Parametrisering og plots af Tycho Brahe Planetarium

Data om Planetarium:

http://www.planetariet.dk/bes%C3%B8g-os/om-planetariet/historien-om-planetariet/bygningen

Højde: 38 m
Diameter: 26 m
Taghældning: 30°

NB: Højeste punkt på Tycho Brahe Planetarium placeres i $(0, 0, 38)$.

```
> restart
> with(plots) :
```

Randen i bunden (cirkel)

Parametrisering med 1 parameter i 3D, hvor $u \in [0; 2\cdot\pi]$:

```
> r1 := u → ⟨a·cos(u) + a, a·sin(u), 0⟩ :
  'r1(u)'=r1(u)
```

$$r1(u) = \begin{bmatrix} a\cos(u) + a \\ a\sin(u) \\ 0 \end{bmatrix} \quad \textbf{(1.1)}$$

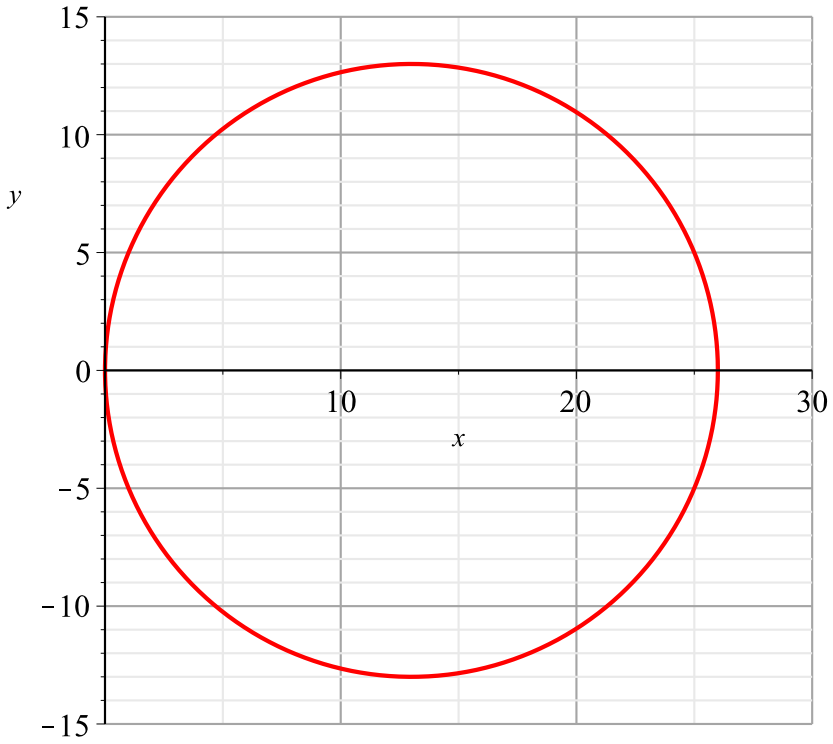
For at plotte vælges en værdi for a (radius i cirklen):

```
> a := 13
```

$$a := 13 \quad \textbf{(1.2)}$$

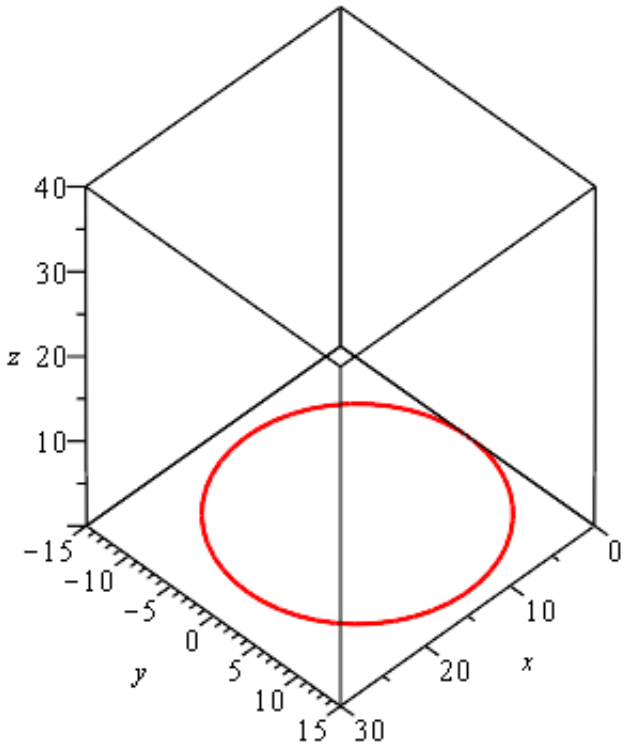
Plot i 2D:

```
> plot([r1(u)[1], r1(u)[2], u = 0..2·π], labels = [x, y], view = [0..30, -15..15], thickness = 2, color
      = red, gridlines, scaling = constrained);
```



Plot i 3D:

> $spacecurve(r1(u), u = 0..2\cdot\pi, axes = box, labels = [x, y, z], view = [0..30, -15..15, 0..40], color = red, thickness = 2, scaling = constrained)$



Bunden (cirkelskive)

> *unassign*('*a*')

Parametrisering med 2 parametre i 3D, hvor $u \in [0; 2\cdot\pi]$ og $v \in [0; 1]$:

> $r2 := (u, v) \to \langle v\cdot a\cdot\cos(u) + a, v\cdot a\cdot\sin(u), 0\rangle$:

'$r2(u, v)$'$= r2(u, v)$

$$r2(u, v) = \begin{bmatrix} v\,a\cos(u) + a \\ v\,a\sin(u) \\ 0 \end{bmatrix} \quad \textbf{(2.1)}$$

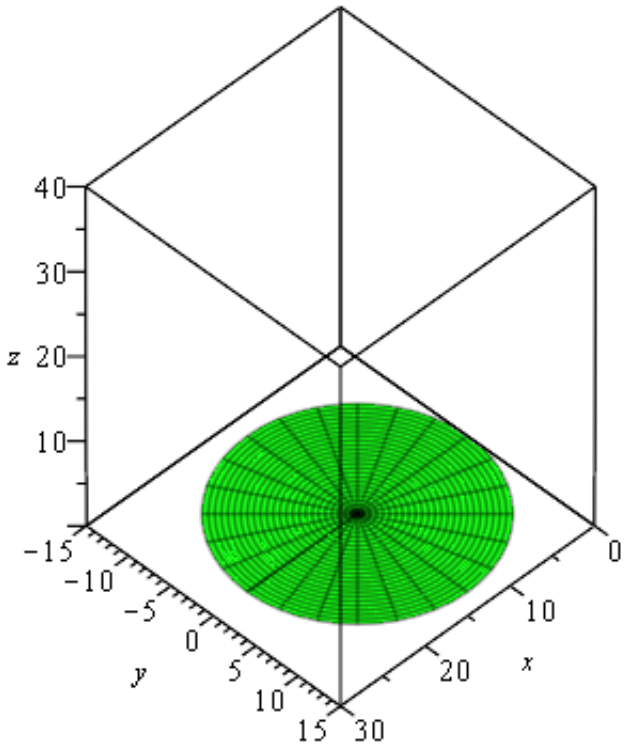
For at plotte vælges en værdi for a (radius i cirklen):

> $a := 13$

$$a := 13 \quad \textbf{(2.2)}$$

Plot i 3D:

> *plot3d*$(r2(u, v), u = 0..2\cdot\pi, v = 0..1, labels = [x, y, z], axes = box, view = [0..30, -15..15, 0..40], scaling = constrained, color = green)$



Kanten langs toppen (løftet cirkel)

> *unassign*('*a*')

Hældningskoefficienten for tagfladen, når vinklen med vandret er 30° :

> $\alpha := \tan\left(\frac{-30}{180}\cdot\pi\right)$

$$\alpha := -\frac{1}{3}\sqrt{3} \qquad \textbf{(3.1)}$$

Ligning for plan, som indeholder tagfladen:

$$z = \alpha\cdot x + h = -\frac{\sqrt{3}}{3}\cdot x + 38$$

Parametrisering med 1 parameter i 3D, hvor $u \in [0; 2\cdot\pi]$:

> $r3 := u \to \langle a\cdot\cos(u) + a,\ a\cdot\sin(u),\ \alpha\cdot(a\cdot\cos(u) + a) + h\rangle$:
> '*r3*(*u*)'=*r3*(*u*)

$$r3(u) = \begin{bmatrix} a\cos(u) + a \\ a\sin(u) \\ -\frac{1}{3}\sqrt{3}\,(a\cos(u) + a) + h \end{bmatrix} \quad \textbf{(3.2)}$$

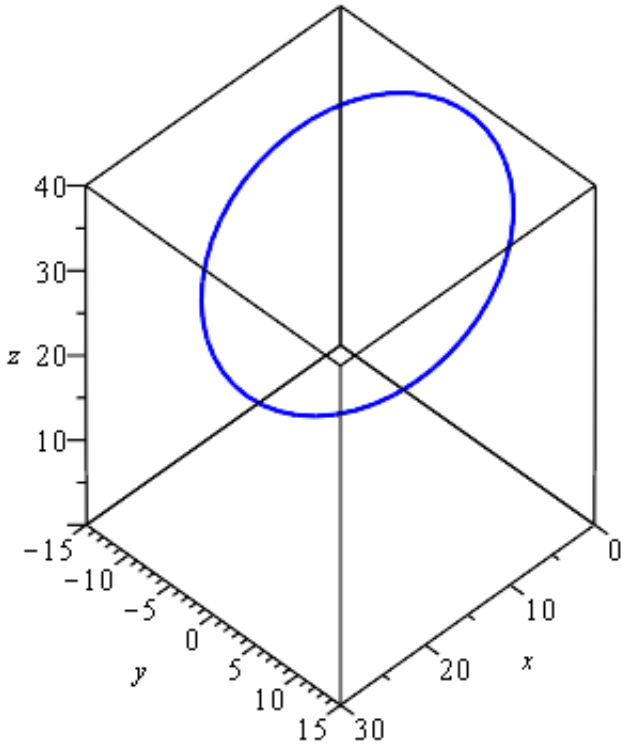
For at plotte vælges en værdi for a (radius i cirklen på jorden) og h (højden):

```
> a := 13;
  h := 38
```

$$a := 13$$
$$h := 38 \quad \textbf{(3.3)}$$

Plot i 3D:

```
> spacecurve(r3(u), u = 0..2·π, axes = box, labels = [x, y, z], view = [0..30, -15..15, 0..40],
      color = blue, thickness = 2, scaling = constrained)
```



Skrå tagflade (løftet cirkelskive)

```
> unassign('a');
  unassign('h')
```

Parametrisering med 2 parametre i 3D, hvor $u \in [0; 2\cdot\pi]$ og $v \in [0; 1]$:

> $r4 := (u, v) \to \langle v\cdot a\cdot\cos(u) + a, v\cdot a\cdot\sin(u), \alpha\cdot(v\cdot a\cdot\cos(u) + a) + h\rangle$:
> $'r4(u, v)' = r4(u, v)$

$$r4(u, v) = \begin{bmatrix} v\,a\cos(u) + a \\ v\,a\sin(u) \\ -\frac{1}{3}\sqrt{3}\,(v\,a\cos(u) + a) + h \end{bmatrix} \quad \textbf{(4.1)}$$

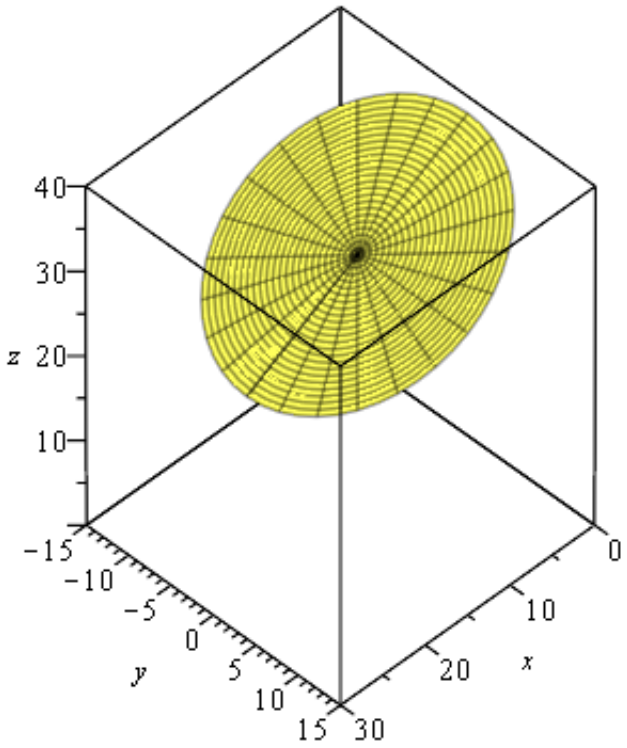
For at plotte vælges en værdi for a (radius i cirklen på jorden) og h (højden):

> $a := 13;$
> $h := 38$

$$a := 13$$
$$h := 38 \quad \textbf{(4.2)}$$

Plot i 3D:

> $plot3d(r4(u, v), u = 0..2\cdot\pi, v = 0..1, labels = [x, y, z], axes = box, view = [0..30, -15..15, 0..40], scaling = constrained, color = yellow)$



Vægfladen (afskåret cylinder)

```
> unassign('a');
  unassign('h')
```

Parametrisering med 2 parametre i 3D, hvor $u \in [0; 2\cdot\pi]$ og $v \in [0; 1]$:

```
> r5 := u → ⟨a·cos(u) + a, a·sin(u), v·(α·(a·cos(u) + a) + h)⟩ :
  'r5(u, v)' = r5(u, v)
```

$$r5(u, v) = \begin{bmatrix} a\cos(u) + a \\ a\sin(u) \\ v\left(-\frac{1}{3}\sqrt{3}\,(a\cos(u) + a) + h\right) \end{bmatrix} \quad \textbf{(5.1)}$$

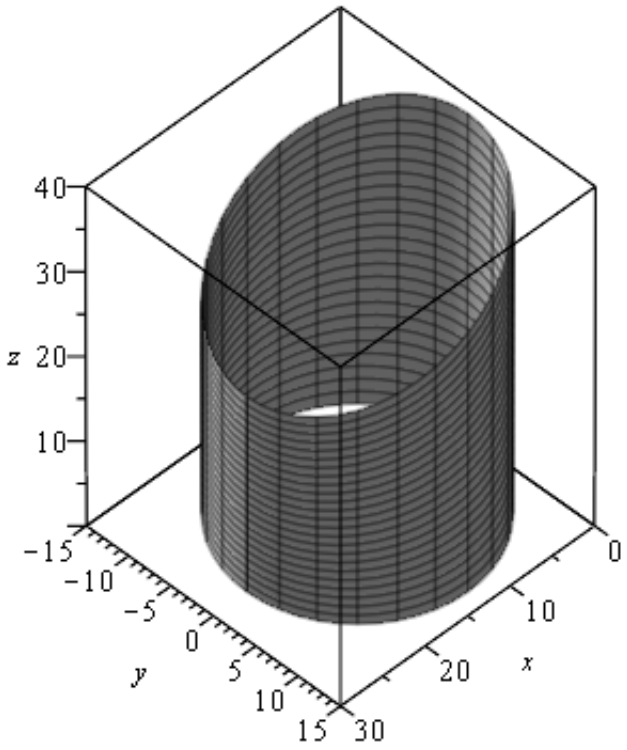
For at plotte vælges en værdi for a (radius i cirklen på jorden) og h (højden):

```
> a := 13;
  h := 38
```

$$a := 13$$
$$h := 38 \quad \textbf{(5.2)}$$

Plot i 3D:

```
> plot3d(r5(u, v), u = 0..2·π, v = 0..1, labels = [x, y, z], axes = box, view = [0..30, -15..15, 0
      ..40], scaling = constrained, color = gray)
```

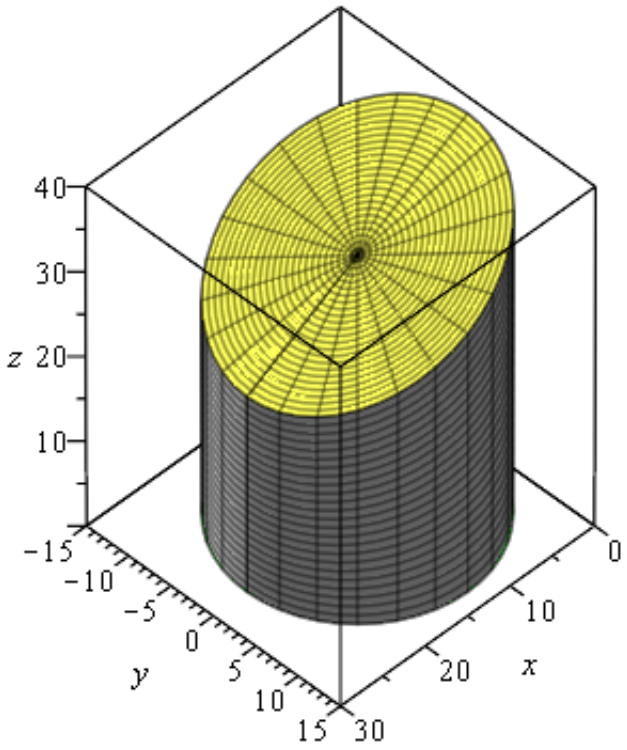


Planetarium (3 stk. 2D-flader)

Bruger ovenstående parametriseringer med 2 parametre i 3D.

Plot i 3D:

```
> bund := plot3d(r2(u, v), u = 0 ..2·π, v = 0 ..1, labels = [x, y, z], axes = box, view = [0 ..30, -15
      ..15, 0 ..40], scaling = constrained, color = green) :
  tag := plot3d(r4(u, v), u = 0 ..2·π, v = 0 ..1, labels = [x, y, z], axes = box, view = [0 ..30, -15
      ..15, 0 ..40], scaling = constrained, color = yellow) :
  sideflade := plot3d(r5(u, v), u = 0 ..2·π, v = 0 ..1, labels = [x, y, z], axes = box, view = [0 ..30,
      -15 ..15, 0 ..40], scaling = constrained, color = gray) :
  display(bund, tag, sideflade)
```



Planetarium (3D-rum)

```
> unassign('a');
  unassign('h')
```

Parametrisering med 3 parametre i 3D, hvor $u \in [0; 2\cdot\pi]$ og $v \in [0; 1]$ og $w \in [0; 1]$:

```
> r6 := (u, v, w) → ⟨w·a·cos(u) + a, w·a·sin(u), v·(α·(w·a·cos(u) + a) + h)⟩ :
  'r6(u, v, w)' = r6(u, v, w)
```

$$r6(u, v, w) = \begin{bmatrix} w\,a\cos(u) + a \\ w\,a\sin(u) \\ v\left(-\frac{1}{3}\sqrt{3}\,(w\,a\cos(u) + a) + h\right) \end{bmatrix} \quad \textbf{(7.1)}$$

For at plotte vælges en værdi for a (radius i cirklen på jorden) og h (højden):

```
> a := 13;
  h := 38
```

$$a := 13$$
$$h := 38 \quad \textbf{(7.2)}$$

```
> with(Integrator8) :
```

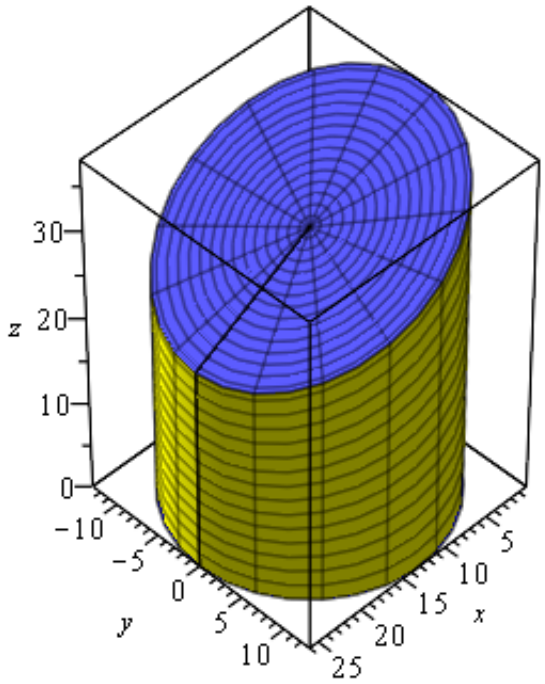
> $B := [0,\ 2\cdot\pi, 0,\ 1, 0, 1]$

$$B := [0, 2\,\pi, 0, 1, 0, 1] \quad \textbf{(7.3)}$$

> $net := [10, 10, 10]$

$$net := [10, 10, 10] \quad \textbf{(7.4)}$$

> $bygning := Integrator8[sideFlader](r6, B, net)$:

Plot i 3D:

> $display(bygning, axes = box, labels = [x, y, z])$