

Analytisk funktion

En C^∞ -funktion, hvis Taylor-række konvergerer imod funktionen i en omegn af punktet x_0 , siges at være en **analytisk funktion**.

Læs nærmere: http://en.wikipedia.org/wiki/Analytic_function

Ikke alle funktion, som kan differentieres uendelig mange gange er analytiske!

Modeksempel:

Definer følgende funktion:

$$f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ e^{-\frac{1}{x^2}} & \text{for } x > 0 \end{cases}$$

Denne funktion $f \in C^\infty$, men er IKKE en analytisk funktion, da Taylor-rækken ud fra ud fra $x_0 = 0$ er identisk med 0!

> restart

Venstre del af funktionen:

> fv := x → 0

$$fv := x \rightarrow 0 \quad (1)$$

Højre del af funktionen:

> fh := x → exp $\left(-\frac{1}{x^2}\right)$

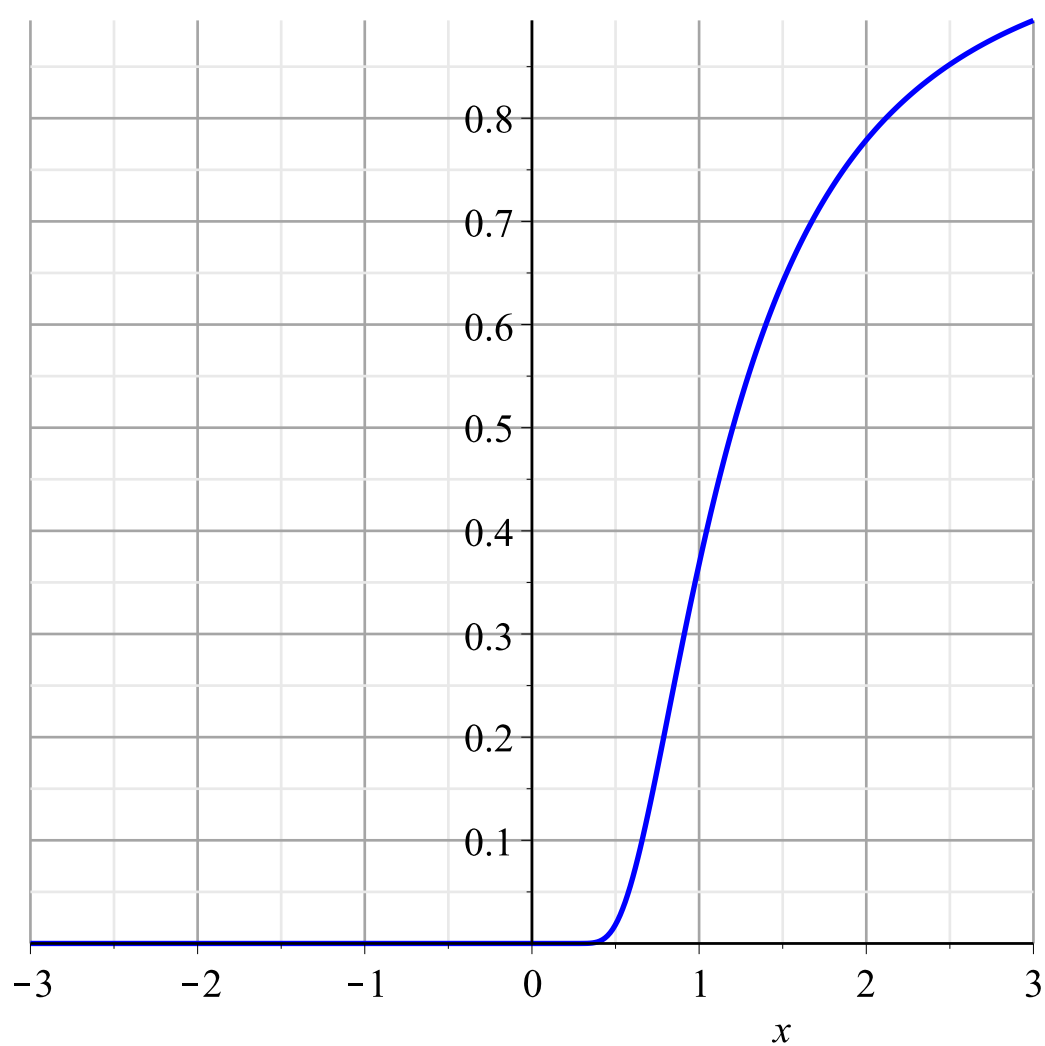
$$fh := x \rightarrow e^{-\frac{1}{x^2}} \quad (2)$$

> with(plots) :

> plot1 := plot(fv(x), x = -3 .. 0, color = blue, thickness = 2) :

> plot2 := plot(fh(x), x = 0 .. 3, color = blue, thickness = 2) :

> display(plot1, plot2, gridlines)

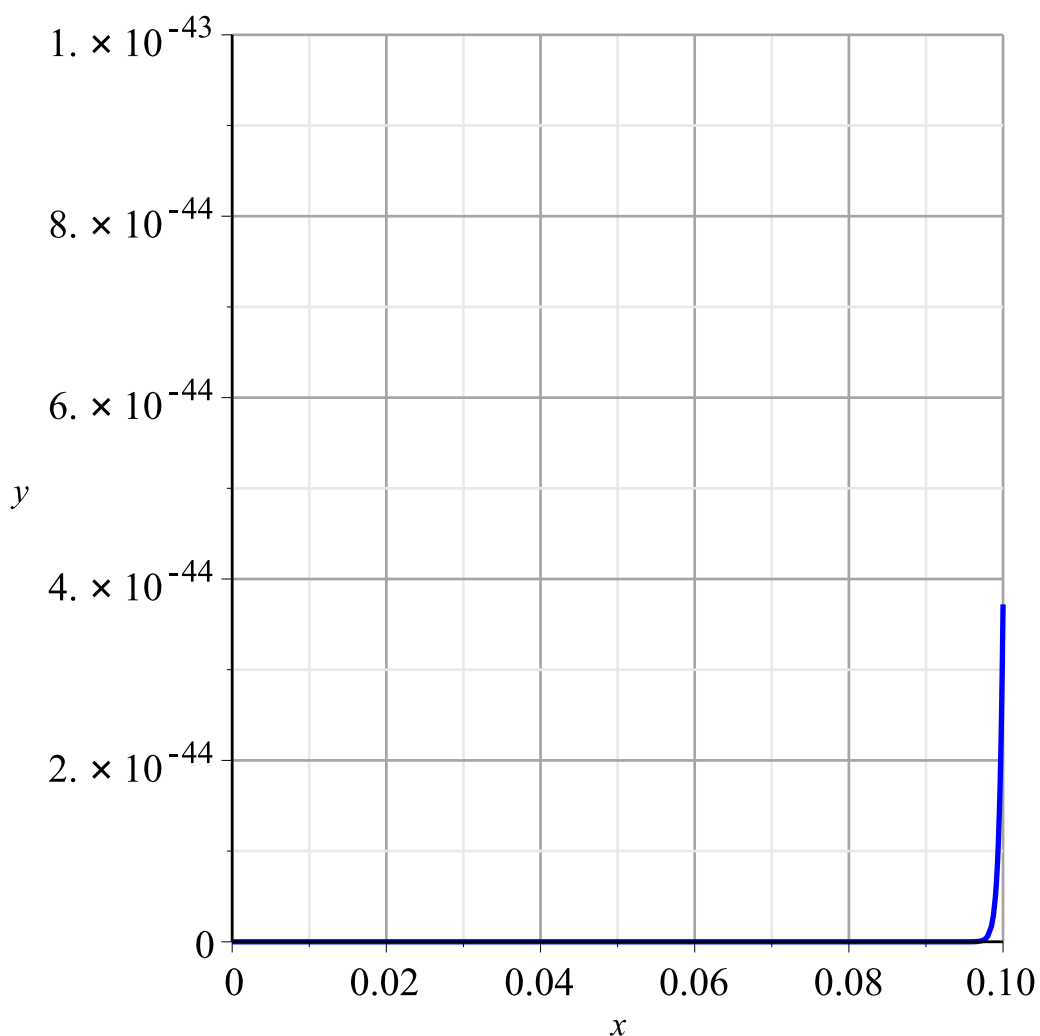


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> fh(0.1)
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$3.720075976 \cdot 10^{-44}$

(3)

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> plot(fh(x), x=0..0.1, y=0..10-43, color=blue, thickness=2, gridlines)
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Funktionen er så flad i $x_0 = 0$, at alle afledede i dette punkt er 0!

Derfor er Taylorrækken ud fra $x_0 = 0$ identisk med nulpolynomiet.

Det betyder, at Taylorrækken ud fra $x_0 = 0$ ALDRIG konvergerer imod $f(x)$ for noget $x > 0$.

$$\lim_{x \rightarrow 0^+} (f(x)) = 0 \tag{4}$$

Dvs. $f(0) = 0$, og f er kontinuert i 0.

$$f_1 := \text{unapply}\left(\frac{d}{dx} f(x), x\right)$$

$$f_1 := x \rightarrow \frac{2 e^{-\frac{1}{x^2}}}{x^3} \tag{5}$$

$$\lim_{x \rightarrow 0^+} (f_1(x)) = 0 \tag{6}$$

$$\lim_{x \rightarrow 0^+} \left(\frac{f(x) - 0}{x - 0} \right) = 0 \tag{7}$$

Dvs. $f'(0) = 0$, og f' er kontinuert i 0.

$$\text{> } f_2 := \text{unapply}\left(\frac{d}{dx} f_1(x), x\right)$$

$$f_2 := x \rightarrow -\frac{6 e^{-\frac{1}{x^2}}}{x^4} + \frac{4 e^{-\frac{1}{x^2}}}{x^6} \quad (8)$$

$$\text{> } \lim_{x \rightarrow 0^+} (f_2(x))$$

$$0 \quad (9)$$

$$\text{> } \lim_{x \rightarrow 0^+} \left(\frac{f_1(x) - 0}{x - 0}\right)$$

$$0 \quad (10)$$

Dvs. $f''(0) = 0$, og f'' er kontinuert i 0.

Osv.

$$\text{> for } n \text{ from 2 by 1 to 10 do } f_{n+1} := \text{unapply}\left(\frac{d}{dx} f_n(x), x\right) \text{ end do}$$

$$f_3 := x \rightarrow \frac{24 e^{-\frac{1}{x^2}}}{x^5} - \frac{36 e^{-\frac{1}{x^2}}}{x^7} + \frac{8 e^{-\frac{1}{x^2}}}{x^9}$$

$$f_4 := x \rightarrow -\frac{120 e^{-\frac{1}{x^2}}}{x^6} + \frac{300 e^{-\frac{1}{x^2}}}{x^8} - \frac{144 e^{-\frac{1}{x^2}}}{x^{10}} + \frac{16 e^{-\frac{1}{x^2}}}{x^{12}}$$

$$f_5 := x \rightarrow \frac{720 e^{-\frac{1}{x^2}}}{x^7} - \frac{2640 e^{-\frac{1}{x^2}}}{x^9} + \frac{2040 e^{-\frac{1}{x^2}}}{x^{11}} - \frac{480 e^{-\frac{1}{x^2}}}{x^{13}} + \frac{32 e^{-\frac{1}{x^2}}}{x^{15}}$$

$$f_6 := x \rightarrow -\frac{5040 e^{-\frac{1}{x^2}}}{x^8} + \frac{25200 e^{-\frac{1}{x^2}}}{x^{10}} - \frac{27720 e^{-\frac{1}{x^2}}}{x^{12}} + \frac{10320 e^{-\frac{1}{x^2}}}{x^{14}} - \frac{1440 e^{-\frac{1}{x^2}}}{x^{16}}$$

$$+ \frac{64 e^{-\frac{1}{x^2}}}{x^{18}}$$

$$f_7 := x \rightarrow \frac{40320 e^{-\frac{1}{x^2}}}{x^9} - \frac{262080 e^{-\frac{1}{x^2}}}{x^{11}} + \frac{383040 e^{-\frac{1}{x^2}}}{x^{13}} - \frac{199920 e^{-\frac{1}{x^2}}}{x^{15}} + \frac{43680 e^{-\frac{1}{x^2}}}{x^{17}}$$

$$- \frac{4032 e^{-\frac{1}{x^2}}}{x^{19}} + \frac{128 e^{-\frac{1}{x^2}}}{x^{21}}$$

$$f_8 := x \rightarrow -\frac{362880 e^{-\frac{1}{x^2}}}{x^{10}} + \frac{2963520 e^{-\frac{1}{x^2}}}{x^{12}} - \frac{5503680 e^{-\frac{1}{x^2}}}{x^{14}} + \frac{3764880 e^{-\frac{1}{x^2}}}{x^{16}}$$

$$\begin{aligned}
 & -\frac{1142400 e^{-\frac{1}{x^2}}}{x^{18}} + \frac{163968 e^{-\frac{1}{x^2}}}{x^{20}} - \frac{10752 e^{-\frac{1}{x^2}}}{x^{22}} + \frac{256 e^{-\frac{1}{x^2}}}{x^{24}} \\
 f_9 := x \rightarrow & \frac{3628800 e^{-\frac{1}{x^2}}}{x^{11}} - \frac{36288000 e^{-\frac{1}{x^2}}}{x^{13}} + \frac{82978560 e^{-\frac{1}{x^2}}}{x^{15}} - \frac{71245440 e^{-\frac{1}{x^2}}}{x^{17}} \\
 & + \frac{28092960 e^{-\frac{1}{x^2}}}{x^{19}} - \frac{5564160 e^{-\frac{1}{x^2}}}{x^{21}} + \frac{564480 e^{-\frac{1}{x^2}}}{x^{23}} - \frac{27648 e^{-\frac{1}{x^2}}}{x^{25}} + \frac{512 e^{-\frac{1}{x^2}}}{x^{27}} \\
 f_{10} := x \rightarrow & -\frac{39916800 e^{-\frac{1}{x^2}}}{x^{12}} + \frac{479001600 e^{-\frac{1}{x^2}}}{x^{14}} - \frac{1317254400 e^{-\frac{1}{x^2}}}{x^{16}} \\
 & + \frac{1377129600 e^{-\frac{1}{x^2}}}{x^{18}} - \frac{676257120 e^{-\frac{1}{x^2}}}{x^{20}} + \frac{173033280 e^{-\frac{1}{x^2}}}{x^{22}} - \frac{24111360 e^{-\frac{1}{x^2}}}{x^{24}} \\
 & + \frac{1820160 e^{-\frac{1}{x^2}}}{x^{26}} - \frac{69120 e^{-\frac{1}{x^2}}}{x^{28}} + \frac{1024 e^{-\frac{1}{x^2}}}{x^{30}} \\
 f_{11} := x \rightarrow & \frac{479001600 e^{-\frac{1}{x^2}}}{x^{13}} - \frac{6785856000 e^{-\frac{1}{x^2}}}{x^{15}} + \frac{22034073600 e^{-\frac{1}{x^2}}}{x^{17}} \\
 & - \frac{27422841600 e^{-\frac{1}{x^2}}}{x^{19}} + \frac{16279401600 e^{-\frac{1}{x^2}}}{x^{21}} - \frac{5159246400 e^{-\frac{1}{x^2}}}{x^{23}} \\
 & + \frac{924739200 e^{-\frac{1}{x^2}}}{x^{25}} - \frac{95546880 e^{-\frac{1}{x^2}}}{x^{27}} + \frac{5575680 e^{-\frac{1}{x^2}}}{x^{29}} - \frac{168960 e^{-\frac{1}{x^2}}}{x^{31}} \\
 & + \frac{2048 e^{-\frac{1}{x^2}}}{x^{33}}
 \end{aligned}$$

(11)

> for n from 2 by 1 to 10 do $\lim_{x \rightarrow 0^+} (f_{n+1}(x))$ end do

0
0
0
0
0
0
0
0
0
0

(12)

> for n from 2 by 1 to 10 do $\lim_{x \rightarrow 0^+} \left(\frac{f_n(x) - 0}{x - 0} \right)$ end do

⌊

0
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0

(13)