

Analytisk funktion

En C^∞ -funktion, hvis Taylor-række konvergerer imod funktionen i en omegn af punktet x_0 , siges at være en **analytisk funktion**.

Læs nærmere: http://en.wikipedia.org/wiki/Analytic_function

Ikke alle funktion, som kan differentieres uendelig mange gange er analytiske!

Modeksempel:

Definer følgende funktion:

$$f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ e^{-\frac{1}{x^2}} & \text{for } x > 0 \end{cases}$$

Denne funktion $f \in C^\infty$, men er IKKE en analytisk funktion, da Taylor-rækken ud fra ud fra $x_0 = 0$ er identisk med 0!

> restart;

Venstre del af funktionen:

> fv := x → 0;

$$fv := x \rightarrow 0$$

(1)

Højre del af funktionen:

> fh := x → exp(-1/x^2);

$$fh := x \rightarrow e^{-\frac{1}{x^2}}$$

(2)

> with(plots) :

> plot1 := plot(fv(x), x = -3 .. 0, color = blue, thickness = 2);

$$plot1 := PLOT(...)$$

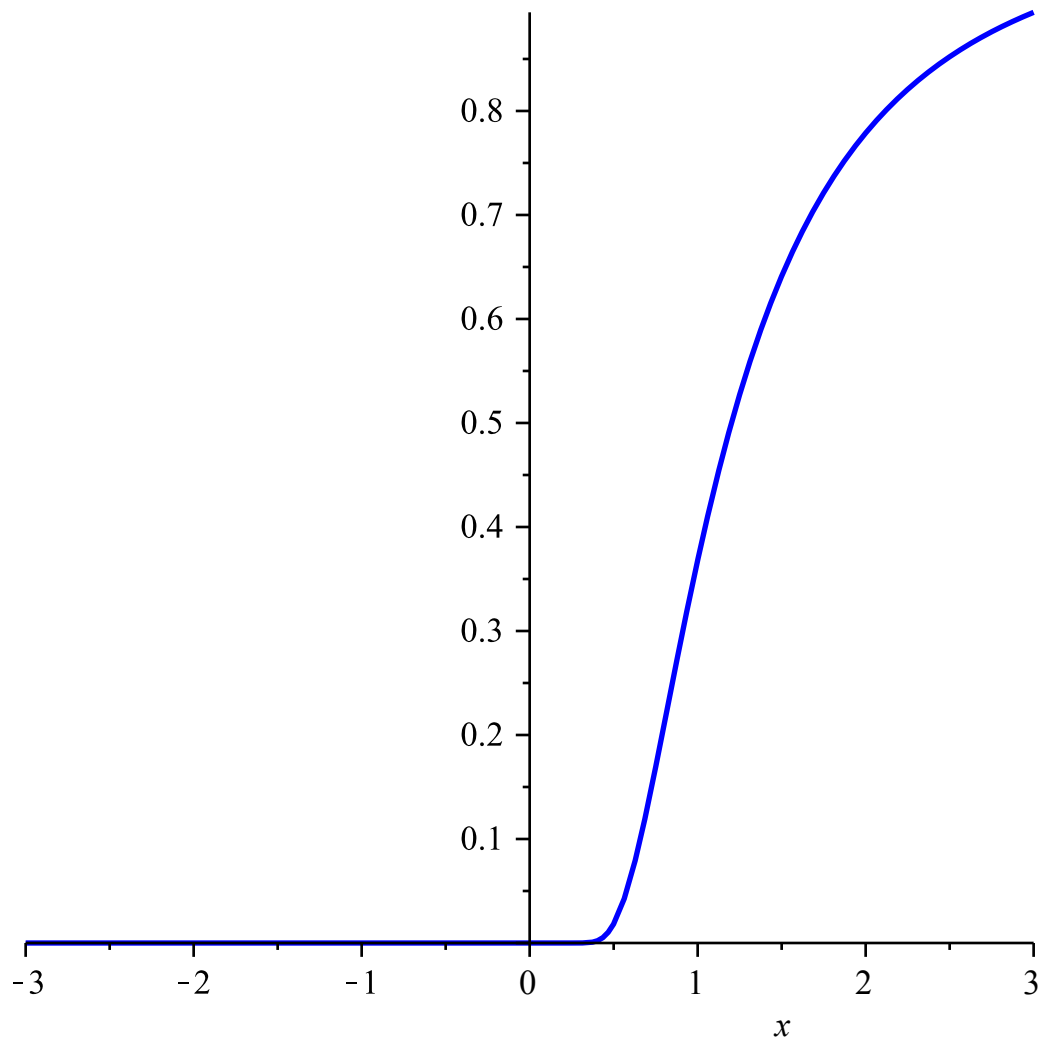
(3)

> plot2 := plot(fh(x), x = 0 .. 3, color = blue, thickness = 2);

$$plot2 := PLOT(...)$$

(4)

> display(plot1, plot2);

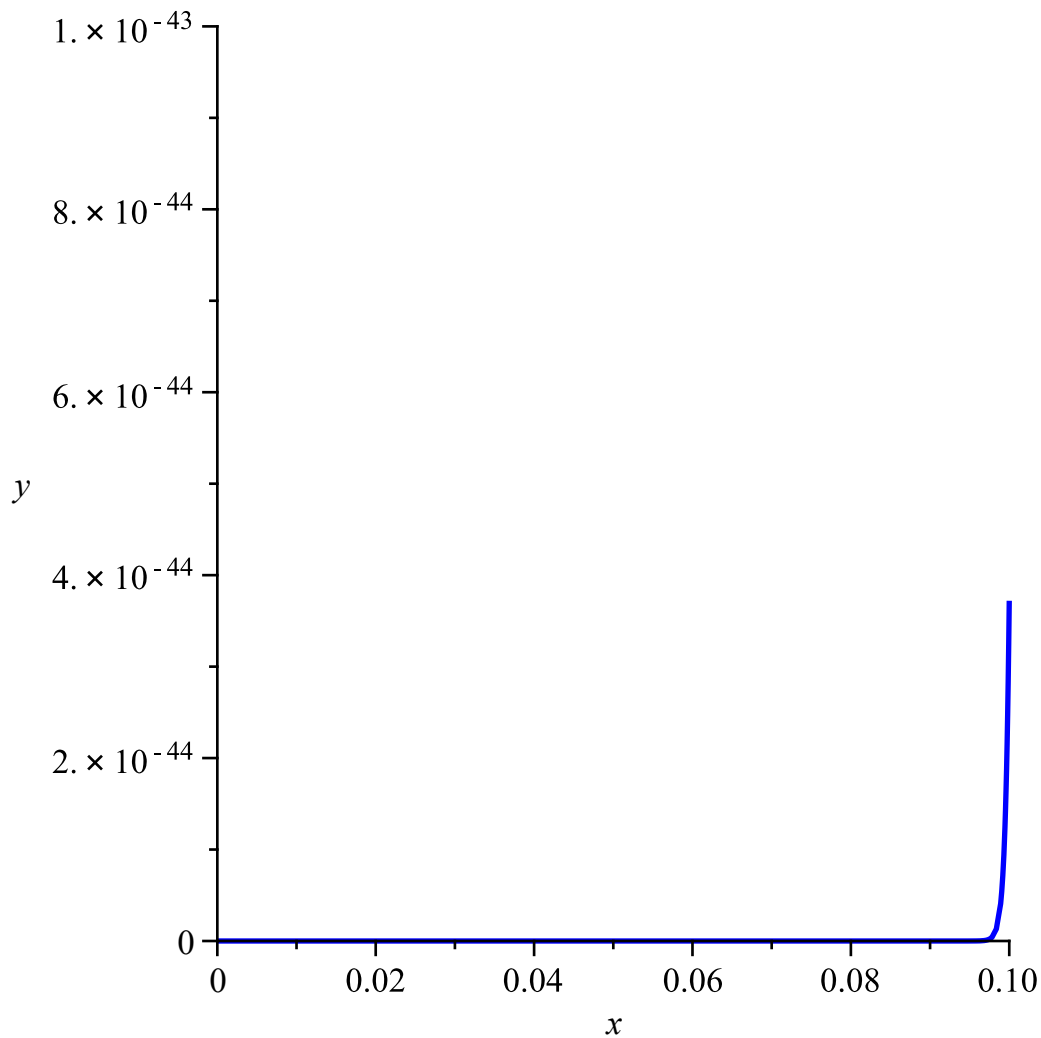


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> fh(0.1);
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$3.720075976 \cdot 10^{-44}$

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> plot(fh(x), x=0..0.1, y=0..10-43, color=blue, thickness=2);
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(5)



Funktionen er så flad i $x_0 = 0$, at alle afledede i dette punkt er 0!

Derfor er Taylorrækken ud fra $x_0 = 0$ identisk med nulpolynomiet.

Det betyder, at Taylorrækken ud fra $x_0 = 0$ ALDRIG konvergerer imod $f(x)$ for noget $x > 0$.

$$> \lim_{x \rightarrow 0^+} (fh(x));$$

0

(6)

Dvs. $f(0) = 0$, og f er kontinuert i 0.

$$> f_1 := \text{unapply}\left(\frac{d}{dx} fh(x), x\right);$$

$$f_1 := x \rightarrow \frac{2e^{-\frac{1}{x^2}}}{x^3}$$

(7)

$$> \lim_{x \rightarrow 0^+} (f_1(x));$$

0

(8)

$$\gt \lim_{x \rightarrow 0^+} \left(\frac{fh(x) - 0}{x - 0} \right);$$

0

(9)

Dvs. $f'(0) = 0$, og f' er kontinuert i 0.

$$\gt f_2 := \text{unapply} \left(\frac{d}{dx} f_1(x), x \right);$$

$$f_2 := x \rightarrow -\frac{6 e^{-\frac{1}{x^2}}}{x^4} + \frac{4 e^{-\frac{1}{x^2}}}{x^6}$$

(10)

$$\gt \lim_{x \rightarrow 0^+} (f_2(x));$$

0

(11)

$$\gt \lim_{x \rightarrow 0^+} \left(\frac{f_1(x) - 0}{x - 0} \right);$$

0

(12)

Dvs. $f''(0) = 0$, og f'' er kontinuert i 0.

Osv.

$$\gt \text{for } n \text{ from 2 by 1 to 10 do } f_{n+1} := \text{unapply} \left(\frac{d}{dx} f_n(x), x \right) \text{ end do};$$

$$f_3 := x \rightarrow \frac{24 e^{-\frac{1}{x^2}}}{x^5} - \frac{36 e^{-\frac{1}{x^2}}}{x^7} + \frac{8 e^{-\frac{1}{x^2}}}{x^9}$$

$$f_4 := x \rightarrow -\frac{120 e^{-\frac{1}{x^2}}}{x^6} + \frac{300 e^{-\frac{1}{x^2}}}{x^8} - \frac{144 e^{-\frac{1}{x^2}}}{x^{10}} + \frac{16 e^{-\frac{1}{x^2}}}{x^{12}}$$

$$f_5 := x \rightarrow \frac{720 e^{-\frac{1}{x^2}}}{x^7} - \frac{2640 e^{-\frac{1}{x^2}}}{x^9} + \frac{2040 e^{-\frac{1}{x^2}}}{x^{11}} - \frac{480 e^{-\frac{1}{x^2}}}{x^{13}} + \frac{32 e^{-\frac{1}{x^2}}}{x^{15}}$$

$$f_6 := x \rightarrow -\frac{5040 e^{-\frac{1}{x^2}}}{x^8} + \frac{25200 e^{-\frac{1}{x^2}}}{x^{10}} - \frac{27720 e^{-\frac{1}{x^2}}}{x^{12}} + \frac{10320 e^{-\frac{1}{x^2}}}{x^{14}} - \frac{1440 e^{-\frac{1}{x^2}}}{x^{16}}$$

$$+ \frac{64 e^{-\frac{1}{x^2}}}{x^{18}}$$

$$f_7 := x \rightarrow \frac{40320 e^{-\frac{1}{x^2}}}{x^9} - \frac{262080 e^{-\frac{1}{x^2}}}{x^{11}} + \frac{383040 e^{-\frac{1}{x^2}}}{x^{13}} - \frac{199920 e^{-\frac{1}{x^2}}}{x^{15}} + \frac{43680 e^{-\frac{1}{x^2}}}{x^{17}}$$

$$\begin{aligned}
 & -\frac{4032 e^{-\frac{1}{x^2}}}{x^{19}} + \frac{128 e^{-\frac{1}{x^2}}}{x^{21}} \\
 f_8 := x \rightarrow & -\frac{362880 e^{-\frac{1}{x^2}}}{x^{10}} + \frac{2963520 e^{-\frac{1}{x^2}}}{x^{12}} - \frac{5503680 e^{-\frac{1}{x^2}}}{x^{14}} + \frac{3764880 e^{-\frac{1}{x^2}}}{x^{16}} \\
 & -\frac{1142400 e^{-\frac{1}{x^2}}}{x^{18}} + \frac{163968 e^{-\frac{1}{x^2}}}{x^{20}} - \frac{10752 e^{-\frac{1}{x^2}}}{x^{22}} + \frac{256 e^{-\frac{1}{x^2}}}{x^{24}} \\
 f_9 := x \rightarrow & \frac{3628800 e^{-\frac{1}{x^2}}}{x^{11}} - \frac{36288000 e^{-\frac{1}{x^2}}}{x^{13}} + \frac{82978560 e^{-\frac{1}{x^2}}}{x^{15}} - \frac{71245440 e^{-\frac{1}{x^2}}}{x^{17}} \\
 & + \frac{28092960 e^{-\frac{1}{x^2}}}{x^{19}} - \frac{5564160 e^{-\frac{1}{x^2}}}{x^{21}} + \frac{564480 e^{-\frac{1}{x^2}}}{x^{23}} - \frac{27648 e^{-\frac{1}{x^2}}}{x^{25}} + \frac{512 e^{-\frac{1}{x^2}}}{x^{27}} \\
 f_{10} := x \rightarrow & -\frac{39916800 e^{-\frac{1}{x^2}}}{x^{12}} + \frac{479001600 e^{-\frac{1}{x^2}}}{x^{14}} - \frac{1317254400 e^{-\frac{1}{x^2}}}{x^{16}} + \frac{1377129600 e^{-\frac{1}{x^2}}}{x^{18}} \\
 & -\frac{676257120 e^{-\frac{1}{x^2}}}{x^{20}} + \frac{173033280 e^{-\frac{1}{x^2}}}{x^{22}} - \frac{24111360 e^{-\frac{1}{x^2}}}{x^{24}} + \frac{1820160 e^{-\frac{1}{x^2}}}{x^{26}} \\
 & -\frac{69120 e^{-\frac{1}{x^2}}}{x^{28}} + \frac{1024 e^{-\frac{1}{x^2}}}{x^{30}} \\
 f_{11} := x \rightarrow & \frac{479001600 e^{-\frac{1}{x^2}}}{x^{13}} - \frac{6785856000 e^{-\frac{1}{x^2}}}{x^{15}} + \frac{22034073600 e^{-\frac{1}{x^2}}}{x^{17}} \\
 & -\frac{27422841600 e^{-\frac{1}{x^2}}}{x^{19}} + \frac{16279401600 e^{-\frac{1}{x^2}}}{x^{21}} - \frac{5159246400 e^{-\frac{1}{x^2}}}{x^{23}} \\
 & + \frac{924739200 e^{-\frac{1}{x^2}}}{x^{25}} - \frac{95546880 e^{-\frac{1}{x^2}}}{x^{27}} + \frac{5575680 e^{-\frac{1}{x^2}}}{x^{29}} - \frac{168960 e^{-\frac{1}{x^2}}}{x^{31}} \\
 & + \frac{2048 e^{-\frac{1}{x^2}}}{x^{33}}
 \end{aligned}$$

(13)

> for n from 2 by 1 to 10 do $\lim_{x \rightarrow 0^+} (f_{n+1}(x))$ end do;

0

0

0
0
0
0
0
0
0

(14)

> for n from 2 by 1 to 10 do $\lim_{x \rightarrow 0^+} \left(\frac{f_n(x) - 0}{x - 0} \right)$ end do;

0
0
0
0
0
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0
0
0
0

(15)