Snoet reb (4)

$restart$
$with(plots):$
$with(VektorAnalyse4):$

$assume(R > 0, h > 0): interface(showassumed = 0):$

Udledning af parametriseringen af helix udvidet med en cirkulær tykkelse

Inspiration til parametrisering af helix med en cirkulær tykkelse:
https://math.stackexchange.com/questions/461547/whats-the-equation-of-helix-surface

Parametrisering af helix (snoet spiral) om z-aksen:

$r_{helix}(u) := \langle R \cdot \cos(u + u_{start}), R \cdot \sin(u + u_{start}), h \cdot u \rangle :$

$$r_{helix}(u) = \begin{bmatrix} R\cos(u + u_{start}) \\ R\sin(u + u_{start}) \\ h\,u \end{bmatrix}$$

Tangent til helixpunkt (hastighedsvektor):

$H := unapply(diff(r_{helix}(u), u), u) :$

$$H(u) = \begin{bmatrix} -R\sin(u + u_{start}) \\ R\cos(u + u_{start}) \\ h \end{bmatrix}$$

Normeret (enhedvektor i samme retning):

$H_{enhed}(u) := simplify\left(\frac{H(u)}{\sqrt{prik(H(u), H(u))}}\right) :$

$$H_{enhed}(u) = \begin{bmatrix} -\frac{R\sin(u + u_{start})}{\sqrt{R^2 + h^2}} \\ \frac{R\cos(u + u_{start})}{\sqrt{R^2 + h^2}} \\ \frac{h}{\sqrt{R^2 + h^2}} \end{bmatrix}$$

Accelerationsvektor til helixpunkt:

$A := unapply(diff(H(u), u), u) :$

$A(u) =$

$$\begin{bmatrix} -R\cos(u+u_{start}) \\ -R\sin(u+u_{start}) \\ 0 \end{bmatrix}$$

Normeret (enhedvektor i samme retning):

$$A_{enhed}(u) := simplify\left(\frac{A(u)}{\sqrt{prik(A(u),A(u))}}\right):$$

$$A_{enhed}(u) = \begin{bmatrix} -\cos(u+u_{start}) \\ -\sin(u+u_{start}) \\ 0 \end{bmatrix}$$

Normalvektor til hastighedsvektor og accelerationsvektor:

$$N(u) := kryds(H(u), A(u)):$$

$$N(u) = \begin{bmatrix} h\,R\sin(u+u_{start}) \\ -h\,R\cos(u+u_{start}) \\ R^2\cos(u+u_{start})^2 + R^2\sin(u+u_{start})^2 \end{bmatrix}$$

Normeret (enhedvektor i samme retning):

$$N_{enhed}(u) := simplify\left(\frac{N(u)}{\sqrt{prik(N(u),N(u))}}\right):$$

$$N_{enhed}(u) = \begin{bmatrix} \dfrac{h\sin(u+u_{start})}{\sqrt{R^2+h^2}} \\ -\dfrac{h\cos(u+u_{start})}{\sqrt{R^2+h^2}} \\ \dfrac{R}{\sqrt{R^2+h^2}} \end{bmatrix}$$

Cirkulær udvidelse af helix-snoningen, hvor der dannes en cirkel ortogonalt på helixen:

De 2 enhedsvektorer $N_{enhed}(u)$ og $H_{enhed}(u)$ danner de 2 ortogonale retninger, som er ortogonale til helixen:

$$r_{snoet} := unapply\big(r_{helix}(u) + r\cdot\big(A_{enhed}(u)\cdot\cos(v) + N_{enhed}(u)\cdot\sin(v)\big), [u, v]\big):$$

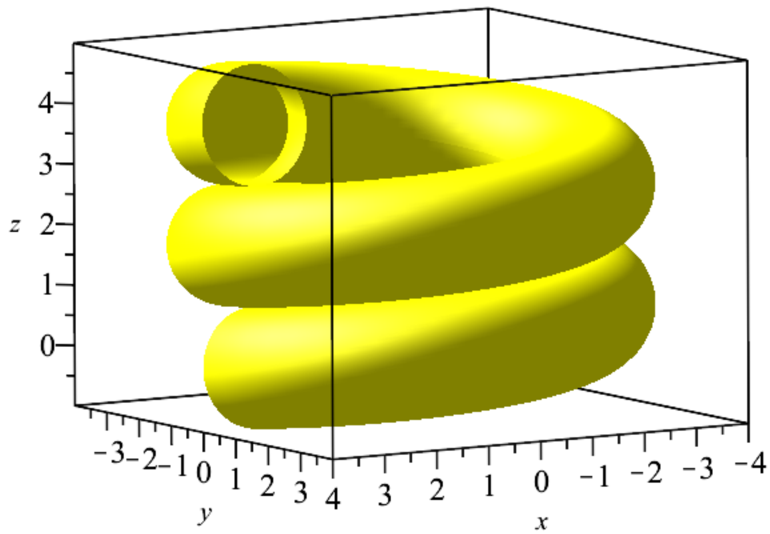
$$r_{snoet}(u,v) = \begin{bmatrix} R\cos(u+u_{start}) + r\left(-\cos(v)\cos(u+u_{start}) + \dfrac{\sin(v)\,h\sin(u+u_{start})}{\sqrt{R^2+h^2}}\right) \\ R\sin(u+u_{start}) + r\left(-\cos(v)\sin(u+u_{start}) - \dfrac{\sin(v)\,h\cos(u+u_{start})}{\sqrt{R^2+h^2}}\right) \\ h\,u + \dfrac{r\sin(v)\,R}{\sqrt{R^2+h^2}} \end{bmatrix}$$

hvor $u \in [0; 2\cdot\pi\cdot n]$, $v \in [0; 2\cdot\pi]$ og n angiver antal omgange i helixen.
R angiver radius i helix-snoningen, og r angiver radius i den cirkulære udvidelse.

1 snoning tætpakket

$P_1 := plot3d\left(subs\left(R=3, r=1, h=\frac{1}{\pi}, u_{start}=0, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z],\right.$
$\left. scaling=constrained, color=yellow, style=patchnogrid, numpoints=10000\right):$

$display(P_1)$



Udledning af formel for stigning pr. omgang, når de 2 snoninger skal være tætpakket

Figur med indtegnet punkt på grønne helix og en bundlinje på den gule helix:

$S := 2:$

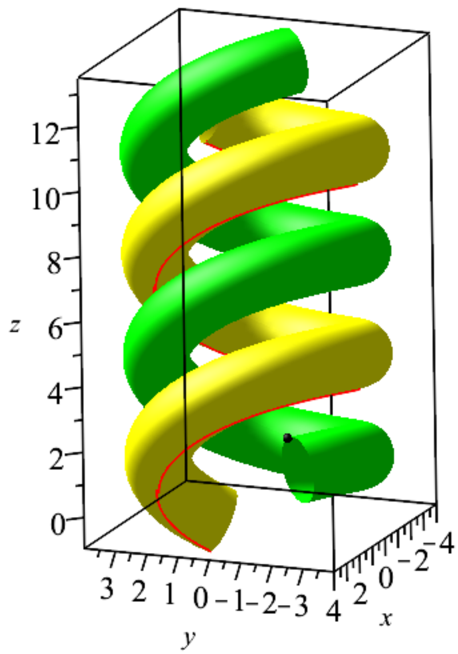
$P_1 := plot3d\left(subs\left(R=3, r=1, h=1, u_{start}=0, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z],\right.$
$\left. scaling=constrained, color=yellow, style=patchnogrid, numpoints=10000\right):$

$P_2 := plot3d\left(subs\left(R=3, r=1, h=1, u_{start}=1\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,\right.$
$\left. z], scaling=constrained, color=green, style=patchnogrid, numpoints=10000\right):$

$PUNKT_{grøn} := pointplot3d\left(\left[\left[vop\left(simplify\left(subs\left(R=3, r=1, h=1, u_{start}=1\cdot\frac{2\cdot\pi}{S}, r_{snoet}\left(0, \frac{\pi}{2}\right)\right)\right)\right)\right]\right],\right.$
$\left. symbol=solidsphere, symbolsize=10, color=black\right):$

$$KURVE_{gul} := spacecurve\left(\left[vop\left(subs\left(R=3, r=1, h=1, u_{start}=0, r_{snoet}\left(u, \frac{3\cdot\pi}{2}\right)\right)\right)\right], u=0..2\cdot\pi\cdot 2, thickness=3, color=red\right):$$

$$display(P_1, P_2, PUNKT_{grøn}, KURVE_{gul})$$



Det sorte punkt på den grønne udvidede helix skal flyttes op til den røde kurve på den udvidede gule helix.
NB: minimum er IKKE ved π, fordi cirklerne ikke er lodrette!

$$Difference := unapply\left(subs\left(R=3, r=1, u_{start}=0, r_{snoet}\left(u, \frac{3\cdot\pi}{2}\right)\right) - simplify\left(subs\left(R=3, r=1, u_{start}=1\cdot\frac{2\cdot\pi}{S}, r_{snoet}\left(0, \frac{\pi}{2}\right)\right)\right), [u, h]\right):$$

$$Difference(u, h) = \begin{bmatrix} 3\cos(u) - \frac{h\sin(u)}{\sqrt{h^2+9}} + 3 \\ 3\sin(u) + \frac{h\cos(u)}{\sqrt{h^2+9}} - \frac{h}{\sqrt{h^2+9}} \\ h\,u - \frac{6}{\sqrt{h^2+9}} \end{bmatrix}$$

$$L := fsolve(\{Difference(u, h)[1]=0, Difference(u, h)[3]=0\}, \{h=0..2, u=0..2\cdot\pi\}) =$$
$$\{h=0.6514057475, u=3.000367376\}$$

NB: kan ikke løses eksakt, derfor anvendes den numeriske løser på 2 af ligningerne (ikke 3 - for så findes der ikke en løsning).

$U := rhs(L[2]) : H := rhs(L[1]) :$

$$evalf(Difference(U, H)) = \begin{bmatrix} 0. \\ -8. \times 10^{-10} \\ 0. \end{bmatrix}$$

Dvs. med den givne H-værdi vil det sorte punkt ligge på den gule helix.

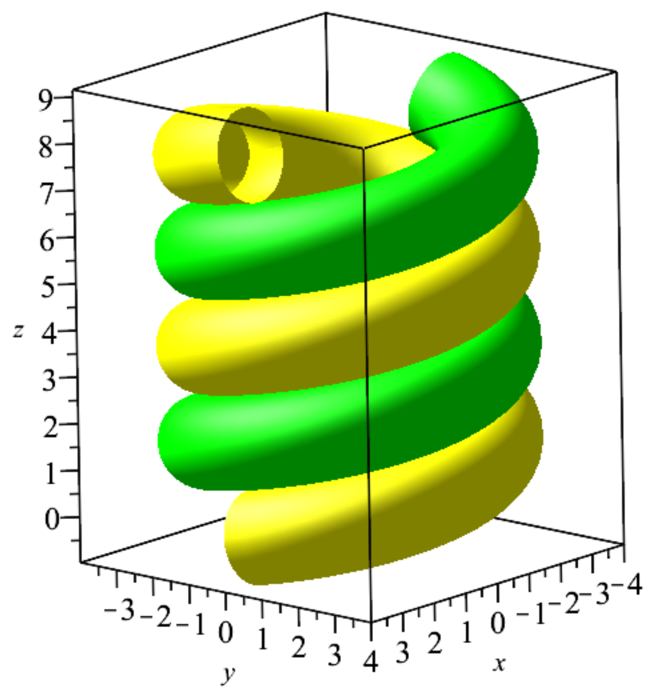
2 snoninger tætpakket

$S := 2 :$

$P_1 := plot3d\left(subs\left(R=3, r=1, h=H, u_{start}=0, r_{snoet}(u, v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x, y, z], scaling=constrained, color=yellow, style=patchnogrid, numpoints=10000\right) :$

$P_2 := plot3d\left(subs\left(R=3, r=1, h=H, u_{start}=1\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u, v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x, y, z], scaling=constrained, color=green, style=patchnogrid, numpoints=10000\right) :$

$display(P_1, P_2)$



Udledning af formel for stigning pr. omgang, når de 3 snoninger skal være tætpakket

Figur med indtegnet punkt på grønne helix og en bundlinje på den gule helix:

$S := 3:$

$P_1 := plot3d\left(subs\left(R=3, r=1, h=\frac{3}{2}, u_{start}=0, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z], scaling=constrained, color=yellow, style=patchnogrid, numpoints=10000\right):$

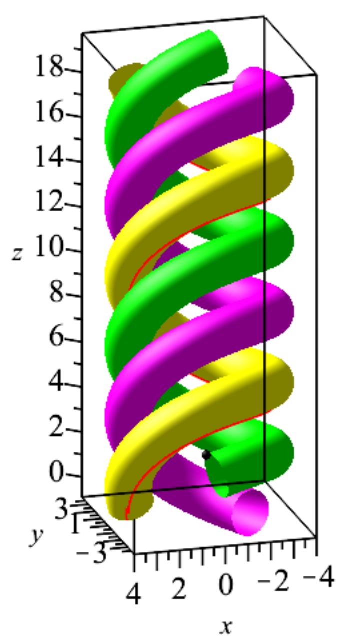
$P_2 := plot3d\left(subs\left(R=3, r=1, h=\frac{3}{2}, u_{start}=1\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z], scaling=constrained, color=green, style=patchnogrid, numpoints=10000\right):$

$P_3 := plot3d\left(subs\left(R=3, r=1, h=\frac{3}{2}, u_{start}=2\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u,v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x,y,z], scaling=constrained, color=magenta, style=patchnogrid, numpoints=10000\right):$

$PUNKT_{grøn} := pointplot3d\left(\left[\left[vop\left(simplify\left(subs\left(R=3, r=1, h=\frac{3}{2}, u_{start}=1\cdot\frac{2\cdot\pi}{S}, r_{snoet}\left(0,\frac{\pi}{2}\right)\right)\right)\right)\right]\right], symbol=solidsphere, symbolsize=10, color=black\right):$

$KURVE_{gul} := spacecurve\left(\left[vop\left(subs\left(R=3, r=1, h=\frac{3}{2}, u_{start}=0, r_{snoet}\left(u,\frac{3\cdot\pi}{2}\right)\right)\right)\right], u=0..2\cdot\pi\cdot 2, thickness=3, color=red\right):$

$display(P_1, P_2, P_3, PUNKT_{grøn}, KURVE_{gul})$



Det sorte punkt på den grønne udvidede helix skal flyttes op til den røde kurve på den udvidede gule helix.
NB: minimum er IKKE ved π, fordi cirklerne ikke er lodrette!

$$Difference := unapply\left(subs\left(R=3, r=1, u_{start}=0, r_{snoet}\left(u, \frac{3\cdot\pi}{2}\right)\right) - simplify\left(subs\left(R=3, r=1, u_{start}=1\cdot\frac{2\cdot\pi}{S}, r_{snoet}\left(0, \frac{\pi}{2}\right)\right)\right), [u, h]\right):$$

$$Difference(u, h) = \begin{bmatrix} 3\cos(u) - \frac{h\sin(u)}{\sqrt{h^2+9}} + \frac{3}{2} - \frac{h\sqrt{3}}{2\sqrt{h^2+9}} \\ 3\sin(u) + \frac{h\cos(u)}{\sqrt{h^2+9}} - \frac{3\sqrt{3}}{2} - \frac{h}{2\sqrt{h^2+9}} \\ h\,u - \frac{6}{\sqrt{h^2+9}} \end{bmatrix}$$

$$L := fsolve(\{Difference(u, h)[1]=0, Difference(u, h)[3]=0\}, \{h=0..2, u=0..2\cdot\pi\}) = \{h=1.006901211, u=1.883058643\}$$

NB: kan ikke løses eksakt, derfor anvendes den numeriske løser på 2 af ligningerne (ikke 3 - for så findes der ikke en løsning).

$$U := rhs(L[2]) : H := rhs(L[1]) :$$

$$evalf(Difference(U, H)) = \begin{bmatrix} 1.0\times 10^{-9} \\ 0. \\ 0. \end{bmatrix}$$

Dvs. med den givne H-værdi vil det sorte punkt ligge på den gule helix.

3 snoninger tætpakket

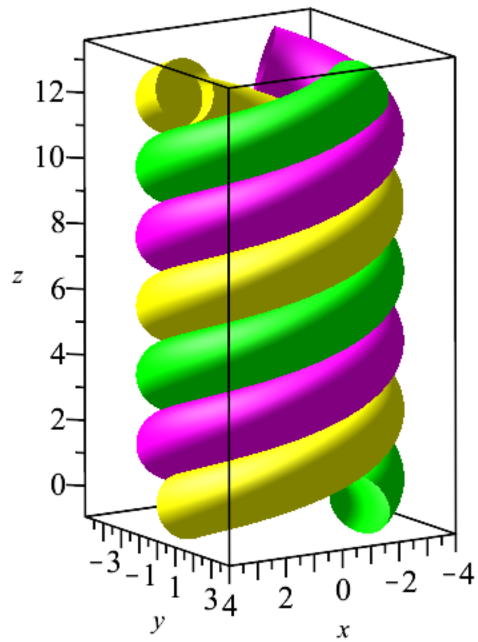
$$S := 3 :$$

$$P_1 := plot3d(subs(R=3, r=1, h=H, u_{start}=0, r_{snoet}(u, v)), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x, y, z], scaling=constrained, color=yellow, style=patchnogrid, numpoints=10000) :$$

$$P_2 := plot3d\left(subs\left(R=3, r=1, h=H, u_{start}=1\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u, v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x, y, z], scaling=constrained, color=green, style=patchnogrid, numpoints=10000\right) :$$

$$P_3 := plot3d\left(subs\left(R=3, r=1, h=H, u_{start}=2\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u, v)\right), u=0..2\cdot\pi\cdot 2, v=0..2\cdot\pi, labels=[x, y, z], scaling=constrained, color=magenta, style=patchnogrid, numpoints=10000\right) :$$

$$display(P_1, P_2, P_3)$$



Udledning af formel for stigning pr. omgang, når de 4 snoninger skal være tætpakket

Figur med indtegnet punkt på grønne helix og en bundlinje på den gule helix:

$S := 4:$

$P_1 := plot3d\left(subs\left(R = 3, r = 1, h = 2, u_{start} = 0, r_{snoet}(u, v)\right), u = 0..2\cdot\pi\cdot 2, v = 0..2\cdot\pi, labels = [x, y, z], scaling = constrained, color = yellow, style = patchnogrid, numpoints = 10000\right):$

$P_2 := plot3d\left(subs\left(R = 3, r = 1, h = 2, u_{start} = 1\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u, v)\right), u = 0..2\cdot\pi\cdot 2, v = 0..2\cdot\pi, labels = [x, y, z], scaling = constrained, color = green, style = patchnogrid, numpoints = 10000\right):$

$P_3 := plot3d\left(subs\left(R = 3, r = 1, h = 2, u_{start} = 2\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u, v)\right), u = 0..2\cdot\pi\cdot 2, v = 0..2\cdot\pi, labels = [x, y, z], scaling = constrained, color = magenta, style = patchnogrid, numpoints = 10000\right):$

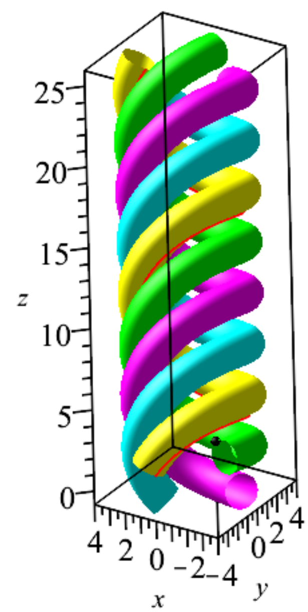
$P_4 := plot3d\left(subs\left(R = 3, r = 1, h = 2, u_{start} = 3\cdot\frac{2\cdot\pi}{S}, r_{snoet}(u, v)\right), u = 0..2\cdot\pi\cdot 2, v = 0..2\cdot\pi, labels = [x, y, z], scaling = constrained, color = cyan, style = patchnogrid, numpoints = 10000\right):$

$PUNKT_{grøn} := pointplot3d\left(\left[\left[vop\left(simplify\left(subs\left(R = 3, r = 1, h = 2, u_{start} = 1\cdot\frac{2\cdot\pi}{S}, r_{snoet}\left(0, \frac{\pi}{2}\right)\right)\right)\right)\right]\right],\right.$

$symbol = solidsphere, symbolsize = 10, color = black\Big)$:

$KURVE_{gul} := spacecurve\left(\left[vop\left(subs\left(R=3, r=1, h=2, u_{start}=0, r_{snoet}\left(u, \frac{3\cdot\pi}{2}\right)\right)\right)\right], u = 0..2\cdot\pi\cdot 2, thickness = 3, color = red\right)$:

$display(P_1, P_2, P_3, P_4, PUNKT_{grøn}, KURVE_{gul})$



Det sorte punkt på den grønne udvidede helix skal flyttes op til den røde kurve på den udvidede gule helix.
NB: minimum er IKKE ved π, fordi cirklerne ikke er lodrette!

$Difference := unapply\left(subs\left(R=3, r=1, u_{start}=0, r_{snoet}\left(u, \frac{3\cdot\pi}{2}\right)\right) - simplify\left(subs\left(R=3, r=1, u_{start}=1\cdot\frac{2\cdot\pi}{S}, r_{snoet}\left(0, \frac{\pi}{2}\right)\right)\right), [u, h]\right)$:

$$Difference(u,h) = \begin{bmatrix} 3\cos(u) - \frac{h\sin(u)}{\sqrt{h^2+9}} - \frac{h}{\sqrt{h^2+9}} \\ 3\sin(u) + \frac{h\cos(u)}{\sqrt{h^2+9}} - 3 \\ h\,u - \frac{6}{\sqrt{h^2+9}} \end{bmatrix}$$

$L := fsolve(\{Difference(u,h)[1] = 0, Difference(u,h)[3] = 0\}, \{h = 0..2, u = 0..2\cdot\pi\}) =$

$\{h = 1.404140097, u = 1.290047317\}$

NB: kan ikke løses eksakt, derfor anvendes den numeriske løser på 2 af ligningerne (ikke 3 - for så findes der ikke en løsning).

$U := rhs(L[2]) : H := rhs(L[1]) :$

$$evalf(Difference(U, H)) = \begin{bmatrix} 1. \times 10^{-10} \\ 0. \\ 0. \end{bmatrix}$$

Dvs. med den givne H-værdi vil det sorte punkt ligge på den gule helix.

4 snoninger tætpakket

$S := 4 :$

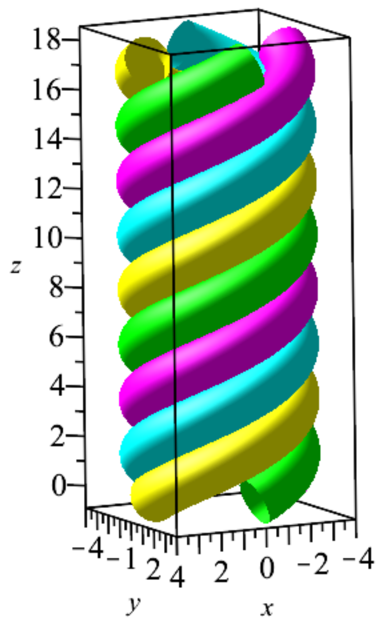
$P_1 := plot3d\left(subs\left(R = 3, r = 1, h = H, u_{start} = 0, r_{snoet}(u, v)\right), u = 0 ..2 \cdot \pi \cdot 2, v = 0 ..2 \cdot \pi, labels = [x, y, z], scaling = constrained, color = yellow, style = patchnogrid, numpoints = 10000\right) :$

$P_2 := plot3d\left(subs\left(R = 3, r = 1, h = H, u_{start} = 1 \cdot \frac{2 \cdot \pi}{S}, r_{snoet}(u, v)\right), u = 0 ..2 \cdot \pi \cdot 2, v = 0 ..2 \cdot \pi, labels = [x, y, z], scaling = constrained, color = green, style = patchnogrid, numpoints = 10000\right) :$

$P_3 := plot3d\left(subs\left(R = 3, r = 1, h = H, u_{start} = 2 \cdot \frac{2 \cdot \pi}{S}, r_{snoet}(u, v)\right), u = 0 ..2 \cdot \pi \cdot 2, v = 0 ..2 \cdot \pi, labels = [x, y, z], scaling = constrained, color = magenta, style = patchnogrid, numpoints = 10000\right) :$

$P_4 := plot3d\left(subs\left(R = 3, r = 1, h = H, u_{start} = 3 \cdot \frac{2 \cdot \pi}{S}, r_{snoet}(u, v)\right), u = 0 ..2 \cdot \pi \cdot 2, v = 0 ..2 \cdot \pi, labels = [x, y, z], scaling = constrained, color = cyan, style = patchnogrid, numpoints = 10000\right) :$

$display(P_1, P_2, P_3, P_4)$



18
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