

Eksempel på funktion af 2 variable, hvor rækkefølgen af differentiation efter x hhv. y IKKE giver det samme ved ombytning.

$$f(x, y) = \begin{cases} x \cdot y \cdot \frac{x^2 - y^2}{x^2 + y^2} & \text{for } x \neq 0 \wedge y \neq 0 \\ 0 & \text{for } x = 0 \wedge y = 0 \end{cases}$$

Eksemplet er hentet fra bogen "Counterexamples in Analysis":

http://books.google.com/books?id=cDAMh5n4lkkC&printsec=frontcover&hl=da&source=gbs_navlinks_s#v=onepage&q=&f=false

> restart

1. ordens afledede:

$\frac{\partial f}{\partial x}$ beregnes (på nær i (0, 0)) :

> diff $\left(x \cdot y \cdot \frac{x^2 - y^2}{x^2 + y^2}, x \right)$

$$\frac{y(x^2 - y^2)}{x^2 + y^2} + \frac{2x^2y}{x^2 + y^2} - \frac{2x^2y(x^2 - y^2)}{(x^2 + y^2)^2} \quad (1)$$

$\frac{\partial f}{\partial x}(0, y)$ beregnes (på nær i (0, 0)) :

> subs(x=0, (1))

$$-y \quad (2)$$

$\frac{\partial f}{\partial x}(0, 0) = 0$, da $|f(x, y) - f(0, 0)| \leq |x| \cdot |y|$

$\frac{\partial f}{\partial y}$ beregnes (på nær i (0, 0)) :

> diff $\left(x \cdot y \cdot \frac{x^2 - y^2}{x^2 + y^2}, y \right)$

$$\frac{x(x^2 - y^2)}{x^2 + y^2} - \frac{2xy^2}{x^2 + y^2} - \frac{2xy^2(x^2 - y^2)}{(x^2 + y^2)^2} \quad (3)$$

$\frac{\partial f}{\partial y}(x, 0)$ beregnes (på nær i (0, 0)) :

> subs(y=0, (3))

$$x \quad (4)$$

$\frac{\partial f}{\partial y}(0, 0) = 0$, da $|f(x, y) - f(0, 0)| \leq |x| \cdot |y|$

2. ordens afledede:

$$\begin{aligned}
 & \text{Beregning af } \frac{\partial^2 f}{\partial y \cdot \partial x}(0, 0) : \\
 & \lim_{y \rightarrow 0} \left(\frac{\frac{\partial f}{\partial x}(0, y) - \frac{\partial f}{\partial x}(0, 0)}{y} \right) \\
 & > \lim_{y \rightarrow 0} \left(\frac{(2) - 0}{y} \right) \qquad \qquad \qquad -1 \qquad \qquad \qquad (5) \\
 & \text{Dvs. } \frac{\partial^2 f}{\partial y \cdot \partial x}(0, 0) = -1 \\
 & \text{Beregning af } \frac{\partial^2 f}{\partial x \cdot \partial y}(0, 0) : \\
 & \lim_{x \rightarrow 0} \left(\frac{\frac{\partial f}{\partial y}(x, 0) - \frac{\partial f}{\partial y}(0, 0)}{x} \right) \\
 & > \lim_{x \rightarrow 0} \left(\frac{(4) - 0}{x} \right) \qquad \qquad \qquad 1 \qquad \qquad \qquad (6) \\
 & \text{Dvs. } \frac{\partial^2 f}{\partial x \cdot \partial y}(0, 0) = 1 \\
 & \text{Altså er } \frac{\partial^2 f}{\partial y \cdot \partial x}(0, 0) \neq \frac{\partial^2 f}{\partial x \cdot \partial y}(0, 0)
 \end{aligned}$$