Snoet reb (2)

restart
with(*plots*) :

Rotation om z-aksen:
https://en.wikipedia.org/wiki/Rotation_matrix#In_three_dimensions

$$R_z(\theta) := \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}:$$

4 kordeller tæt snoet 2 omgange, tæt

$n := 4$:
$FARVER := [blue, green, red, cyan]$:
$omgange := 2$:
$k := \frac{n}{\pi}$:

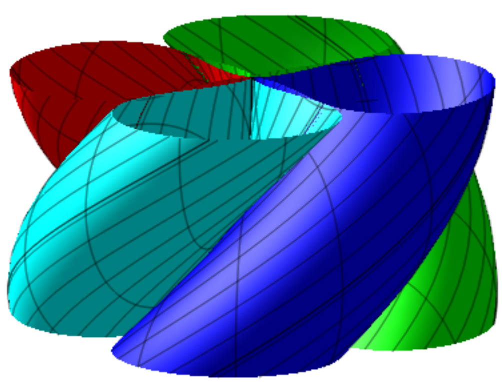
Cirkel (lodret):
$c(u) := \langle 1\cdot\cos(u), 0, 1\cdot\sin(u)\rangle + \langle 1, 0, 1\rangle$:

Torus og helix:
for i **from** 0 **to** $n - 1$ **do**
$t[i] := unapply\left(R_z\left(v + \frac{2\cdot\pi}{n}\cdot i\right) \cdot c(u), [u, v]\right)$:
$h[i] := unapply(t[i](u, v) + \langle 0, 0, k\cdot v\rangle, [u, v])$:
end do:

Grafer

$REB_1 := display\big(seq\big(plot3d\big(h[i](u, v), u = 0..2\cdot\pi, v = 0..2\cdot\pi\cdot omgange, color = FARVER[i + 1], labels = [x, y, z], scaling = constrained, numpoints = 20000, axes = none\big), i = 0..n - 1\big)\big)$

$display\left(REB_1, view = [-2..2, -2..2, 2..4]\right)$



Vandret snit

z-koordinaten af h'erne er alle den samme, nemlig:

for i **from** 0 **to** $n-1$ **do** $h[i](u,v)[3]$ **end do**

$$\sin(u)+1+\frac{4\,v}{\pi}$$

$$\sin(u)+1+\frac{4\,v}{\pi}$$

$$\sin(u)+1+\frac{4\,v}{\pi}$$

$$\sin(u)+1+\frac{4\,v}{\pi} \qquad \textbf{(1.2.1)}$$

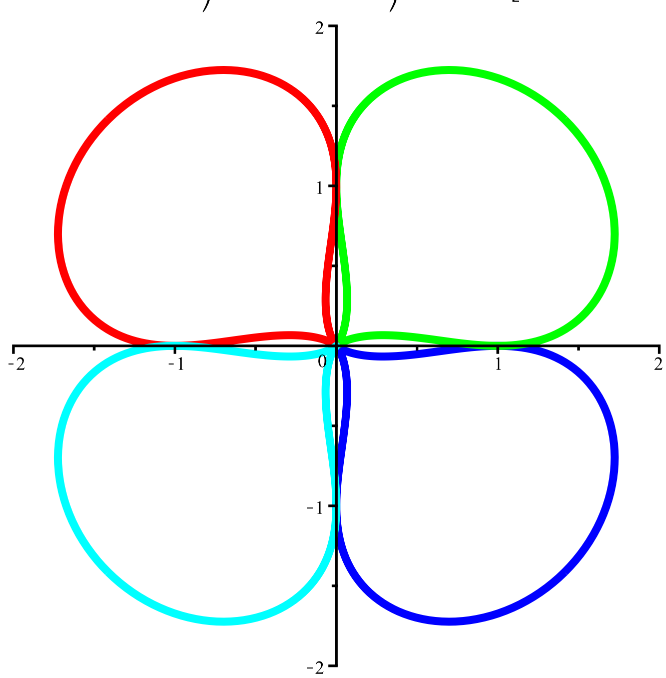
Ønsker et vandret snit, dvs. z-koordinaten ønskes konstant, f.eks. 8:

$$v_{vandret} := solve(h[0](u,v)[3]=8, v) = -\frac{1}{4}\sin(u)\,\pi+\frac{7}{4}\pi$$

$$h[0]\left(u, v_{vandret}\right) = \begin{bmatrix} \cos\left(\frac{\sin(u)\,\pi}{4}+\frac{\pi}{4}\right)(\cos(u)+1) \\ -\sin\left(\frac{\sin(u)\,\pi}{4}+\frac{\pi}{4}\right)(\cos(u)+1) \\ \sin(u)+1+\frac{4\left(-\frac{\sin(u)\,\pi}{4}+\frac{7\,\pi}{4}\right)}{\pi} \end{bmatrix}$$

Vandret snit (z=8):

$display\left(seq\left(plot\left(\left[h[i]\left(u, v_{vandret}\right)[1], h[i]\left(u, v_{vandret}\right)[2], u=0..2\cdot\pi\right], color=FARVER[i+1], thickness=3\right), i=0..n-1\right), view=[-2..2, -2..2]\right)$



4 kordeller tæt snoet 2 omgange, med luft

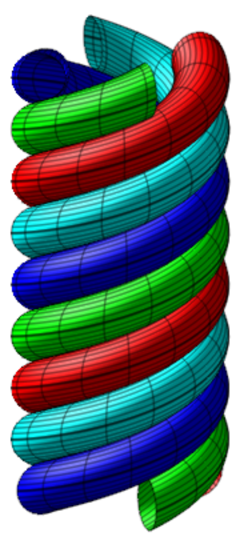
$n := 4:$
$FARVER := [blue, green, red, cyan]:$
$omgange := 2:$
$k := \frac{n}{\pi}:$

Cirkel:
$c(u) := \langle 1\cdot\cos(u), 0, 1\cdot\sin(u)\rangle + \langle 3, 0, 1\rangle :$

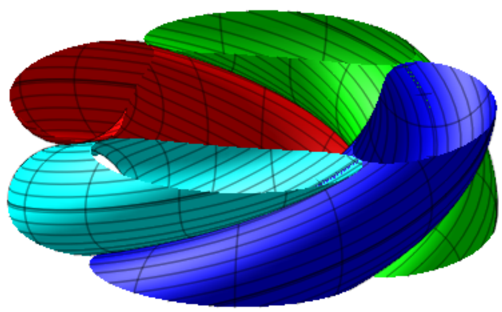
Torus og helix:
for i **from** 0 **to** $n-1$ **do**
$t[i] := unapply\left(R_z\left(v + \frac{2\cdot\pi}{n}\cdot i\right) \cdot c(u), [u, v]\right):$
$h[i] := unapply(t[i](u, v) + \langle 0, 0, k\cdot v\rangle, [u, v]):$
end do:

Grafer

$REB_2 := display\big(seq\big(plot3d\big(h[i](u, v), u = 0..2\cdot\pi, v = 0..2\cdot\pi\cdot omgange, color = FARVER[i+1], labels = [x, y, z], scaling = constrained, numpoints = 20000, axes = none\big), i = 0..n-1\big)\big)$



$display\big(REB_2, view = [-4..4, -4..4, 2..4]\big)$



Vandret snit

z-koordinaten af h'erne er alle den samme, nemlig:

for i **from** 0 **to** $n-1$ **do** $h[i](u,v)[3]$ **end do**

$$\sin(u)+1+\frac{4\,v}{\pi}$$

$$\sin(u)+1+\frac{4\,v}{\pi}$$

$$\sin(u)+1+\frac{4\,v}{\pi}$$

$$\sin(u)+1+\frac{4\,v}{\pi} \qquad \textbf{(2.2.1)}$$

Ønsker et vandret snit, dvs. z-koordinaten ønskes konstant, f.eks. 8:

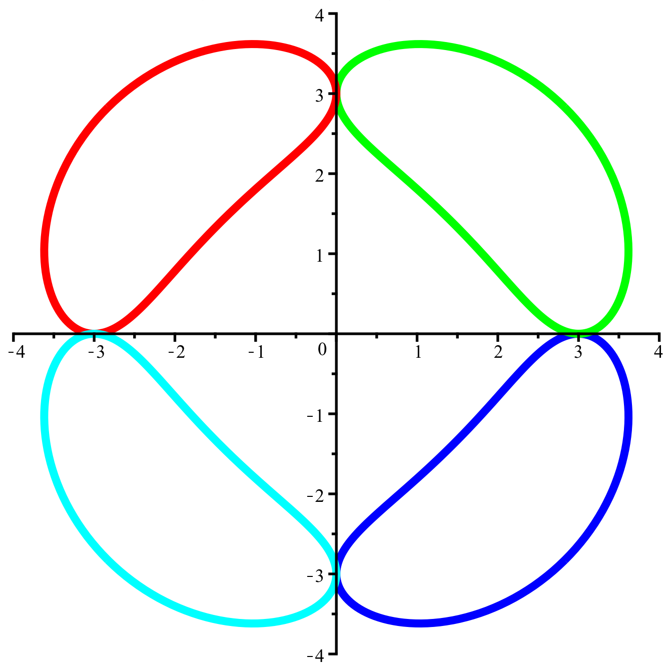
$v_{vandret} := solve(h[0](u,v)[3]=8,v) = -\frac{1}{4}\sin(u)\,\pi+\frac{7}{4}\,\pi$

$h[0]\left(u, v_{vandret}\right) =$

$$\left[\begin{array}{c}\cos\left(\frac{\sin(u)\,\pi}{4}+\frac{\pi}{4}\right)(\cos(u)+3)\\ -\sin\left(\frac{\sin(u)\,\pi}{4}+\frac{\pi}{4}\right)(\cos(u)+3)\\ \sin(u)+1+\frac{4\left(-\frac{\sin(u)\,\pi}{4}+\frac{7\,\pi}{4}\right)}{\pi}\end{array}\right]$$

Vandret snit (z=8):

$display\Big(seq\Big(plot\Big(\big[h[i]\big(u, v_{vandret}\big)[1], h[i]\big(u, v_{vandret}\big)[2], u=0..2\cdot\pi\big], color=FARVER[i+1], thickness=3\Big), i=0..n-1\Big), view=[-4..4, -4..4]\Big)$



6 kordeller snoet 2 omgange, tæt

$n := 6 :$
$FARVER := [blue, green, red, cyan, gold, black] :$
$omgange := 2 :$

$k := \frac{n}{\pi} :$

Cirkel:
$c(u) := \langle 1\cdot\cos(u), 0, 1\cdot\sin(u)\rangle + \langle 1, 0, 1\rangle :$

Torus og helix:
for i **from** 0 **to** $n-1$ **do**

$t[i] := unapply\left(R_z\left(v+\frac{2\cdot\pi}{n}\cdot i\right)\cdot c(u), [u, v]\right) :$

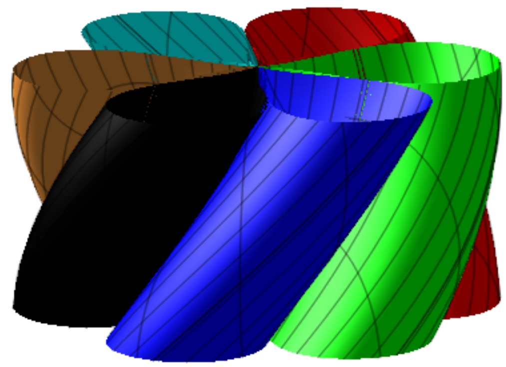
$h[i] := unapply(t[i](u, v) + \langle 0, 0, k\cdot v\rangle, [u, v]) :$

end do:

Grafer

$REB_3 := display(seq(plot3d(h[i](u, v), u = 0 ..2 \cdot \pi, v = 0 ..2 \cdot \pi \cdot omgange, color = FARVER[i + 1], labels = [x, y, z], scaling = constrained, numpoints = 20000, axes = none), i = 0 ..n - 1))$

$display(REB_3, view = [-2 ..2, -2 ..2, 2 ..4])$



Vandret snit

z-koordinaten af h'erne er alle den samme, nemlig:

for i **from** 0 **to** $n-1$ **do** $h[i](u,v)[3]$ **end do**

$$\sin(u)+1+\frac{6\,v}{\pi}$$

$$\sin(u)+1+\frac{6\,v}{\pi}$$

$$\sin(u)+1+\frac{6\,v}{\pi}$$

$$\sin(u)+1+\frac{6\,v}{\pi}$$

$$\sin(u)+1+\frac{6\,v}{\pi}$$

$$\sin(u)+1+\frac{6\,v}{\pi} \qquad \textbf{(3.2.1)}$$

Ønsker et vandret snit, dvs. z-koordinaten ønskes konstant, f.eks. 8:

$v_{vandret} := solve(h[0](u,v)[3]=8, v) = -\frac{1}{6}\sin(u)\,\pi+\frac{7}{6}\,\pi$

$$h[0]\left(u, v_{vandret}\right) = \begin{bmatrix} -\sin\left(\frac{\sin(u)\,\pi}{6} + \frac{\pi}{3}\right)(\cos(u) + 1) \\ -\cos\left(\frac{\sin(u)\,\pi}{6} + \frac{\pi}{3}\right)(\cos(u) + 1) \\ \sin(u) + 1 + \frac{6\left(-\frac{\sin(u)\,\pi}{6} + \frac{7\,\pi}{6}\right)}{\pi} \end{bmatrix}$$

$display\left(seq\left(plot\left(\left[h[i]\left(u, v_{vandret}\right)[1], h[i]\left(u, v_{vandret}\right)[2], u = 0..2\cdot\pi\right], color = FARVER[i+1], thickness = 3\right), i = 0..n-1\right), view = [-2..2, -2..2]\right)$