Løsning af differentialligninger med Maple

Eksempel 11.5 (enkel 1. ordens differentialligning)

> *restart*

> $DiffLign := x'(t) + 2 \cdot x(t) = 30 + 8 \cdot t$

$$DiffLign := \mathrm{D}(x)(t) + 2\,x(t) = 30 + 8\,t \quad (1.1)$$

> $dsolve(DiffLign)$

$$x(t) = 4\,t + 13 + \mathrm{e}^{-2\,t}\,\_C1 \quad (1.2)$$

Fuldstændig løsning: $x(t) = 4 \cdot t + 13 + c \cdot \mathrm{e}^{-2 \cdot t}$ hvor $t \in \mathbb{R}$ og $c \in \mathbb{R}$

Eksempel 11.10 (enkel 1. ordens differentialligning)

> *restart*

> $DiffLign := x'(t) + \frac{2}{t} \cdot x(t) = 8 \cdot t - \frac{10}{t}$

$$DiffLign := \mathrm{D}(x)(t) + \frac{2\,x(t)}{t} = 8\,t - \frac{10}{t} \quad (2.1)$$

> $Bet := x(1) = 2$

$$Bet := x(1) = 2 \quad (2.2)$$

> $dsolve(DiffLign)$

$$x(t) = -5 + 2\,t^2 + \frac{\_C1}{t^2} \quad (2.3)$$

> $dsolve(\{DiffLign, Bet\})$

$$x(t) = -5 + 2\,t^2 + \frac{5}{t^2} \quad (2.4)$$

Den betingede løsning: $x(t) = -5 + 2 \cdot t^2 + \frac{5}{t^2}$ hvor $t \in \mathbb{R}$

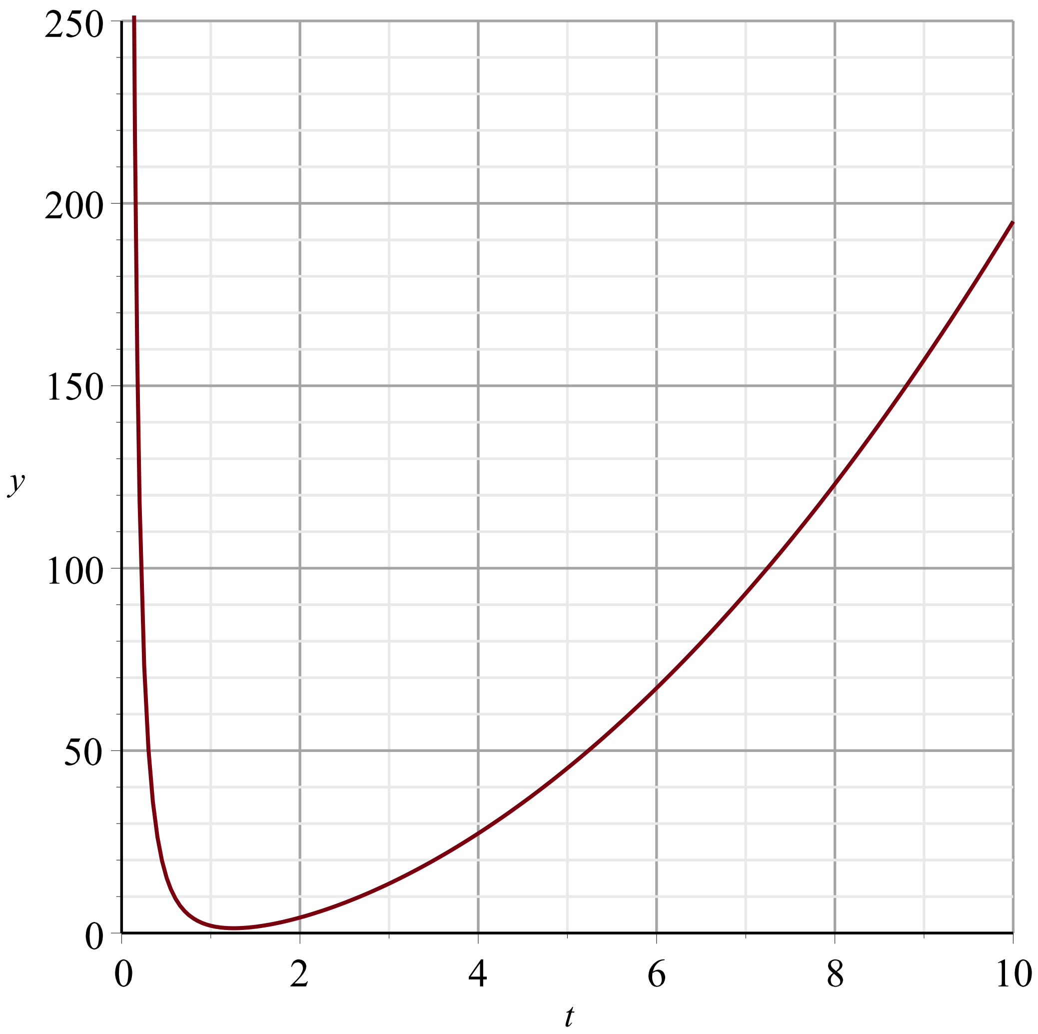
Gemme løsningen i en funktion:

> $x := unapply(rhs(\%), t)$

$$x := t \rightarrow -5 + 2\,t^2 + \frac{5}{t^2} \quad (2.5)$$

Så kan grafen tegnes:

> $plot(x(t), t = 0..10, y = 0..250, gridlines)$



Eksempel 12.8 (system af differentialligninger)

Med teorien (metode 12.4 og 12.7)

> *restart*

> *with*(*LinearAlgebra*) :

> $A := \begin{bmatrix} 16 & -1 \\ 4 & 12 \end{bmatrix}$

$$A := \begin{bmatrix} 16 & -1 \\ 4 & 12 \end{bmatrix} \qquad \textbf{(3.1.1)}$$

> *Eigenvectors*(*A*, *output* = *list*)

$$\left[\left[14, 2, \left\{\left[\begin{array}{c} \frac{1}{2} \\ 1 \end{array}\right]\right\}\right]\right] \qquad \textbf{(3.1.2)}$$

Dobbeltrod 14 med $am = 2$ og $gm = 1$.

Metode 12.7:

> $\lambda :=$ **(3.1.2)**[1, 1]

(3.1.3)

$$\lambda := 14 \quad (3.1.3)$$

> $v := \mathbf{(3.1.2)}[1, 3, 1]$

$$v := \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} \quad (3.1.4)$$

> $LinearSolve(A - \lambda \cdot IdentityMatrix(2), v)$

$$\begin{bmatrix} \_t_1 \\ 2\,\_t_1 - \frac{1}{2} \end{bmatrix} \quad (3.1.5)$$

Vælger en løsning b :

> $b := subs(\_t[1] = 0, \mathbf{(3.1.5)})$

$$b := \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix} \quad (3.1.6)$$

> $u_1 := unapply(e^{\lambda \cdot t} \cdot v, t)$:

> $u_1(t)$

$$\begin{bmatrix} \frac{1}{2}\, e^{14\,t} \\ e^{14\,t} \end{bmatrix} \quad (3.1.7)$$

> $u_2 := unapply(t \cdot e^{\lambda \cdot t} \cdot v + e^{\lambda \cdot t} \cdot b, t)$:

> $u_2(t)$

$$\begin{bmatrix} \frac{1}{2}\, t\, e^{14\,t} \\ t\, e^{14\,t} - \frac{1}{2}\, e^{14\,t} \end{bmatrix} \quad (3.1.8)$$

Metode 12.4:

> $x := unapply(c_1 \cdot u_1(t) + c_2 \cdot u_2(t), t)$:

> $x(t)$

$$\begin{bmatrix} \frac{1}{2}\, c_1\, e^{14\,t} + \frac{1}{2}\, c_2\, t\, e^{14\,t} \\ c_1\, e^{14\,t} + c_2 \left(t\, e^{14\,t} - \frac{1}{2}\, e^{14\,t} \right) \end{bmatrix} \quad (3.1.9)$$

> $x_1 := unapply(\mathbf{(3.1.9)}[1], t)$:

> $x_1(t)$

$$\frac{1}{2}\, c_1\, e^{14\,t} + \frac{1}{2}\, c_2\, t\, e^{14\,t} \quad (3.1.10)$$

> $x_2 := unapply(\mathbf{(3.1.9)}[2], t)$:

> $x_2(t)$

$$c_1 e^{14t} + c_2\left(t e^{14t} - \frac{1}{2} e^{14t}\right) \quad \textbf{(3.1.11)}$$

Den fuldstændige løsning: $x_1(t) = \frac{1}{2}\cdot c_1\cdot e^{14\cdot t} + \frac{1}{2}\cdot c_2\cdot t\cdot e^{14\cdot t}$ og

$x_2(t) = c_1\cdot e^{14\cdot t} + c_2\cdot\left(t\cdot e^{14\cdot t} - \frac{1}{2}\cdot e^{14\cdot t}\right)$ hvor $t\in\mathbb{R}$, $c_1\in\mathbb{R}$, $c_2\in\mathbb{R}$

Direkte med Maple

> *restart*

> $A := \begin{bmatrix} 16 & -1 \\ 4 & 12 \end{bmatrix}$

$$A := \begin{bmatrix} 16 & -1 \\ 4 & 12 \end{bmatrix} \quad \textbf{(3.2.1)}$$

> $A.\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

$$\begin{bmatrix} 16\, x_1(t) - x_2(t) \\ 4\, x_1(t) + 12\, x_2(t) \end{bmatrix} \quad \textbf{(3.2.2)}$$

> $DiffLign1 := x_1'(t) = \textbf{(3.2.2)}[1, 1]$

$$DiffLign1 := \mathrm{D}(x_1)(t) = 16\, x_1(t) - x_2(t) \quad \textbf{(3.2.3)}$$

> $DiffLign2 := x_2'(t) = \textbf{(3.2.2)}[2, 1]$

$$DiffLign2 := \mathrm{D}(x_2)(t) = 4\, x_1(t) + 12\, x_2(t) \quad \textbf{(3.2.4)}$$

> $dsolve(\{DiffLign1, DiffLign2\})$

$$\{x_1(t) = e^{14t}(\_C2\, t + \_C1), x_2(t) = e^{14t}(2\,\_C2\, t + 2\,\_C1 - \_C2)\} \quad \textbf{(3.2.5)}$$

> $subs(\_C1 = c_1, \_C2 = c_2, \textbf{(3.2.5)})$

$$\{x_1(t) = e^{14t}(c_2 t + c_1), x_2(t) = e^{14t}(2\, c_2 t + 2\, c_1 - c_2)\} \quad \textbf{(3.2.6)}$$

> $x_1 := unapply(rhs(\textbf{(3.2.6)}[1]), t) :$

> $x_1(t)$

$$e^{14t}(c_2 t + c_1) \quad \textbf{(3.2.7)}$$

> $x_2 := unapply(rhs(\textbf{(3.2.6)}[2]), t) :$

> $x_2(t)$

$$e^{14t}(2\, c_2 t + 2\, c_1 - c_2) \quad \textbf{(3.2.8)}$$

Den fuldstændige løsning: $x_1(t) = e^{14\cdot t}\cdot(c_2\cdot t + c_1)$ og $x_2(t) = e^{14\cdot t}\cdot(2\cdot c_2\cdot t + 2\cdot c_1 - c_2)$

hvor $t\in\mathbb{R}$, $c_1\in\mathbb{R}$, $c_2\in\mathbb{R}$

Eksempel 13.18 (enkel 2. ordens differentialligning)

> *restart*

> $DiffLign := x''(t) - 2\cdot x'(t) - 2\cdot x(t) = 19\cdot e^{4\cdot t}\cdot \cos(t) - 35\cdot e^{4\cdot t}\cdot \sin(t)$

$$DiffLign := D^{(2)}(x)(t) - 2\,D(x)(t) - 2\,x(t) = 19\,e^{4t}\cos(t) - 35\,e^{4t}\sin(t) \quad \textbf{(4.1)}$$

> $dsolve(DiffLign)$

$$x(t) = e^{\left(1+\sqrt{3}\right)t}\,\_C2 + e^{-\left(\sqrt{3}-1\right)t}\,\_C1 + e^{4t}\left(5\cos(t) - \sin(t)\right) \quad \textbf{(4.2)}$$

Den fuldstændige løsning: $x(t) = c_2\cdot e^{\left(1+\sqrt{3}\right)\cdot t} + c_1\cdot e^{\left(1-\sqrt{3}\right)\cdot t} + e^{4\cdot t}\cdot\left(5\cdot\cos(t) - \sin(t)\right)$ hvor $t \in \mathbb{R}$, $c_1 \in \mathbb{C}$, $c_2 \in \mathbb{C}$