

## Amplitude af svingningen $a \cdot \cos(x) + b \cdot \sin(x)$

Et udtryk af formen  $a \cdot \cos(x) + b \cdot \sin(x)$  kan omskrives.

$$a \cdot \cos(x) + b \cdot \sin(x) = \sqrt{a^2 + b^2} \cdot \left( \frac{a}{\sqrt{a^2 + b^2}} \cdot \cos(x) + \frac{b}{\sqrt{a^2 + b^2}} \cdot \sin(x) \right)$$

Da koefficienterne  $\begin{bmatrix} \frac{a}{\sqrt{a^2 + b^2}} \\ \frac{b}{\sqrt{a^2 + b^2}} \end{bmatrix}$  udgør en enheds-vektor, findes der en vinkel  $v$ , så vektoren er  $\begin{bmatrix} \cos(v) \\ \sin(v) \end{bmatrix}$ .

Derfor kan vi omskrive videre:

$$a \cdot \cos(x) + b \cdot \sin(x) = \sqrt{a^2 + b^2} \cdot \left( \frac{a}{\sqrt{a^2 + b^2}} \cdot \cos(x) + \frac{b}{\sqrt{a^2 + b^2}} \cdot \sin(x) \right) = \sqrt{a^2 + b^2} \cdot (\cos(v) \cdot \cos(x) + \sin(v) \cdot \sin(x))$$

En **trigonometrisk sumformel** anvendes:

$$\cos(\alpha - \beta) = \cos(\alpha) \cdot \cos(\beta) + \sin(\alpha) \cdot \sin(\beta)$$

[http://en.wikipedia.org/wiki/Double-angle\\_formula#Angle\\_sum\\_and\\_difference\\_identities](http://en.wikipedia.org/wiki/Double-angle_formula#Angle_sum_and_difference_identities)

Så kan omskrivningen fortsættes:

$$a \cdot \cos(x) + b \cdot \sin(x) = \sqrt{a^2 + b^2} \cdot \left( \frac{a}{\sqrt{a^2 + b^2}} \cdot \cos(x) + \frac{b}{\sqrt{a^2 + b^2}} \cdot \sin(x) \right) = \sqrt{a^2 + b^2} \cdot (\cos(v) \cdot \cos(x) + \sin(v) \cdot \sin(x)) = \sqrt{a^2 + b^2} \cdot \cos(v - x)$$

**Dette er en svingning med amplituden  $\sqrt{a^2 + b^2}$**

**NB:**

**Sumformlen kan vises via omskrivning med komplekse tal:**

$$\cos(\alpha - \beta) =$$

$$\operatorname{Re}(e^{i \cdot (\alpha - \beta)}) =$$

$$\operatorname{Re}(e^{i \cdot \alpha} \cdot e^{-i \cdot \beta}) =$$

$$\operatorname{Re}((\cos(\alpha) + i \cdot \sin(\alpha)) \cdot (\cos(-\beta) + i \cdot \sin(-\beta))) =$$

$$\operatorname{Re}((\cos(\alpha) + i \cdot \sin(\alpha)) \cdot (\cos(\beta) - i \cdot \sin(\beta))) =$$

$$\operatorname{Re}((\cos(\alpha) \cdot \cos(\beta) + \sin(\alpha) \cdot \sin(\beta)) + i \cdot (\sin(\alpha) \cdot \cos(\beta) - \cos(\alpha) \cdot \sin(\beta))) =$$

$$\cos(\alpha) \cdot \cos(\beta) + \sin(\alpha) \cdot \sin(\beta)$$