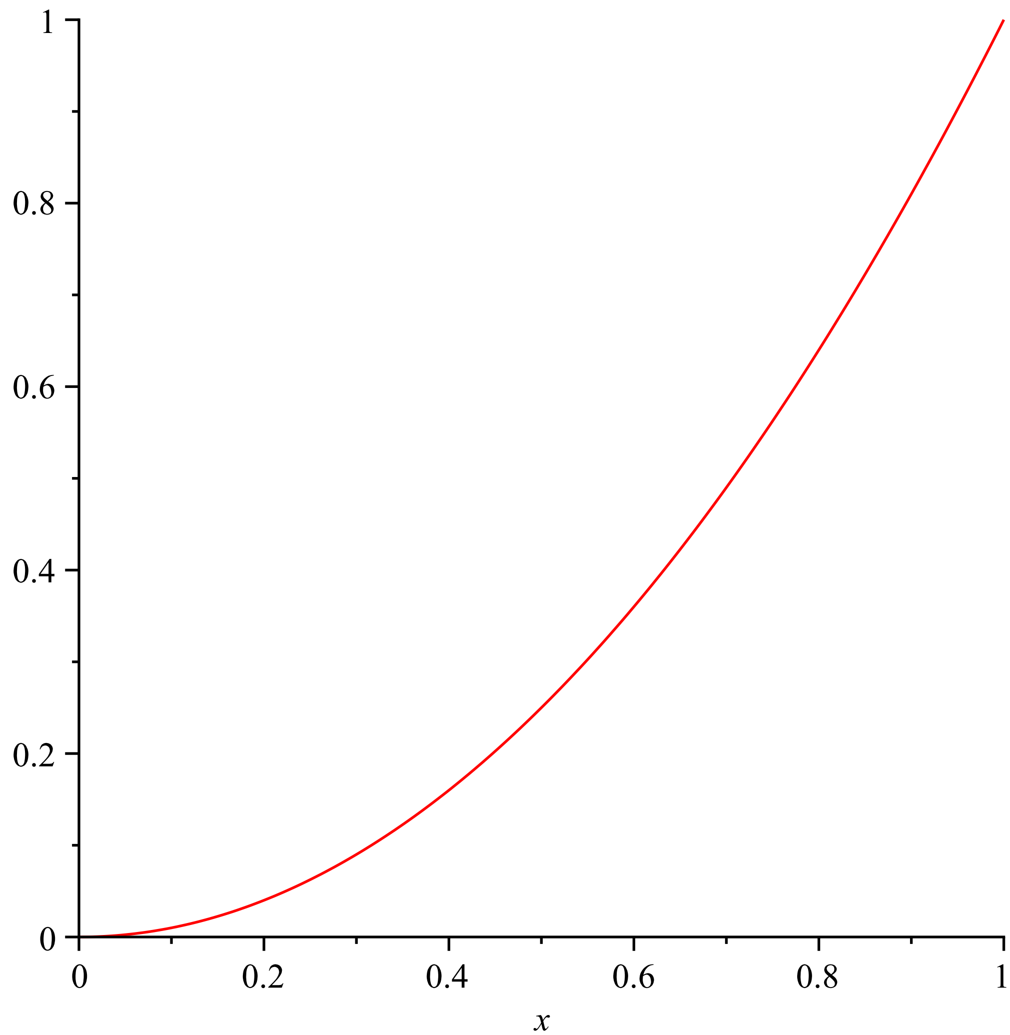
Buelængde

> $restart$

> $f := x \rightarrow x^2$

$$f := x \rightarrow x^2 \quad (1)$$

> $plot(f(x), x = 0..1)$



Generel formel for buelængde:

$$Buelængde = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Se formel, bevis og eksempler:

http://en.wikipedia.org/wiki/Arc_length

http://archives.math.utk.edu/visual.calculus/5/arclength.1/

> $f'(x)$

$$2\,x \quad (2)$$

> $\sqrt{1+(f'(x))^2}$

$$\sqrt{1+4x^2} \quad (3)$$

> $\int_0^1 \sqrt{1+(f'(x))^2}\,dx$; *evalf*(%)

$$\frac{1}{2}\sqrt{5} - \frac{1}{4}\ln\left(-2+\sqrt{5}\right)$$

$$1.478942857 \quad (4)$$

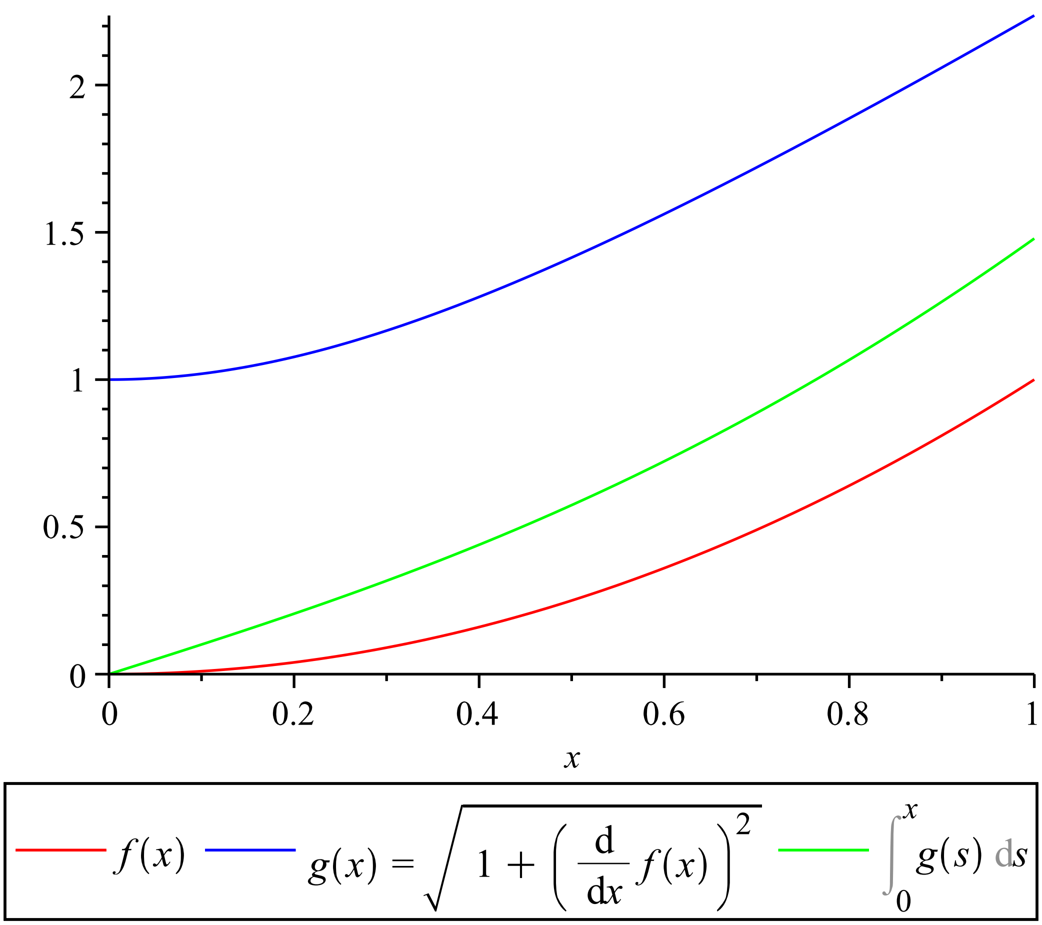
> *with*(*Student*[*Calculus1*]) :

> *ArcLength*($f(x)$, $x = 0\,..1$); *evalf*(%)

$$\frac{1}{2}\sqrt{5} - \frac{1}{4}\ln\left(-2+\sqrt{5}\right)$$

$$1.478942857 \quad (5)$$

> *ArcLength*($f(x)$, $x = 0\,..1$, *output* = *plot*)



The arc length of $f(x) = x^2$ on the interval $[0, 1]$. The coordinate system is Cartesian.

Konklusion: Buelængde af parablen $f(x) = x^2$ for $x \in [0; 1]$ er altså

$$\frac{1}{2}\sqrt{5} - \frac{1}{4}\ln\left(-2+\sqrt{5}\right) \approx 1.48$$