Snoet reb (3)

restart
with(*plots*) :
with(*VektorAnalyse4*) :

$assume(R > 0, h > 0) : interface(showassumed = 0) :$

Udledning af parametriseringen af helix udvidet med en cirkulær tykkelse

Inspiration til parametrisering af helix med en cirkulær tykkelse:
https://math.stackexchange.com/questions/461547/whats-the-equation-of-helix-surface

Parametrisering af helix (snoet spiral) om z-aksen:

$r_{helix}(u) := \langle R \cdot \cos(u + u_{start}), R \cdot \sin(u + u_{start}), h \cdot u \rangle :$

$$r_{helix}(u) = \begin{bmatrix} R\cos(u + u_{start}) \\ R\sin(u + u_{start}) \\ h\,u \end{bmatrix}$$

Tangent til helixpunkt (hastighedsvektor):

$H := unapply(diff(r_{helix}(u), u), u) :$

$$H(u) = \begin{bmatrix} -R\sin(u + u_{start}) \\ R\cos(u + u_{start}) \\ h \end{bmatrix}$$

Normeret (enhedvektor i samme retning):

$H_{enhed}(u) := simplify\left(\frac{H(u)}{\sqrt{prik(H(u), H(u))}}\right) :$

$$H_{enhed}(u) = \begin{bmatrix} -\frac{R\sin(u + u_{start})}{\sqrt{R^2 + h^2}} \\ \frac{R\cos(u + u_{start})}{\sqrt{R^2 + h^2}} \\ \frac{h}{\sqrt{R^2 + h^2}} \end{bmatrix}$$

Accelerationsvektor til helixpunkt:

$A := unapply(diff(H(u), u), u) :$

$A(u) =$

$$\begin{bmatrix} -R\cos(u+u_{start}) \\ -R\sin(u+u_{start}) \\ 0 \end{bmatrix}$$

Normeret (enhedvektor i samme retning):

$$A_{enhed}(u) := simplify\left(\frac{A(u)}{\sqrt{prik(A(u),A(u))}}\right):$$

$$A_{enhed}(u) = \begin{bmatrix} -\cos(u+u_{start}) \\ -\sin(u+u_{start}) \\ 0 \end{bmatrix}$$

Normalvektor til hastighedsvektor og accelerationsvektor:

$$N(u) := kryds(H(u), A(u)):$$

$$N(u) = \begin{bmatrix} h\,R\sin(u+u_{start}) \\ -h\,R\cos(u+u_{start}) \\ R^2\cos(u+u_{start})^2 + R^2\sin(u+u_{start})^2 \end{bmatrix}$$

Normeret (enhedvektor i samme retning):

$$N_{enhed}(u) := simplify\left(\frac{N(u)}{\sqrt{prik(N(u),N(u))}}\right):$$

$$N_{enhed}(u) = \begin{bmatrix} \dfrac{h\sin(u+u_{start})}{\sqrt{R^2+h^2}} \\ -\dfrac{h\cos(u+u_{start})}{\sqrt{R^2+h^2}} \\ \dfrac{R}{\sqrt{R^2+h^2}} \end{bmatrix}$$

Cirkulær udvidelse af helix-snoningen, hvor der dannes en cirkel ortogonalt på helixen:

De 2 enhedsvektorer $N_{enhed}(u)$ og $H_{enhed}(u)$ danner de 2 ortogonale retninger, som er ortogonale til helixen:

$$r_{snoet} := unapply\left(r_{helix}(u) + r\cdot\left(A_{enhed}(u)\cdot\cos(v) + N_{enhed}(u)\cdot\sin(v)\right), [u, v]\right):$$

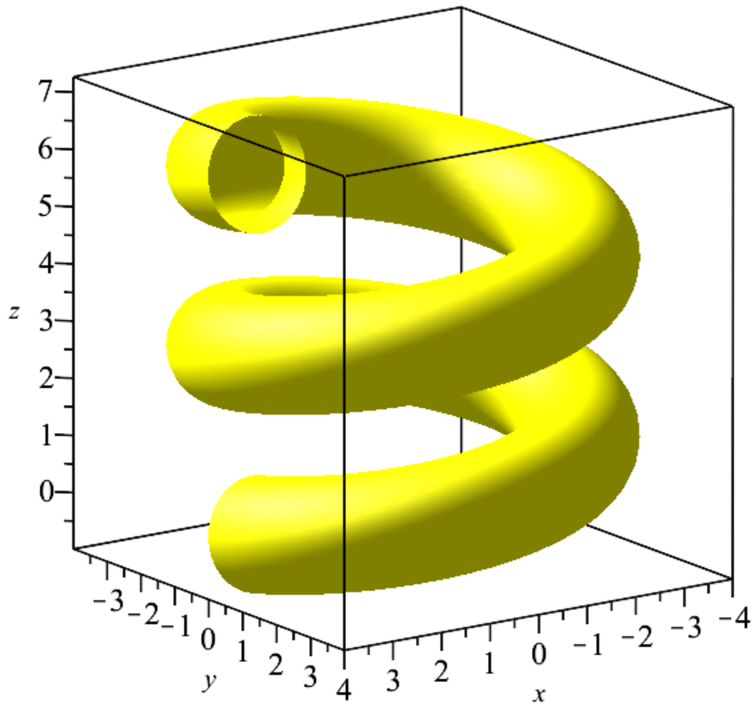
$$r_{snoet}(u,v) = \begin{bmatrix} R\cos(u+u_{start}) + r\left(-\cos(v)\cos(u+u_{start}) + \dfrac{\sin(v)\,h\sin(u+u_{start})}{\sqrt{R^2+h^2}}\right) \\ R\sin(u+u_{start}) + r\left(-\cos(v)\sin(u+u_{start}) - \dfrac{\sin(v)\,h\cos(u+u_{start})}{\sqrt{R^2+h^2}}\right) \\ h\,u + \dfrac{r\sin(v)\,R}{\sqrt{R^2+h^2}} \end{bmatrix}$$

hvor $u \in [0; 2\cdot\pi\cdot n]$, $v \in [0; 2\cdot\pi]$ og n angiver antal omgange i helixen.
R angiver radius i helix-snoningen, og r angiver radius i den cirkulære udvidelse.

1 snoning med luft

$P_1 := plot3d\left(subs\left(R=3, r=1, h=\frac{1}{2}, u_{start}=0, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z], scaling=constrained, color=yellow, style=patchnogrid, numpoints=10000\right):$

$display(P_1)$



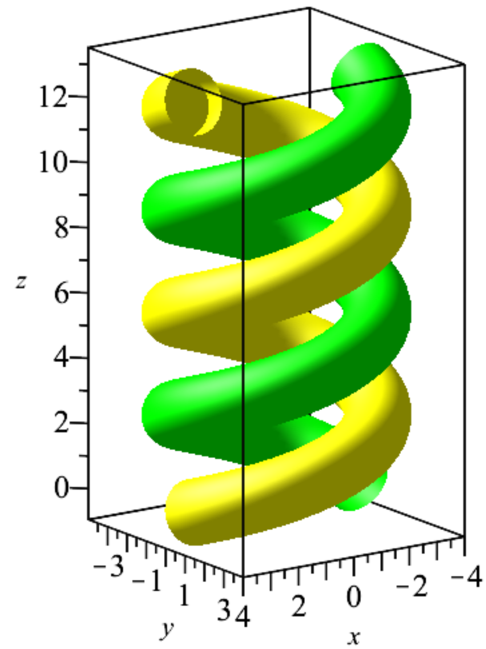
2 snoninger med luft

$S := 2:$

$P_1 := plot3d\left(subs\left(R=3, r=1, h=1, u_{start}=0, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z], scaling=constrained, color=yellow, style=patchnogrid, numpoints=10000\right):$

$P_2 := plot3d\left(subs\left(R=3, r=1, h=1, u_{start}=1\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z], scaling=constrained, color=green, style=patchnogrid, numpoints=10000\right):$

$display(P_1, P_2)$



3 snoninger med luft

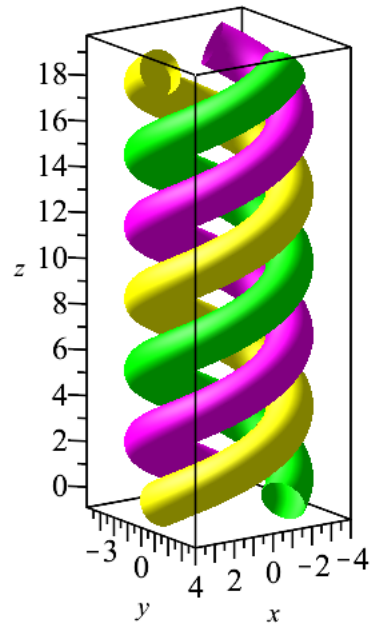
$S := 3 :$

$P_1 := plot3d\left(subs\left(R = 3, r = 1, h = \frac{3}{2}, u_{start} = 0, r_{snoet}(u, v) \right), u = 0 ..2\cdot\pi\cdot 2, v = 0 ..2\cdot\pi, labels = [x, y, z], scaling = constrained, color = yellow, style = patchnogrid, numpoints = 10000 \right) :$

$P_2 := plot3d\left(subs\left(R = 3, r = 1, h = \frac{3}{2}, u_{start} = 1\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u, v) \right), u = 0 ..2\cdot\pi\cdot 2, v = 0 ..2\cdot\pi, labels = [x, y, z], scaling = constrained, color = green, style = patchnogrid, numpoints = 10000 \right) :$

$P_3 := plot3d\left(subs\left(R = 3, r = 1, h = \frac{3}{2}, u_{start} = 2\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u, v) \right), u = 0 ..2\cdot\pi\cdot 2, v = 0 ..2\cdot\pi, labels = [x, y, z], scaling = constrained, color = magenta, style = patchnogrid, numpoints = 10000 \right) :$

$display(P_1, P_2, P_3)$



4 snoninger med luft

$S := 4 :$

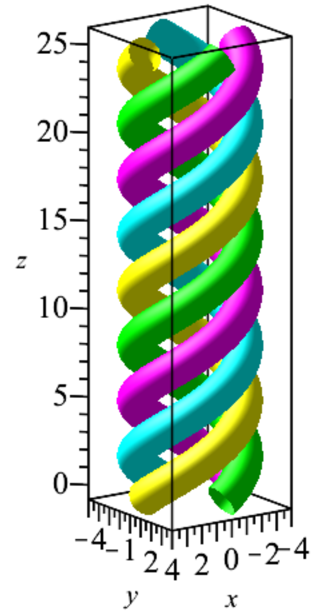
$P_1 := plot3d\left(subs\left(R=3, r=1, h=2, u_{start}=0, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z], scaling=constrained, color=yellow, style=patchnogrid, numpoints=10000\right) :$

$P_2 := plot3d\left(subs\left(R=3, r=1, h=2, u_{start}=1\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z], scaling=constrained, color=green, style=patchnogrid, numpoints=10000\right) :$

$P_3 := plot3d\left(subs\left(R=3, r=1, h=2, u_{start}=2\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z], scaling=constrained, color=magenta, style=patchnogrid, numpoints=10000\right) :$

$P_4 := plot3d\left(subs\left(R=3, r=1, h=2, u_{start}=3\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z], scaling=constrained, color=cyan, style=patchnogrid, numpoints=10000\right) :$

$display(P_1, P_2, P_3, P_4)$


25
20
15
10
5
0
z
−4
−1
2
4
2
0
−2
−4
y
x