

Snoet reb (1)

Inspiration fra 2 artikler:

<https://link.springer.com/content/pdf/10.1007/s00214-009-0639-4.pdf>

<https://iopscience.iop.org/article/10.1209/0295-5075/93/60004/pdf>

restart

with(plots) :

Rotation om z-aksen:

https://en.wikipedia.org/wiki/Rotation_matrix#In_three_dimensions

$$R_z(\theta) := \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} :$$

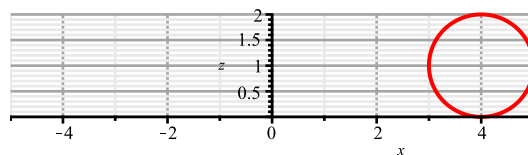
▼ Cirkel i xz-plan

$c(u) := \langle 1 \cdot \cos(u), 0, 1 \cdot \sin(u) \rangle + \langle 4, 0, 1 \rangle :$

$$c(u) = \begin{bmatrix} \cos(u) + 4 \\ 0 \\ \sin(u) + 1 \end{bmatrix}$$

Med parameterområdet $u \in [0; 2 \cdot \pi]$

$plot([c(u)[1], c(u)[3], u=0..2 \cdot \pi], color=red, gridlines, labels=[x, z], view=[-5..5, 0..2], scaling=constrained)$



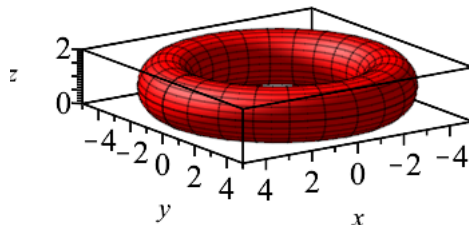
▼ Torus

$t(u, v) := R_z(v) \cdot c(u) :$

$$t(u, v) = \begin{bmatrix} \cos(v) (\cos(u) + 4) \\ \sin(v) (\cos(u) + 4) \\ \sin(u) + 1 \end{bmatrix}$$

Med parameterområderne: $u \in [0; 2 \cdot \pi]$ og $v \in [0; 2 \cdot \pi]$

`plot3d(t(u, v), u=0..2·π, v=0..2·π, color=red, labels=[x, y, z], scaling=constrained)`



▼ Helix

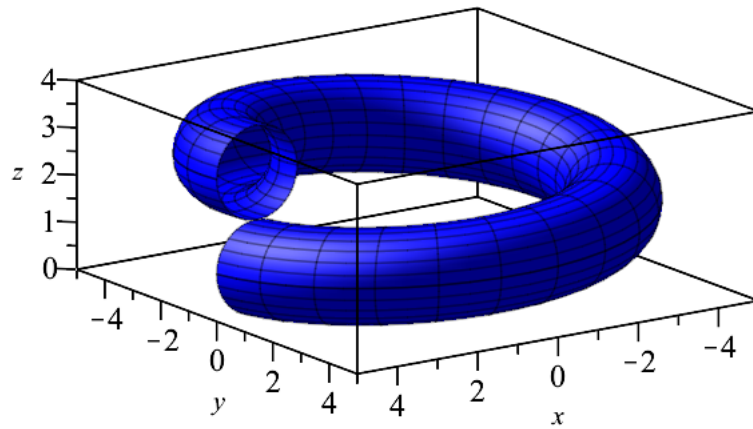
$h(u, v) := t(u, v) + \langle 0, 0, k \cdot v \rangle :$

$$h(u, v) = \begin{bmatrix} \cos(v) (\cos(u) + 4) \\ \sin(v) (\cos(u) + 4) \\ \sin(u) + 1 + k v \end{bmatrix}$$

Med parameterområderne: $u \in [0; 2 \cdot \pi]$ og $v \in [0; 2 \cdot \pi]$

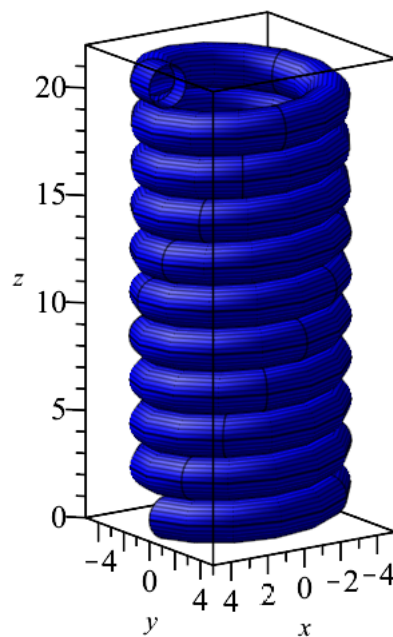
En omgang:

`plot3d(subs(k = 1/π, h(u, v)), u=0..2·π, v=0..2·π, color=blue, labels=[x, y, z], scaling=constrained)`



10 omgange:

```
plot3d(subs(k = 1/pi, h(u, v)), u = 0..2*pi, v = 0..(2*pi)*10, color = blue, labels = [x, y, z], scaling = constrained,
        numpoints = 20000)
```



Med luft imellem:

$$BLÅ := \text{plot3d}\left(\text{subs}\left(k = \frac{2}{\pi}, h(u, v)\right), u = 0 .. 2 \cdot \pi, v = 0 .. (2 \cdot \pi) \cdot 10, \text{color} = \text{blue}, \text{labels} = [x, y, z], \text{scaling} = \text{constrained}, \text{numpoints} = 20000, \text{axes} = \text{none}\right)$$


▼ Helix 2

$$c2(u) := \langle 1 \cdot \cos(u), 0, 1 \cdot \sin(u) \rangle + \langle -4, 0, 1 \rangle :$$

$$c2(u) = \begin{bmatrix} \cos(u) - 4 \\ 0 \\ \sin(u) + 1 \end{bmatrix}$$

Med parameterområdet $u \in [0; 2 \cdot \pi]$

$$t2(u, v) := R_z(v) \cdot c2(u) :$$

$$t2(u, v) = \begin{bmatrix} \cos(v) (\cos(u) - 4) \\ \sin(v) (\cos(u) - 4) \\ \sin(u) + 1 \end{bmatrix}$$

Med parameterområderne: $u \in [0; 2 \cdot \pi]$ og $v \in [0; 2 \cdot \pi]$

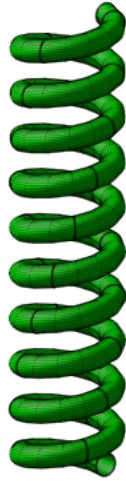
$$h2(u, v) := t2(u, v) + \langle 0, 0, k \cdot v \rangle :$$

$$h2(u, v) = \begin{bmatrix} \cos(v) (\cos(u) - 4) \\ \sin(v) (\cos(u) - 4) \\ \sin(u) + 1 + k v \end{bmatrix}$$

Med parameterområderne: $u \in [0; 2 \cdot \pi]$ og $v \in [0; 2 \cdot \pi]$

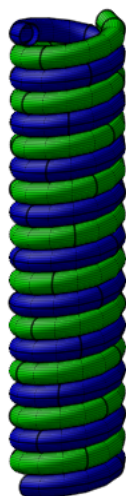
Startet en halv omgang længere fremme:

```
GRØN := plot3d( subs( k =  $\frac{2}{\pi}$ , h2(u, v) ), u = 0 .. 2·π, v = 0 .. (2·π) · 10, color = green, labels = [x, y, z], scaling = constrained, numpoints = 20000, axes = none )
```



▼ Sammensnoning

```
display(BLÅ, GRØN)
```



▼ Reb (2 kordeller med tæt snoning)

$$c1(u) := \langle 1 \cdot \cos(u), 0, 1 \cdot \sin(u) \rangle + \langle 1, 0, 1 \rangle :$$

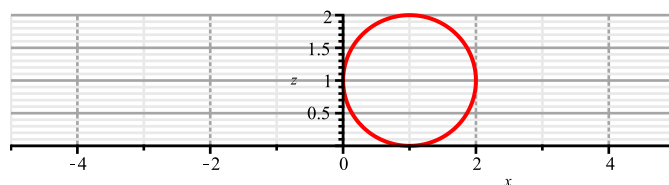
$$t1(u, v) := R_z(v) \cdot c1(u) :$$

$$h1(u, v) := t1(u, v) + \langle 0, 0, k \cdot v \rangle :$$

$$h1(u, v) = \begin{bmatrix} \cos(v) (\cos(u) + 1) \\ \sin(v) (\cos(u) + 1) \\ \sin(u) + 1 + k v \end{bmatrix}$$

Med parameterområderne: $u \in [0; 2 \cdot \pi]$ og $v \in [0; 2 \cdot \pi]$

`plot([c1(u)[1], c1(u)[3], u=0..2·π], color=red, gridlines, labels=[x, z], view=[-5..5, 0..2], scaling=constrained)`



$$c2(u) := \langle 1 \cdot \cos(u), 0, 1 \cdot \sin(u) \rangle + \langle -1, 0, 1 \rangle :$$

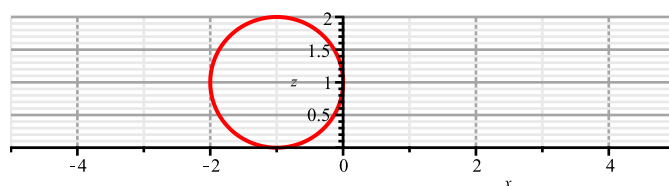
$$t2(u, v) := R_z(v) \cdot c2(u) :$$

$$h2(u, v) := t2(u, v) + \langle 0, 0, k \cdot v \rangle :$$

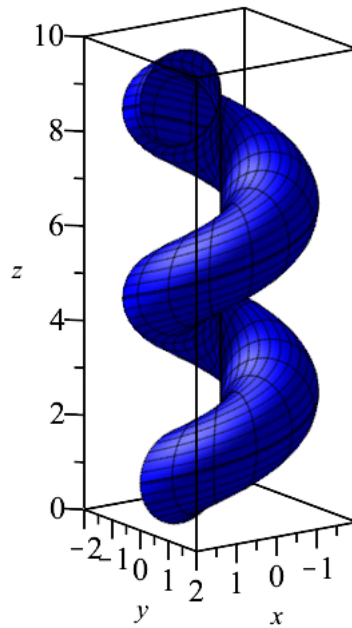
$$h2(u, v) = \begin{bmatrix} \cos(v) (\cos(u) - 1) \\ \sin(v) (\cos(u) - 1) \\ \sin(u) + 1 + k v \end{bmatrix}$$

Med parameterområderne: $u \in [0; 2 \cdot \pi]$ og $v \in [0; 2 \cdot \pi]$

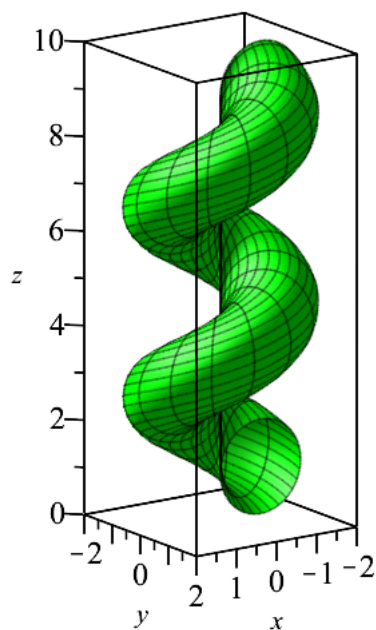
`plot([c2(u)[1], c2(u)[3], u=0..2·π], color=red, gridlines, labels=[x, z], view=[-5..5, 0..2], scaling=constrained)`



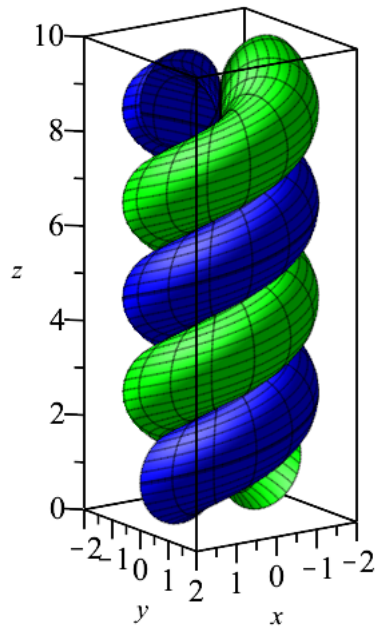
`BLÅ := plot3d(subs(k = 2/π, h1(u, v)), u=0..2·π, v=0..(2·π)·2, color=blue, labels=[x, y, z], scaling=constrained, numpoints=20000)`



$GRØN := plot3d\left(\text{subs}\left(k = \frac{2}{\pi}, h2(u, v)\right), u = 0..2\cdot\pi, v = 0..(2\cdot\pi)\cdot 2, \text{color} = \text{green}, \text{labels} = [x, y, z], \text{scaling} = \text{constrained}, \text{numpoints} = 20000\right)$



$display(BLÅ, GRØN)$



Reb (2 kordeller med tæt snoning)

$$c(u) := \langle 1 \cdot \cos(u), 0, 1 \cdot \sin(u) \rangle + \langle 1, 0, 1 \rangle :$$

$$t1(u, v) := R_z(v) \cdot c(u) :$$

$$h1(u, v) := t1(u, v) + \langle 0, 0, k \cdot v \rangle :$$

$$t2(u, v) := R_z(v + \pi) \cdot c(u) :$$

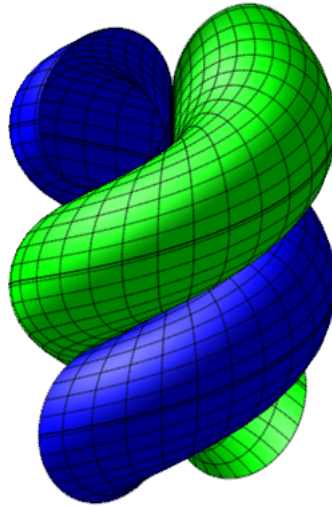
$$h2(u, v) := t2(u, v) + \langle 0, 0, k \cdot v \rangle :$$

Med parameterområderne: $u \in [0; 2 \cdot \pi]$ og $v \in [0; 2 \cdot \pi]$

$$\begin{aligned} BLÅ &:= \text{plot3d}\left(\text{subs}\left(k = \frac{2}{\pi}, h1(u, v)\right), u = 0 .. 2 \cdot \pi, v = 0 .. (2 \cdot \pi), \text{color} = \text{blue}, \text{labels} = [x, y, z], \text{scaling} \right. \\ &\quad \left. = \text{constrained}, \text{numpoints} = 20000, \text{axes} = \text{none}\right) : \end{aligned}$$

$$\begin{aligned} GRØN &:= \text{plot3d}\left(\text{subs}\left(k = \frac{2}{\pi}, h2(u, v)\right), u = 0 .. 2 \cdot \pi, v = 0 .. (2 \cdot \pi), \text{color} = \text{green}, \text{labels} = [x, y, z], \text{scaling} \right. \\ &\quad \left. = \text{constrained}, \text{numpoints} = 20000, \text{axes} = \text{none}\right) : \end{aligned}$$

$$\text{display}(BLÅ, GRØN)$$



Reb (3 kordeller med tæt snoning)

$$c(u) := \langle 1 \cdot \cos(u), 0, 1 \cdot \sin(u) \rangle + \langle 1, 0, 1 \rangle :$$

$$t1(u, v) := R_z(v) \cdot c(u) :$$

$$h1(u, v) := t1(u, v) + \langle 0, 0, k \cdot v \rangle :$$

$$t2(u, v) := R_z\left(v + \frac{2 \cdot \pi}{3}\right) \cdot c(u) :$$

$$h2(u, v) := t2(u, v) + \langle 0, 0, k \cdot v \rangle :$$

$$t3(u, v) := R_z\left(v + \frac{4 \cdot \pi}{3}\right) \cdot c(u) :$$

$$h3(u, v) := t3(u, v) + \langle 0, 0, k \cdot v \rangle :$$

Med parameterområderne: $u \in [0; 2 \cdot \pi]$ og $v \in [0; 2 \cdot \pi]$

$$\begin{aligned} BLÅ := & \text{plot3d}\left(\text{subs}\left(k = \frac{3}{\pi}, h1(u, v)\right), u = 0 .. 2 \cdot \pi, v = 0 .. (2 \cdot \pi) \cdot 2, \text{color} = \text{blue}, \text{labels} = [x, y, z], \text{scaling} \right. \\ & \left. = \text{constrained}, \text{numpoints} = 20000, \text{axes} = \text{none}\right) : \end{aligned}$$

$$\begin{aligned} GRØN := & \text{plot3d}\left(\text{subs}\left(k = \frac{3}{\pi}, h2(u, v)\right), u = 0 .. 2 \cdot \pi, v = 0 .. (2 \cdot \pi) \cdot 2, \text{color} = \text{green}, \text{labels} = [x, y, z], \text{scaling} \right. \\ & \left. = \text{constrained}, \text{numpoints} = 20000, \text{axes} = \text{none}\right) : \end{aligned}$$

```

RØD := plot3d( subs( k =  $\frac{3}{\pi}$ , h3(u, v) ), u = 0 .. 2·π, v = 0 .. (2·π)·2, color = red, labels = [x, y, z], scaling
= constrained, numpoints = 20000, axes = none ):
display( BLÅ, GRØN, RØD)

```



Reb (4 kordeller med tæt snoning)

$$c(u) := \langle 1 \cdot \cos(u), 0, 1 \cdot \sin(u) \rangle + \langle 1, 0, 1 \rangle :$$

$$t1(u, v) := R_z(v) \cdot c(u) :$$

$$h1(u, v) := t1(u, v) + \langle 0, 0, k \cdot v \rangle :$$

$$t2(u, v) := R_z\left(v + \frac{\pi}{2}\right) \cdot c(u) :$$

$$h2(u, v) := t2(u, v) + \langle 0, 0, k \cdot v \rangle :$$

$$t3(u, v) := R_z(v + \pi) \cdot c(u) :$$

$$h3(u, v) := t3(u, v) + \langle 0, 0, k \cdot v \rangle :$$

$$t4(u, v) := R_z\left(v + \frac{3 \cdot \pi}{2}\right) \cdot c(u) :$$

$$h4(u, v) := t4(u, v) + \langle 0, 0, k \cdot v \rangle :$$

Med parameterområderne: $u \in [0; 2 \cdot \pi]$ og $v \in [0; 2 \cdot \pi]$

```

BLÅ := plot3d( subs( k =  $\frac{4}{\pi}$ , h1(u, v) ), u = 0 .. 2·π, v = 0 .. (2·π), color = blue, labels = [x, y, z], scaling

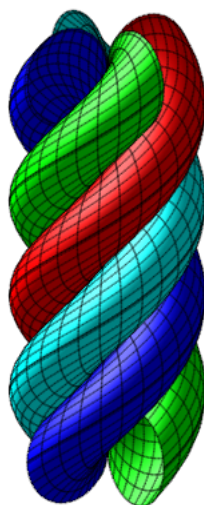
```

```

= constrained, numpoints = 20000, axes = none ):
GRØN := plot3d( subs( k = 4/π, h2(u, v) ), u = 0 .. 2·π, v = 0 .. (2·π), color = green, labels = [x, y, z], scaling
= constrained, numpoints = 20000, axes = none ):
RØD := plot3d( subs( k = 4/π, h3(u, v) ), u = 0 .. 2·π, v = 0 .. (2·π), color = red, labels = [x, y, z], scaling
= constrained, numpoints = 20000, axes = none ):
CYAN := plot3d( subs( k = 4/π, h4(u, v) ), u = 0 .. 2·π, v = 0 .. (2·π), color = cyan, labels = [x, y, z], scaling
= constrained, numpoints = 20000, axes = none ):

display( BLÅ, GRØN, RØD, CYAN )

```



Generel formel (n kordeller i et reb)

6 liner med hver sin farve

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n := 6 :
FARVER := [blue, green, red, cyan, gold, magenta] :

```

Cirkel:

$$c(u) := \langle 1 \cdot \cos(u), 0, 1 \cdot \sin(u) \rangle + \langle 1, 0, 1 \rangle :$$

Torus og helix:

for i **from** 0 **to** $n - 1$ **do**

$$t[i](u, v) := R_z \left(v + \frac{2 \cdot \pi}{n} \cdot i \right) \cdot c(u) :$$

$$h[i](u, v) := t[i](u, v) + \langle 0, 0, k \cdot v \rangle :$$

end do:

```
display( seq( plot3d( subs( k =  $\frac{n}{\pi}$ , h[i](u, v) ), u = 0 .. 2 * pi, v = 0 .. (2 * pi) * 1, color = FARVER[i + 1], labels
= [x, y, z], scaling = constrained, numpoints = 20000, axes = none ), i = 0 .. n - 1 ) )
```

