

Spiraler (9)

Inspireret af artikel:

"Spirals on surfaces of revolution" af Cristian Lazureanu:

<http://elib.mi.sanu.ac.rs/files/journals/vm/57/vmn57p2-10.pdf>

restart

with(plots) :

with(VektorAnalyse4) :

Rotation om z-aksen:

https://en.wikipedia.org/wiki/Rotation_matrix#In_three_dimensions

$$R_z(\theta) := \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} :$$

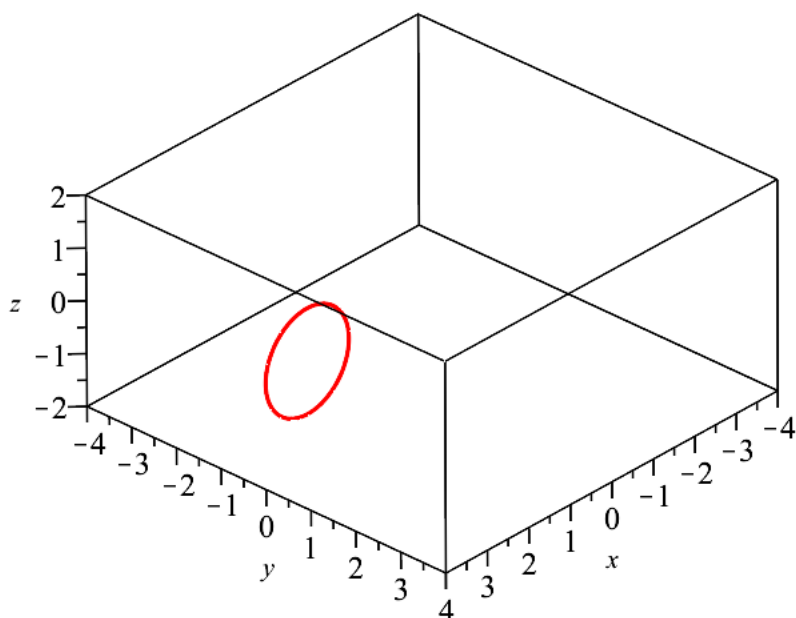
▼ Torus

Parametrisering af lodret cirkel, dvs. beliggende i xz-planen:

$C(u) := \langle r \cdot \cos(u), 0, r \cdot \sin(u) \rangle + \langle c, 0, 0 \rangle :$

hvor $u \in [0; 2 \cdot \pi]$

$P_{\text{cirkel}} := \text{spacecurve}([\text{vop}(\text{subs}(r=1, c=3, C(u)))], u=0 .. 2 \cdot \pi, \text{color} = \text{red}, \text{thickness} = 3, \text{labels} = [x, y, z], \text{scaling} = \text{constrained}, \text{view} = [-4 .. 4, -4 .. 4, -2 .. 2])$

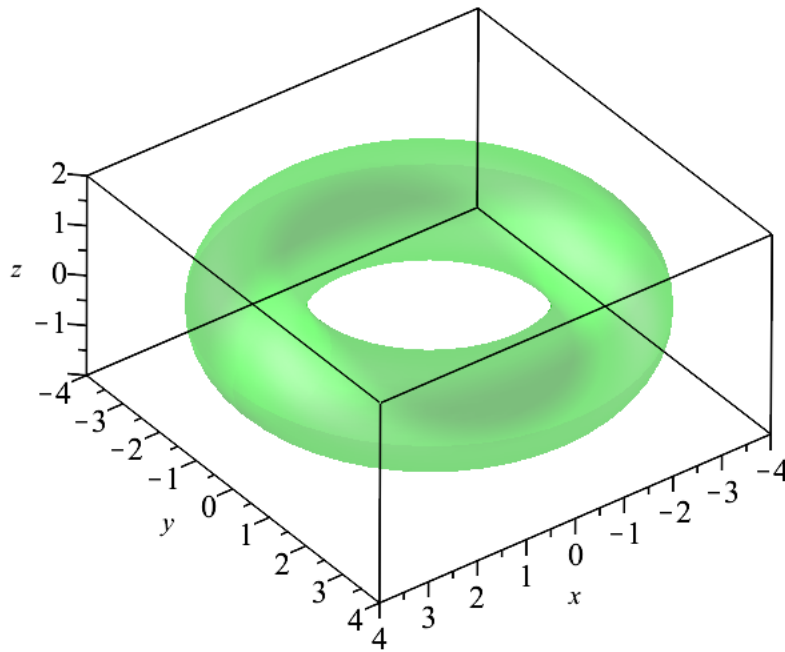


En **torus** fås ved at dreje cirklen om z-aksen med vinklen v :

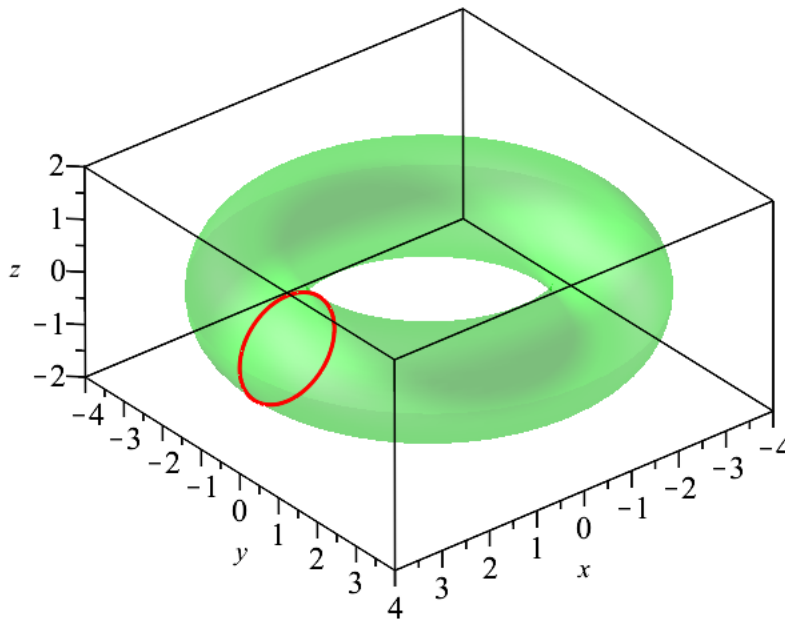
$$T(u, v) := R_z(v) \cdot C(u) :$$

$$T(u, v) = \begin{bmatrix} \cos(v) (r \cos(u) + c) \\ \sin(v) (r \cos(u) + c) \\ r \sin(u) \end{bmatrix}$$

$P_{\text{torus}} := \text{plot3d}(\text{subs}(r=1, c=3, T(u, v)), u=0..2\cdot\pi, v=0..2\cdot\pi, \text{color}=\text{green}, \text{labels}=[x, y, z], \text{scaling}=\text{constrained}, \text{style}=\text{patchnograd}, \text{transparency}=0.7, \text{view}=[-4..4, -4..4, -2..2])$



$display(P_{\text{cirke}} P_{\text{torus}})$



Helix

Helixen skal rotere n gange rundt på torussen.

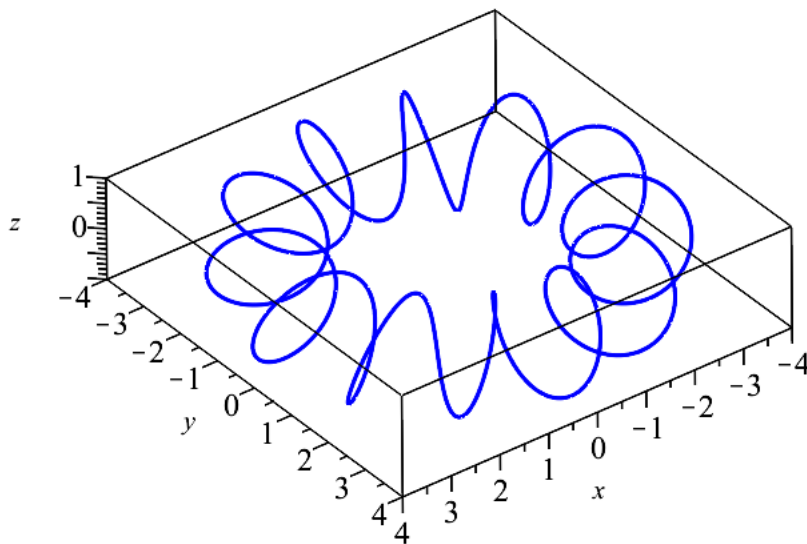
$$H(t) := R_z\left(\frac{u}{n}\right) \cdot C(u) :$$

hvor $u \in [0; 2 \cdot \pi \cdot n]$

$$H(t) = \begin{bmatrix} \cos\left(\frac{u}{n}\right) (r \cos(u) + c) \\ \sin\left(\frac{u}{n}\right) (r \cos(u) + c) \\ r \sin(u) \end{bmatrix}$$

$n := 12 :$

$P_{helix} := \text{spacecurve}([\text{vop}(\text{subs}(r=1, c=3, H(u))), u=0..2 \cdot \pi \cdot n, \text{color}=\text{blue}, \text{thickness}=3, \text{labels}=[x, y, z], \text{scaling}=\text{constrained}, \text{numpoints}=1000)$



Helix på en torus

$\text{display}(P_{torus}, P_{helix})$

