

Operaen i Beijing

<https://www.archdaily.com/1218/national-grand-theater-of-china-paul-andreu>



restart

with(plots) :

with(Integrator8) :

with(VektorAnalyse2) :

with(plots) :

<https://en.wikipedia.org/wiki/Superellipsoid>

$$\left(\left| \frac{x}{A} \right|^r + \left| \frac{y}{B} \right|^r \right)^{t/r} + \left| \frac{z}{C} \right|^t \leq 1.$$

Setting $r = 2$, $t = 2.5$, $A = B = 3$, $C = 4$ one obtains Piet Hein's superegg.

The general superellipsoid has a parametric representation in terms of surface parameters $-\pi/2 < v < \pi/2$, $-\pi < u < \pi$.^[3]

$$x(u, v) = Ac \left(v, \frac{2}{t} \right) c \left(u, \frac{2}{r} \right)$$

$$y(u, v) = Bc \left(v, \frac{2}{t} \right) s \left(u, \frac{2}{r} \right)$$

$$z(u, v) = Cs \left(v, \frac{2}{t} \right)$$

where the auxiliary functions are

$$c(\omega, m) = \operatorname{sgn}(\cos \omega) |\cos \omega|^m$$

$$s(\omega, m) = \operatorname{sgn}(\sin \omega) |\sin \omega|^m$$

and the sign function $\operatorname{sgn}(x)$ is

$$\operatorname{sgn}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ +1, & x > 0. \end{cases}$$

The volume inside this surface can be expressed in terms of beta functions (and Gamma functions, because $\beta(m, n) = \Gamma(m)\Gamma(n) / \Gamma(m + n)$), as:

$$V = \frac{2}{3} ABC \frac{4}{rt} \beta \left(\frac{1}{r}, \frac{1}{r} \right) \beta \left(\frac{2}{t}, \frac{1}{t} \right).$$

Ønsker den øvre halve super ellipsoide!

$assume\left(0 \leq u \leq \pi, -\frac{\pi}{2} \leq v \leq \frac{\pi}{2}\right) : interface(showassumed=0) :$

$r := 2 : t := \frac{5}{2} : A := \frac{213}{2} : B := 46 : C := \frac{144}{2} :$

$$\frac{2}{t} = \frac{4}{5}$$

$$\frac{2}{r} = 1$$

$$sgn(x) := \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} :$$

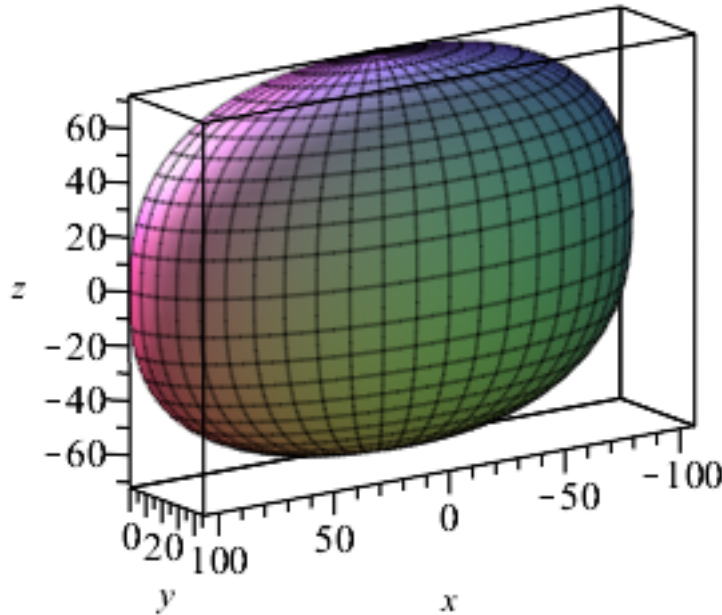
$c(w, m) := sgn(\cos(w)) \cdot |\cos(w)|^m :$

$s(w, m) := sgn(\sin(w)) \cdot |\sin(w)|^m :$

$p(u, v) := \left\langle A \cdot c\left(v, \frac{2}{t}\right) \cdot c\left(u, \frac{2}{r}\right), B \cdot c\left(v, \frac{2}{t}\right) \cdot s\left(u, \frac{2}{r}\right), C \cdot s\left(v, \frac{2}{t}\right) \right\rangle :$

$$p(u, v) = \left[\begin{array}{c} 213 \left(\begin{array}{l} \left(\begin{array}{l} 1 & 0 < \cos(v) \\ 0 & \cos(v) = 0 \\ -1 & \cos(v) < 0 \end{array} \right) |\cos(v)|^{4/5} \\ \left(\begin{array}{l} 1 & 0 < \cos(u) \\ 0 & \cos(u) = 0 \\ -1 & \cos(u) < 0 \end{array} \right) |\cos(u)| \end{array} \right) \\ \hline 2 \\ 46 \left(\begin{array}{l} \left(\begin{array}{l} 1 & 0 < \cos(v) \\ 0 & \cos(v) = 0 \\ -1 & \cos(v) < 0 \end{array} \right) |\cos(v)|^{4/5} \\ \left(\begin{array}{l} 1 & 0 < \sin(u) \\ 0 & \sin(u) = 0 \\ -1 & \sin(u) < 0 \end{array} \right) |\sin(u)| \end{array} \right) \\ 72 \left(\begin{array}{l} \left(\begin{array}{l} 1 & 0 < \sin(v) \\ 0 & \sin(v) = 0 \\ -1 & \sin(v) < 0 \end{array} \right) |\sin(v)|^{4/5} \end{array} \right) \end{array} \right]$$

$plot3d\left(p(u, v), u=0..\pi, v=-\frac{\pi}{2}..\frac{\pi}{2}, labels=[x, y, z], scaling=constrained\right)$

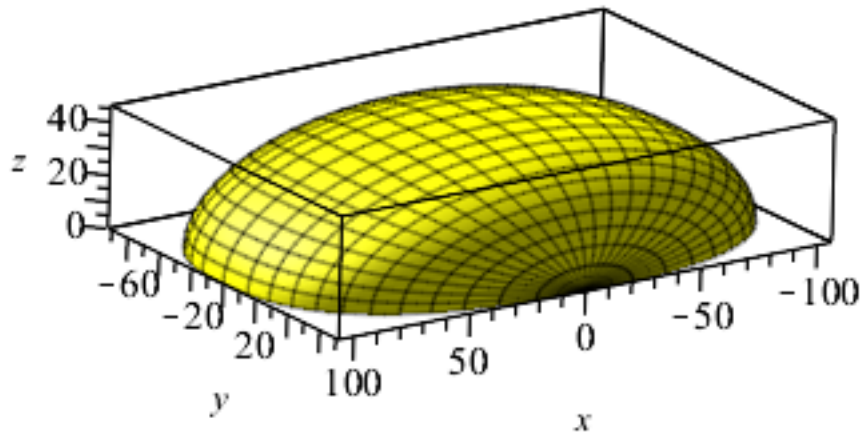


NB: y og z skal ombyttes!

$$p2(u, v) := \left\langle A \cdot c\left(v, \frac{2}{t}\right) \cdot c\left(u, \frac{2}{r}\right), C \cdot s\left(v, \frac{2}{t}\right), B \cdot c\left(v, \frac{2}{t}\right) \cdot s\left(u, \frac{2}{r}\right) \right\rangle :$$

$$p2(u, v) = \left[\begin{array}{c} 213 \left(\begin{array}{l} \left(\begin{array}{ll} 1 & 0 < \cos(v) \\ 0 & \cos(v) = 0 \\ -1 & \cos(v) < 0 \end{array} \right) |\cos(v)|^{4/5} \\ \left(\begin{array}{ll} 1 & 0 < \cos(u) \\ 0 & \cos(u) = 0 \\ -1 & \cos(u) < 0 \end{array} \right) |\cos(u)| \end{array} \right) \\ \hline 2 \\ 72 \left(\begin{array}{l} \left(\begin{array}{ll} 1 & 0 < \sin(v) \\ 0 & \sin(v) = 0 \\ -1 & \sin(v) < 0 \end{array} \right) |\sin(v)|^{4/5} \\ \left(\begin{array}{ll} 1 & 0 < \sin(u) \\ 0 & \sin(u) = 0 \\ -1 & \sin(u) < 0 \end{array} \right) |\sin(u)| \end{array} \right) \\ 46 \left(\begin{array}{l} \left(\begin{array}{ll} 1 & 0 < \cos(v) \\ 0 & \cos(v) = 0 \\ -1 & \cos(v) < 0 \end{array} \right) |\cos(v)|^{4/5} \\ \left(\begin{array}{ll} 1 & 0 < \sin(u) \\ 0 & \sin(u) = 0 \\ -1 & \sin(u) < 0 \end{array} \right) |\sin(u)| \end{array} \right) \end{array} \right]$$

$$T := \text{plot3d}\left(p2(u, v), u = 0 .. \pi, v = -\frac{\pi}{2} .. \frac{\pi}{2}, \text{labels} = [x, y, z], \text{scaling} = \text{constrained}, \text{color} = \text{yellow}\right)$$



Parametrisering af bunden af halv-super-ellipsoiden:

https://en.wikipedia.org/wiki/Superellipse#Mathematical_properties

$$x(t) = |\cos t|^{\frac{2}{n}} \cdot a \operatorname{sgn}(\cos t)$$

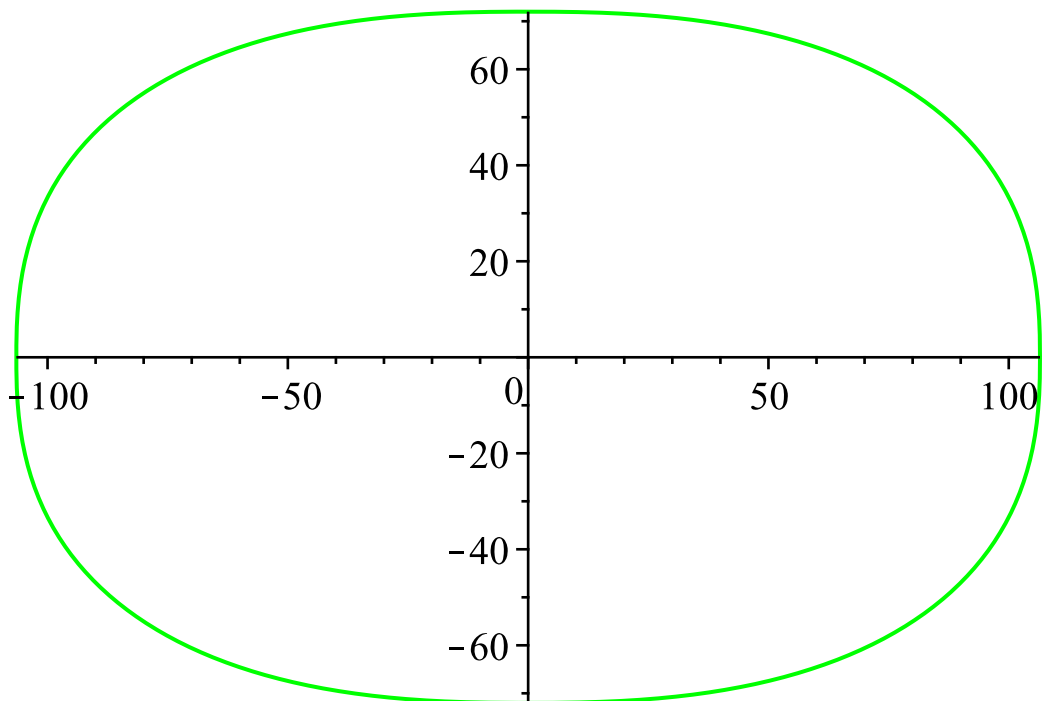
$$y(t) = |\sin t|^{\frac{2}{n}} \cdot b \operatorname{sgn}(\sin t)$$

where the sign function is

$$\operatorname{sgn}(w) = \begin{cases} -1, & w < 0 \\ 0, & w = 0 \\ +1, & w > 0. \end{cases}$$

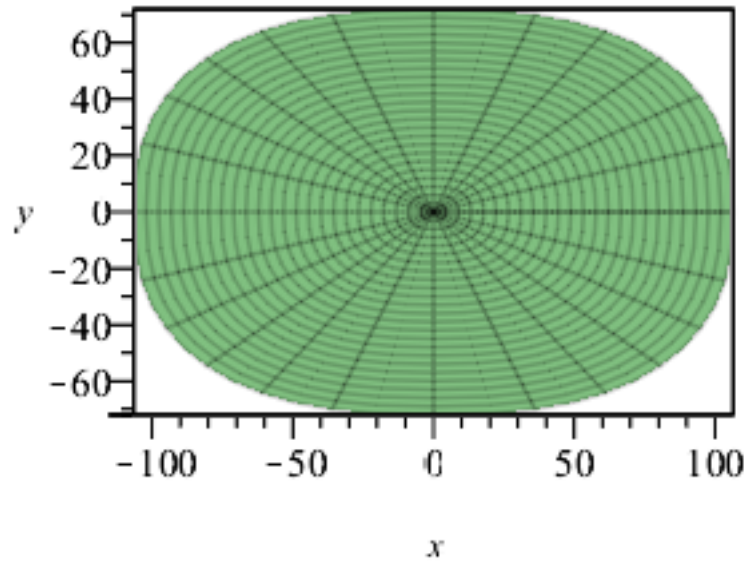
$$p3(u) := \left\langle A \cdot |\cos(u)|^{\frac{4}{5}} \cdot \operatorname{sgn}(\cos(u)), C \cdot |\sin(u)|^{\frac{4}{5}} \cdot \operatorname{sgn}(\sin(u)) \right\rangle :$$

$$\text{plot}([\text{vop}(p3(u)), u = 0 .. 2 \cdot \pi], \text{scaling} = \text{constrained}, \text{color} = \text{green})$$

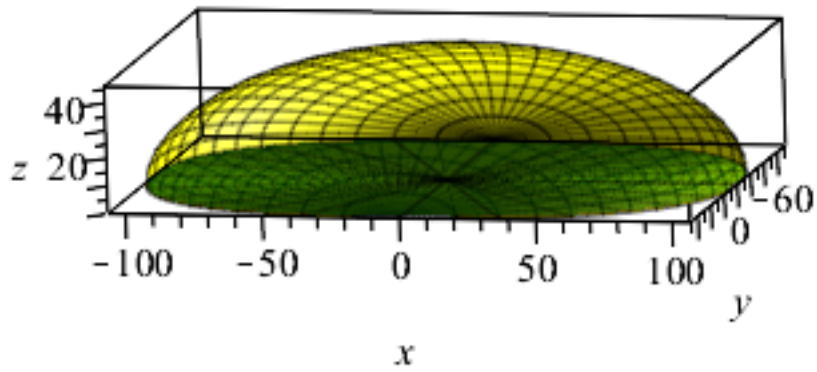


$$p4(u, v) := \left\langle v \cdot A \cdot |\cos(u)|^{\frac{4}{5}} \cdot \text{sgn}(\cos(u)), v \cdot C \cdot |\sin(u)|^{\frac{4}{5}} \cdot \text{sgn}(\sin(u)) \right\rangle :$$

$G := \text{plot3d}(\langle \text{vop}(p4(u, v)), 0 \rangle, u = 0 .. 2 \cdot \pi, v = 0 .. 1, \text{orientation} = [-90, 0], \text{labels} = [x, y, ""], \text{color} = \text{green}, \text{scaling} = \text{constrained}, \text{transparency} = 0.5)$



operaen := display(T, G)

**Fremstilling af STL-fil:**

`Export("operaen.stl", operaen, base = homedir)`

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