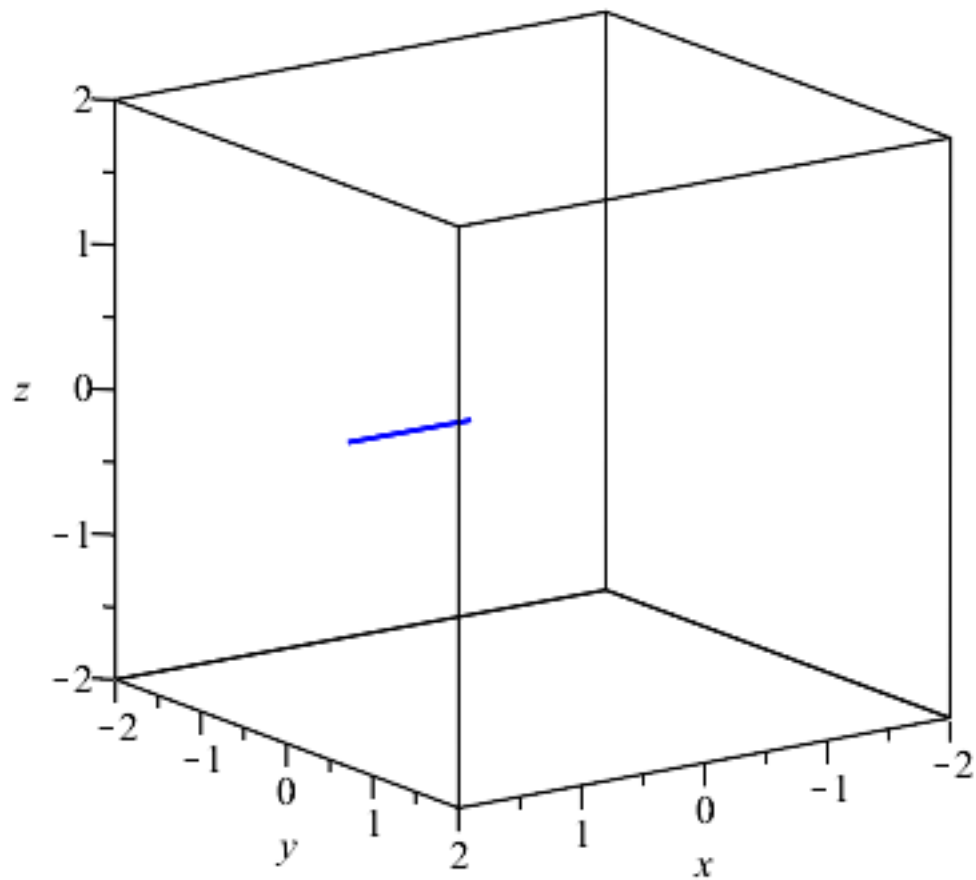
$$Rz(\theta) := \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} :$$

▼ Et fladt Möbius bånd

Et linjestykke i xz-planen:

$$l(u, \theta) := \langle 1, 0, 0 \rangle + u \cdot \left\langle \cos\left(\frac{\theta}{2}\right), 0, \sin\left(\frac{\theta}{2}\right) \right\rangle :$$

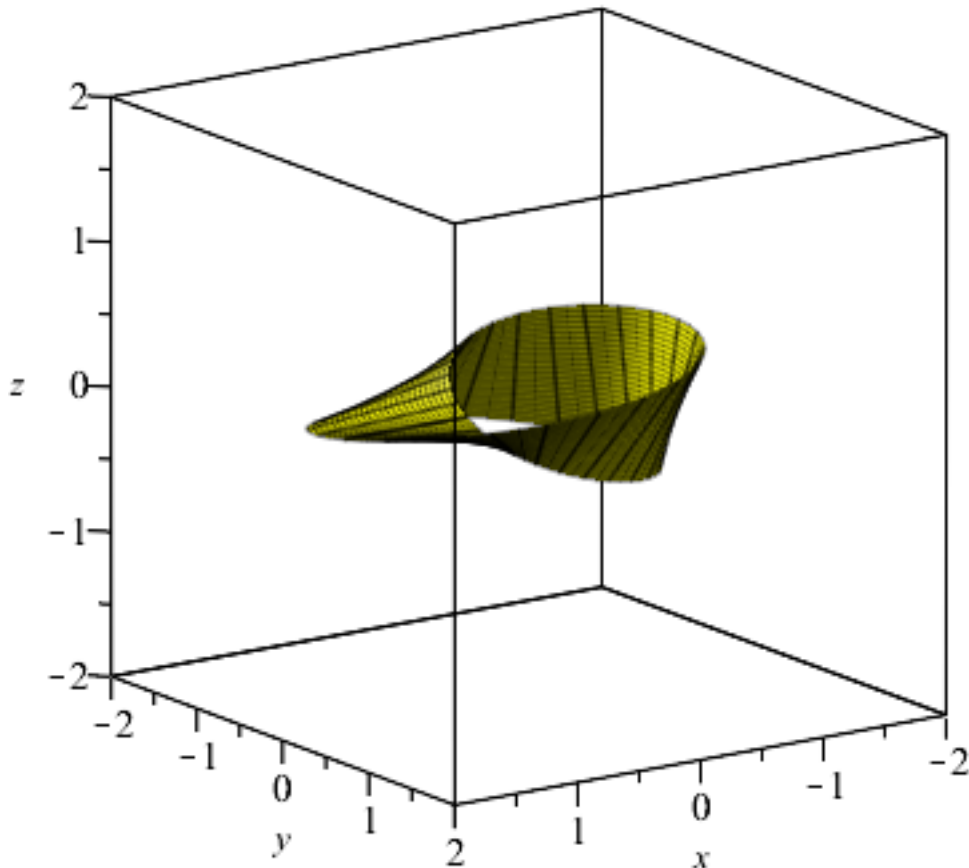
spacecurve(l(u, 0), u = -0.5 .. 0.5, view = [-2 .. 2, -2 .. 2, -2 .. 2], color = blue, labels = [x, y, z])



$b(u, \theta) := Rz(\theta) \cdot l(u, \theta) :$

$$b(u, \theta) = \begin{bmatrix} \cos(\theta) \left(1 + u \cos\left(\frac{\theta}{2}\right) \right) \\ \sin(\theta) \left(1 + u \cos\left(\frac{\theta}{2}\right) \right) \\ u \sin\left(\frac{\theta}{2}\right) \end{bmatrix}$$

$M := \text{plot3d}(b(u, \theta), u = -0.5 .. 0.5, \theta = 0 .. 2 \cdot \pi, \text{view} = [-2 .. 2, -2 .. 2, -2 .. 2], \text{color} = \text{yellow}, \text{labels} = [x, y, z])$



Det ser OK ud, men er for skrøbeligt at 3D-printe!

Et rektangel, som køres rundt

`assume(u, real) : assume(v, real) : assume(theta, real) :`
`interface(showassumed=0) :`

Tykkelsen til linjen $l(u, \theta)$ laves ved at addere et tal gange tværvektoren i xz-planen på:

$$r(u, v, \theta) := \left(\langle 1, 0, 0 \rangle + u \cdot \left\langle \cos\left(\frac{\theta}{2}\right), 0, \sin\left(\frac{\theta}{2}\right) \right\rangle + v \cdot \left\langle -\sin\left(\frac{\theta}{2}\right), 0, \cos\left(\frac{\theta}{2}\right) \right\rangle \right) :$$

$$R(u, v, \theta) := Rz(\theta).r(u, v, \theta) :$$

$$R(u, v, \theta) = \begin{bmatrix} \cos(\theta) \left(1 + u \cos\left(\frac{\theta}{2}\right) - v \sin\left(\frac{\theta}{2}\right) \right) \\ \sin(\theta) \left(1 + u \cos\left(\frac{\theta}{2}\right) - v \sin\left(\frac{\theta}{2}\right) \right) \\ u \sin\left(\frac{\theta}{2}\right) + v \cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

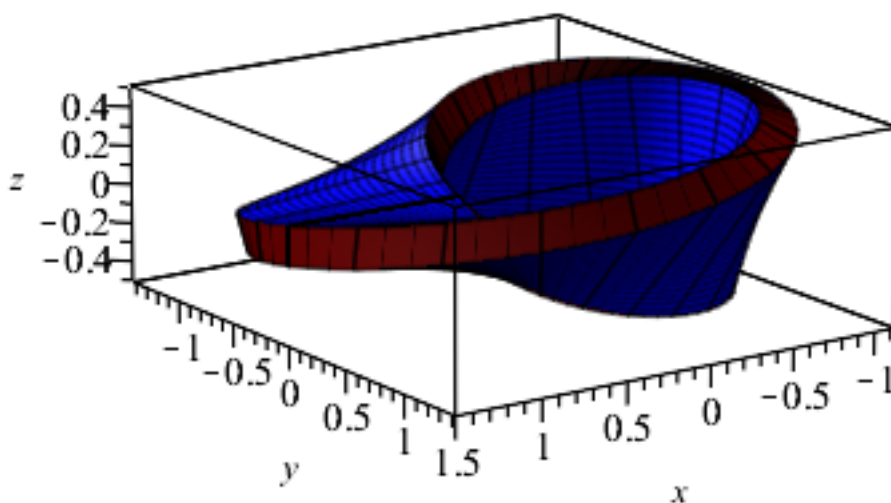
Plottet med Integrator8-pakken fra DTU:

`with(Integrator8) :`

```

B := [-0.5, 0.5, -0.1, 0.1, 0, 2·π]:
net := [20, 20, 20]:
MÖBIUS := sideFlader(R, B, net):
display(MÖBIUS, labels = [x, y, z], axes = box, scaling = constrained)

```



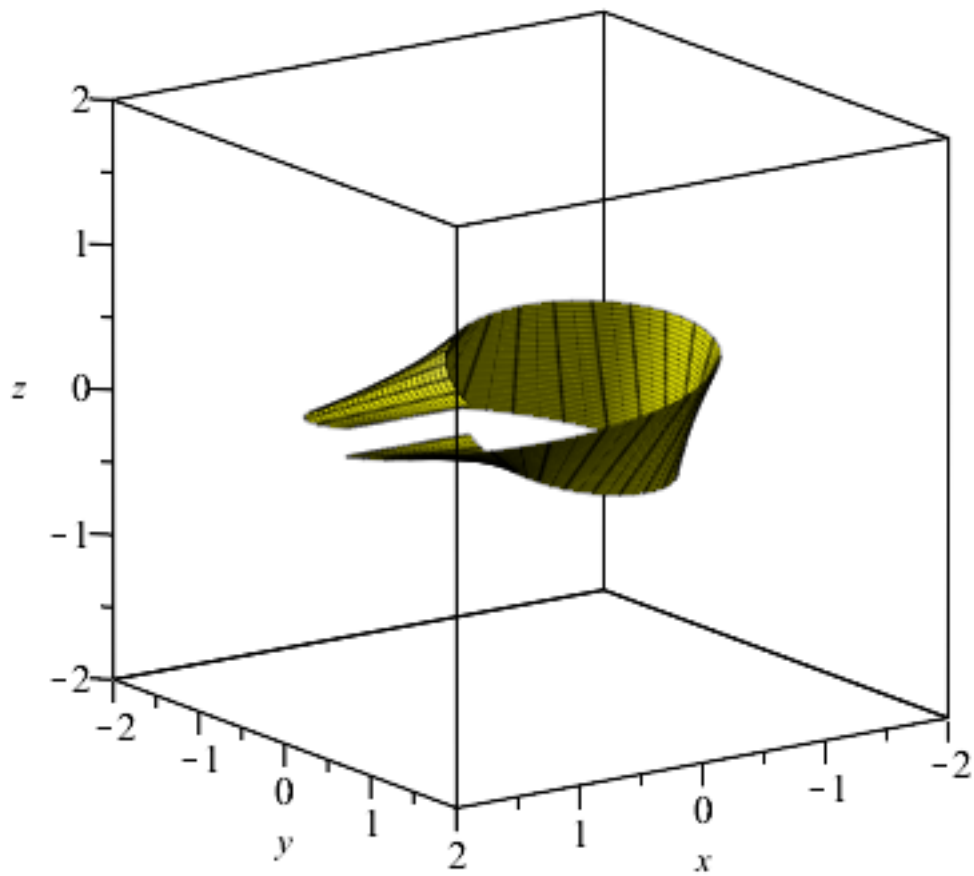
Det ser OK ud, men hvis man ikke har Integrator8-pakken, og gerne selv vil producere den samlede figur?

Plottet i 4 dele med standard "plot3d":

```

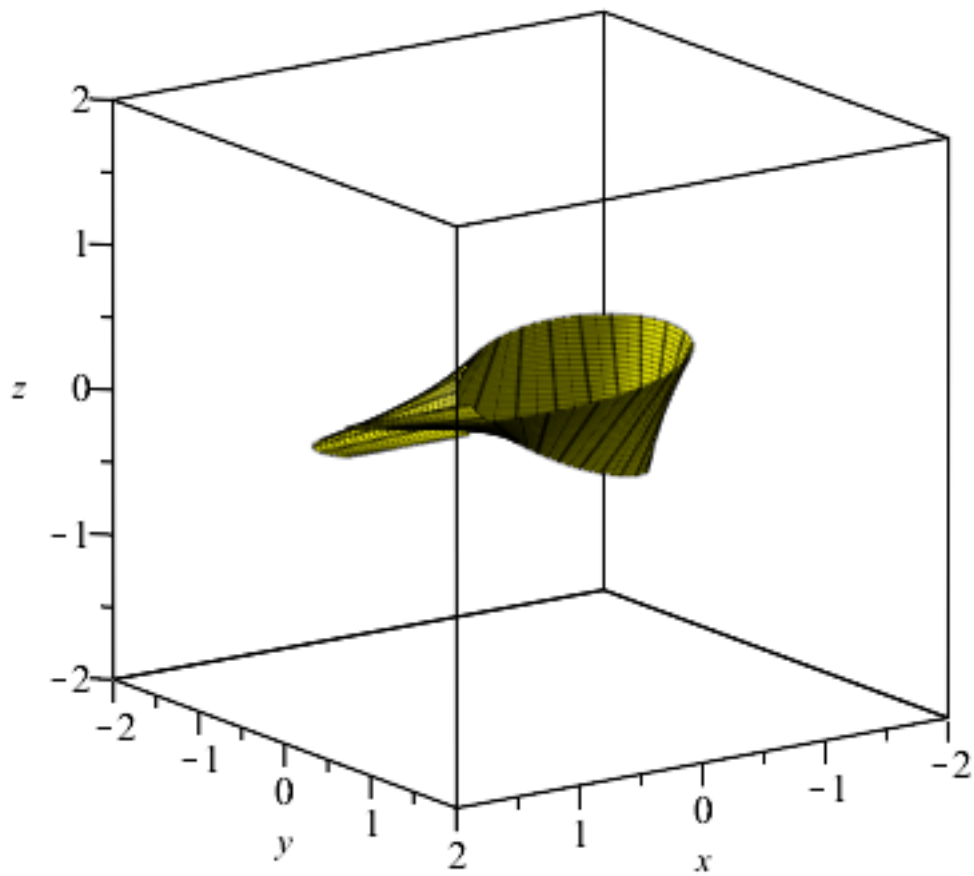
M1 := plot3d(R(u, -0.1, θ), u = -0.5 .. 0.5, θ = 0 .. 2·π, view = [-2 .. 2, -2 .. 2, -2 .. 2], color
= yellow, labels = [x, y, z])

```



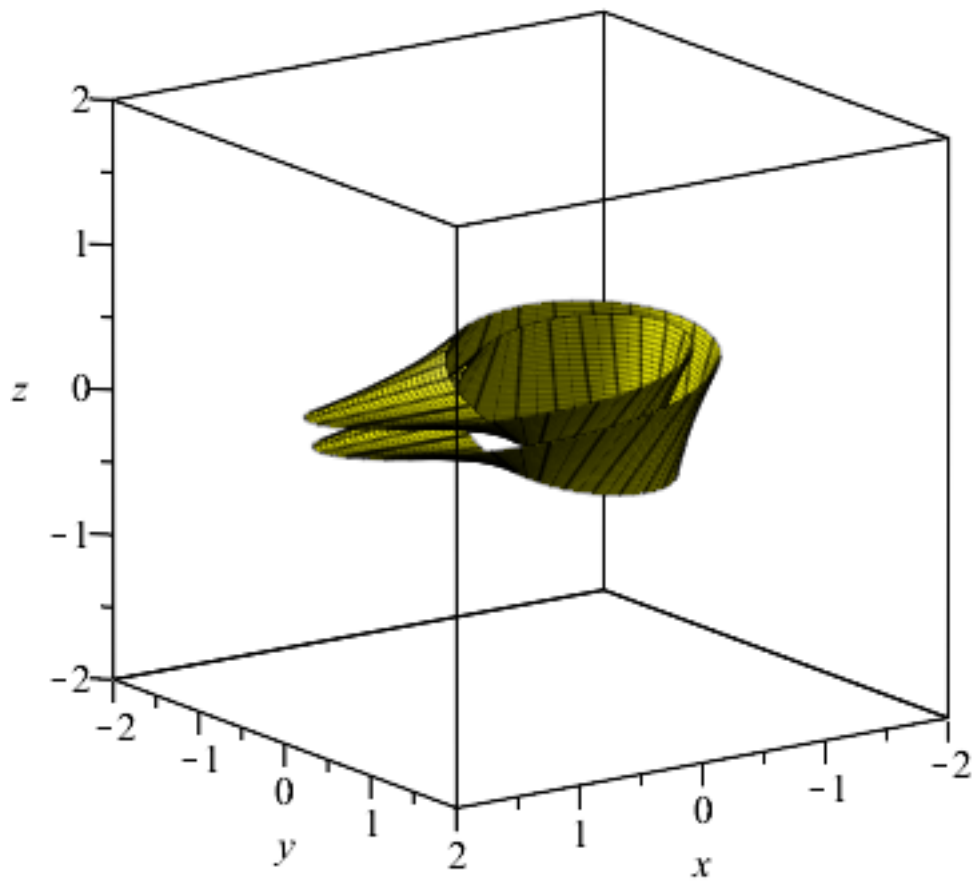
NB: Båndet hænger ikke sammen!

```
M2 := plot3d(R(u, 0.1, θ), u = -0.5 .. 0.5, θ = 0 .. 2 · π, view = [-2 .. 2, -2 .. 2, -2 .. 2], color = yellow, labels = [x, y, z])
```



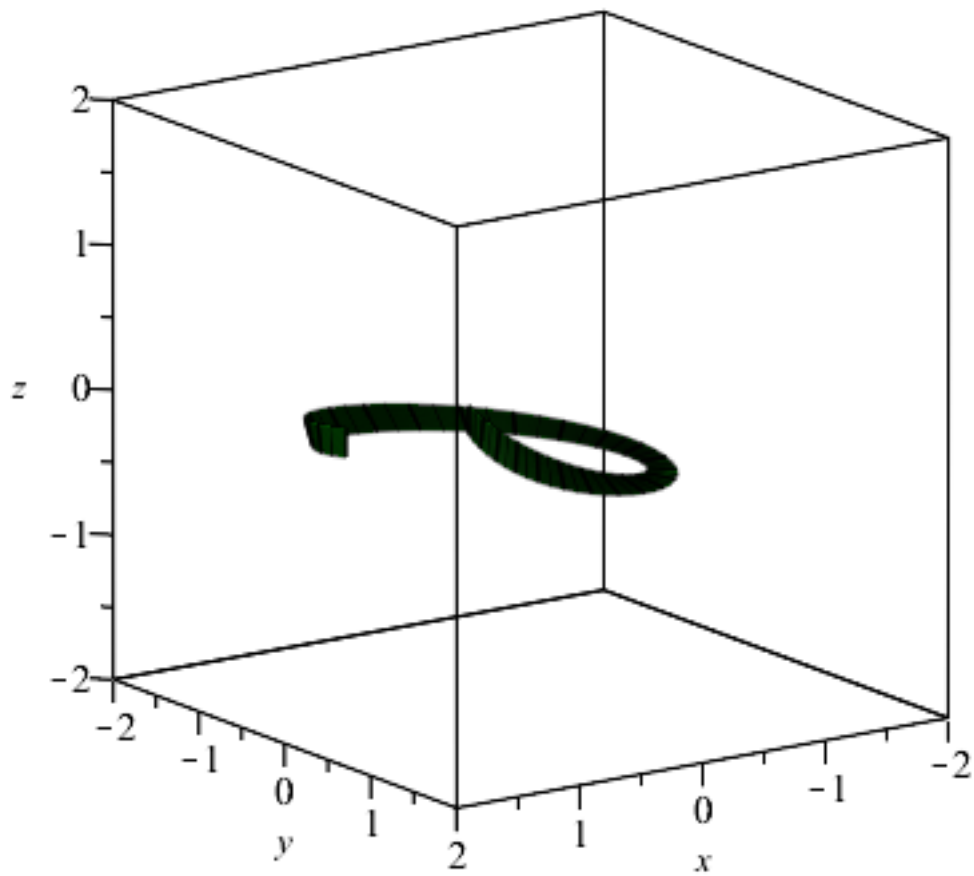
NB: Båndet hænger ikke sammen!

`display(M1, M2)`



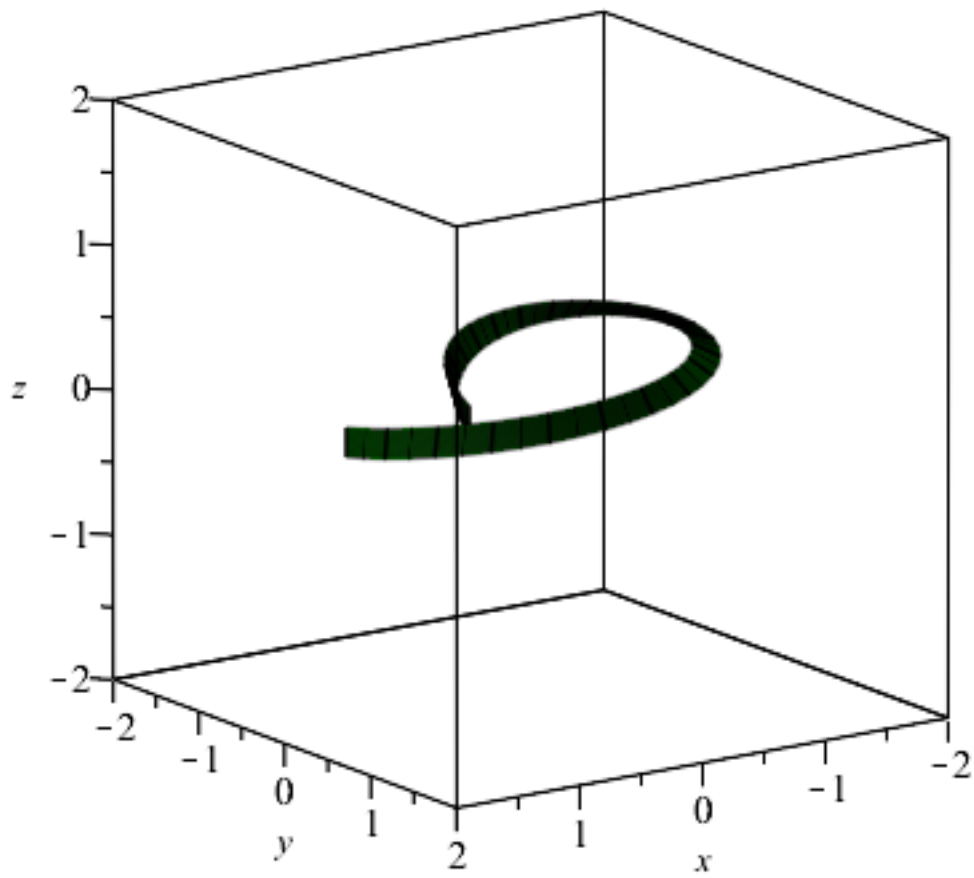
NB: Der er dannet ét sammenhængende bånd!

```
M3 := plot3d(R(-0.5, v, θ), v=-0.1..0.1, θ=0..2·π, view=[-2..2,-2..2,-2..2], color=green, labels=[x,y,z])
```



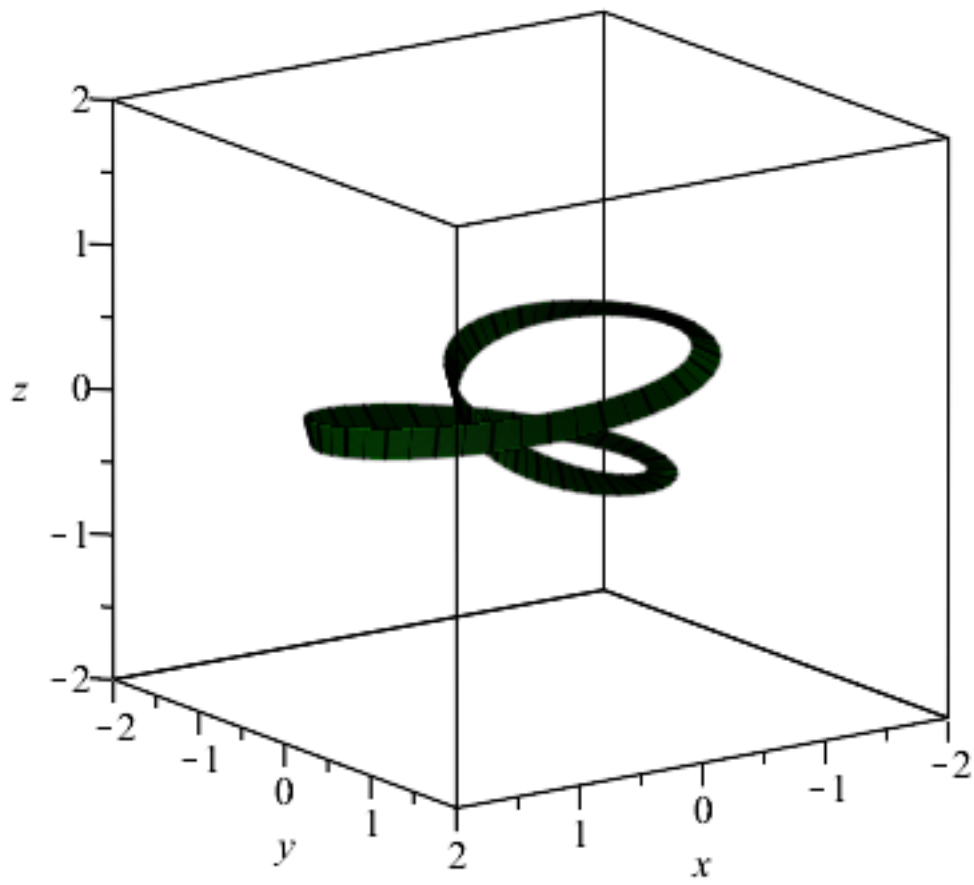
NB: kanten hænger ikke sammen.

```
M4 := plot3d(R(0.5, v, θ), v=-0.1..0.1, θ=0.. 2·π, view=[-2..2,-2..2,-2..2], color  
= green, labels=[x, y, z])
```

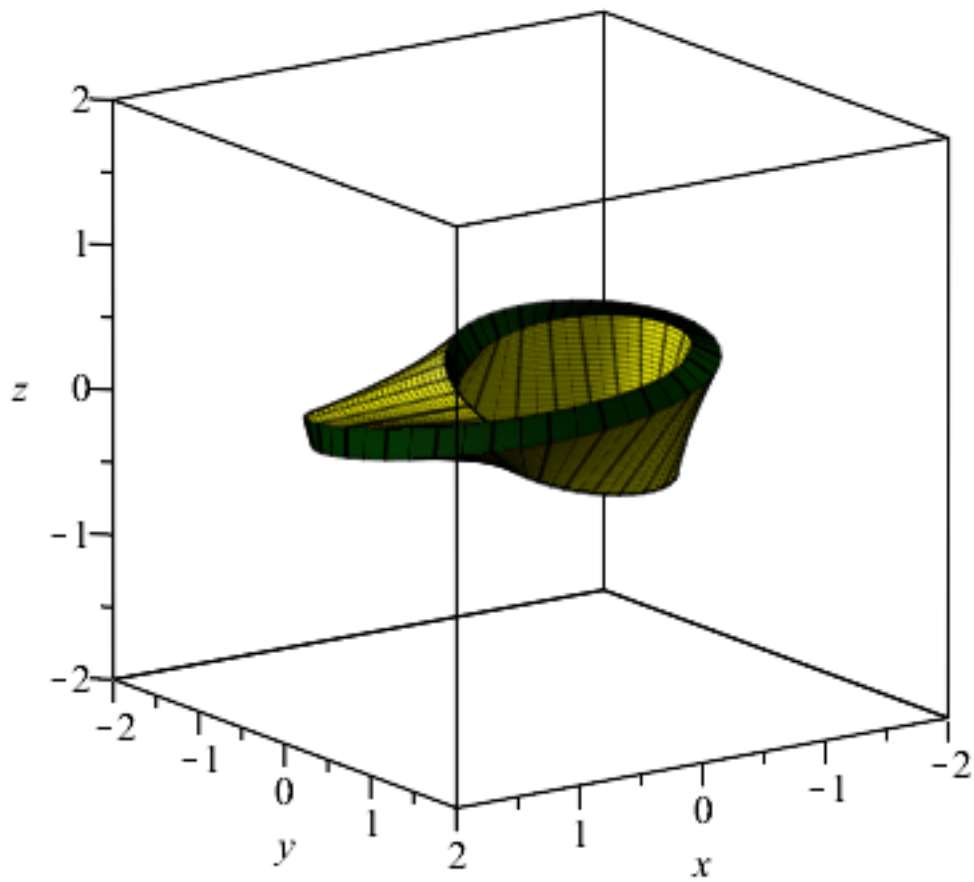
NB: kanten hænger ikke sammen.

display(M3, M4)



NB: Der er dannet én sammenhængende kant!

möbius := display(M1, M2, M3, M4)



NB: Der er dannet et Möbius bånd med en vis tykkelse, som egner sig til 3D-printning!

Eksportering til en STL-fil til 3D-printning:

Export("mobius.stl", möbius, base = homedir)

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(2.1)