Piet Heins superæg

Baggrundsviden

https://en.wikipedia.org/wiki/Superellipsoid

$$\left(\left|\frac{x}{A}\right|^r+\left|\frac{y}{B}\right|^r\right)^{t/r}+\left|\frac{z}{C}\right|^t \leq 1.$$

Setting $r = 2$, $t = 2.5$, $A = B = 3$, $C = 4$ one obtains Piet Hein's superegg.

The general superellipsoid has a parametric representation in terms of surface parameters $-\pi/2 < v < \pi/2$, $-\pi < u < \pi$.[3]

$$x(u,v) = Ac\left(v,\frac{2}{t}\right)c\left(u,\frac{2}{r}\right)$$
$$y(u,v) = Bc\left(v,\frac{2}{t}\right)s\left(u,\frac{2}{r}\right)$$
$$z(u,v) = Cs\left(v,\frac{2}{t}\right)$$

where the auxiliary functions are

$$c(\omega,m) = \operatorname{sgn}(\cos\omega)|\cos\omega|^m$$
$$s(\omega,m) = \operatorname{sgn}(\sin\omega)|\sin\omega|^m$$

and the sign function sgn(x) is

$$\operatorname{sgn}(x) = \begin{cases} -1, & x<0 \\ 0, & x=0 \\ +1, & x>0. \end{cases}$$

The volume inside this surface can be expressed in terms of beta functions (and Gamma functions, because $\beta(m,n) = \Gamma(m)\Gamma(n) / \Gamma(m+n)$), as:

$$V = \frac{2}{3}ABC\frac{4}{rt}\beta\left(\frac{1}{r},\frac{1}{r}\right)\beta\left(\frac{2}{t},\frac{1}{t}\right).$$

Parametrisering af en hel super ellipsoide

restart
with(*plots*) :
with(*Integrator8*) :
with(*VektorAnalyse2*) :
with(*plots*) :

$$r := 2 : t := \frac{5}{2} : A := 3 : B := 3 : C := 4 :$$

$$\frac{2}{t} = \frac{4}{5}$$

$$\frac{2}{r} = 1$$

$$sgn(x) := \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} :$$

$$c(w, m) := sgn(\cos(w))\cdot|\cos(w)|^m :$$

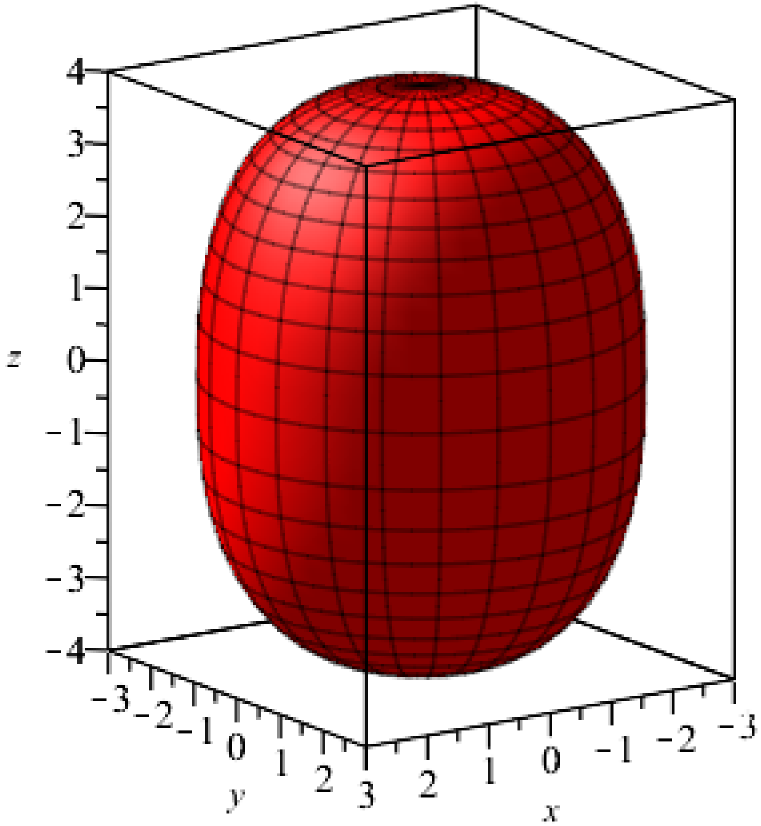
$$s(w, m) := sgn(\sin(w)) \cdot |\sin(w)|^m :$$

$$p(u, v) := \left\langle A \cdot c\left(v, \frac{2}{t}\right) \cdot c\left(u, \frac{2}{r}\right), B \cdot c\left(v, \frac{2}{t}\right) \cdot s\left(u, \frac{2}{r}\right), C \cdot s\left(v, \frac{2}{t}\right) \right\rangle :$$

$$p(u, v) = \begin{bmatrix} 3 \left(\begin{cases} 1 & 0 < \cos(v) \\ 0 & \cos(v) = 0 \\ -1 & \cos(v) < 0 \end{cases} \right) |\cos(v)|^{4/5} \left(\begin{cases} 1 & 0 < \cos(u) \\ 0 & \cos(u) = 0 \\ -1 & \cos(u) < 0 \end{cases} \right) |\cos(u)| \\ 3 \left(\begin{cases} 1 & 0 < \cos(v) \\ 0 & \cos(v) = 0 \\ -1 & \cos(v) < 0 \end{cases} \right) |\cos(v)|^{4/5} \left(\begin{cases} 1 & 0 < \sin(u) \\ 0 & \sin(u) = 0 \\ -1 & \sin(u) < 0 \end{cases} \right) |\sin(u)| \\ 4 \left(\begin{cases} 1 & 0 < \sin(v) \\ 0 & \sin(v) = 0 \\ -1 & \sin(v) < 0 \end{cases} \right) |\sin(v)|^{4/5} \end{bmatrix}$$

En hel super ellipsoide:

$$piethein := plot3d\left(p(u, v), u = -\pi..\pi, v = -\frac{\pi}{2}..\frac{\pi}{2}, labels = [x, y, z], scaling = constrained, color = red\right)$$



Fremstilling af STL-fil:

Export("piethein.stl", *piethein*, *base* = *homedir*)

230484 **(2.1)**