Analytisk funktion

En C^{∞} - funktion, hvis Taylor-række konvergerer imod funktionen i en omegn af punktet x_0, siges at være en **analytisk funktion**.
Læs nærmere: **http://en.wikipedia.org/wiki/Analytic_function**

Ikke alle funktion, som kan differentieres uendelig mange gange, er analytiske!

Modeksempel:

Definer følgende funktion:

$$f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ e^{-\frac{1}{x^2}} & \text{for } x > 0 \end{cases}$$

Denne funktion $f \in C^{\infty}$, men er IKKE en analytisk funktion, da Taylor-rækken ud fra ud fra $x_0 = 0$ er identisk med 0 !

> *restart*

Venstre del af funktionen:

> $fv := x \to 0$

$$fv := x \to 0 \tag{1}$$

Højre del af funktionen:

> $fh := x \to \exp\left(-\frac{1}{x^2}\right)$

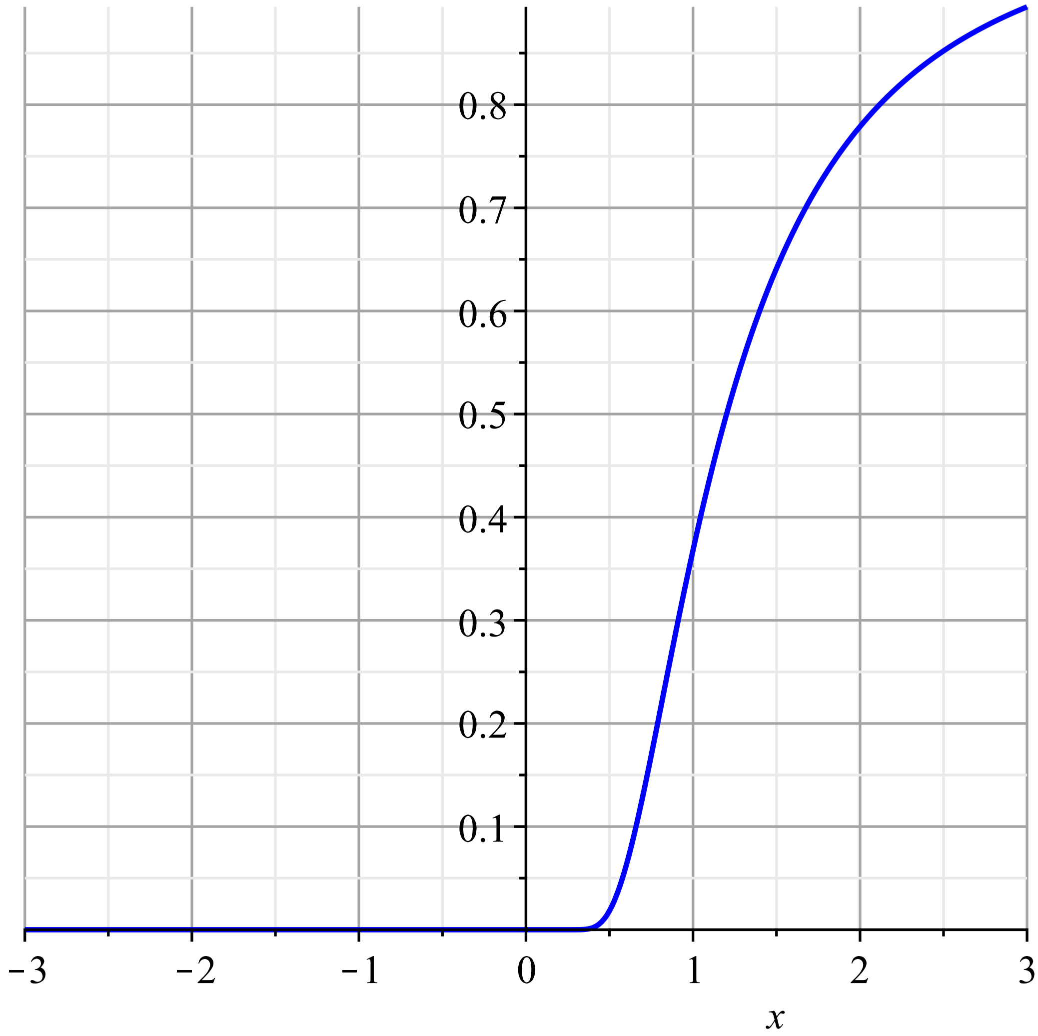
$$fh := x \to e^{-\frac{1}{x^2}} \tag{2}$$

> *with(plots)* :

> *plot1* := *plot*(*fv*(*x*), *x* = -3 ..0, *color* = *blue*, *thickness* = 2) :

> *plot2* := *plot*(*fh*(*x*), *x* = 0 ..3, *color* = *blue*, *thickness* = 2) :

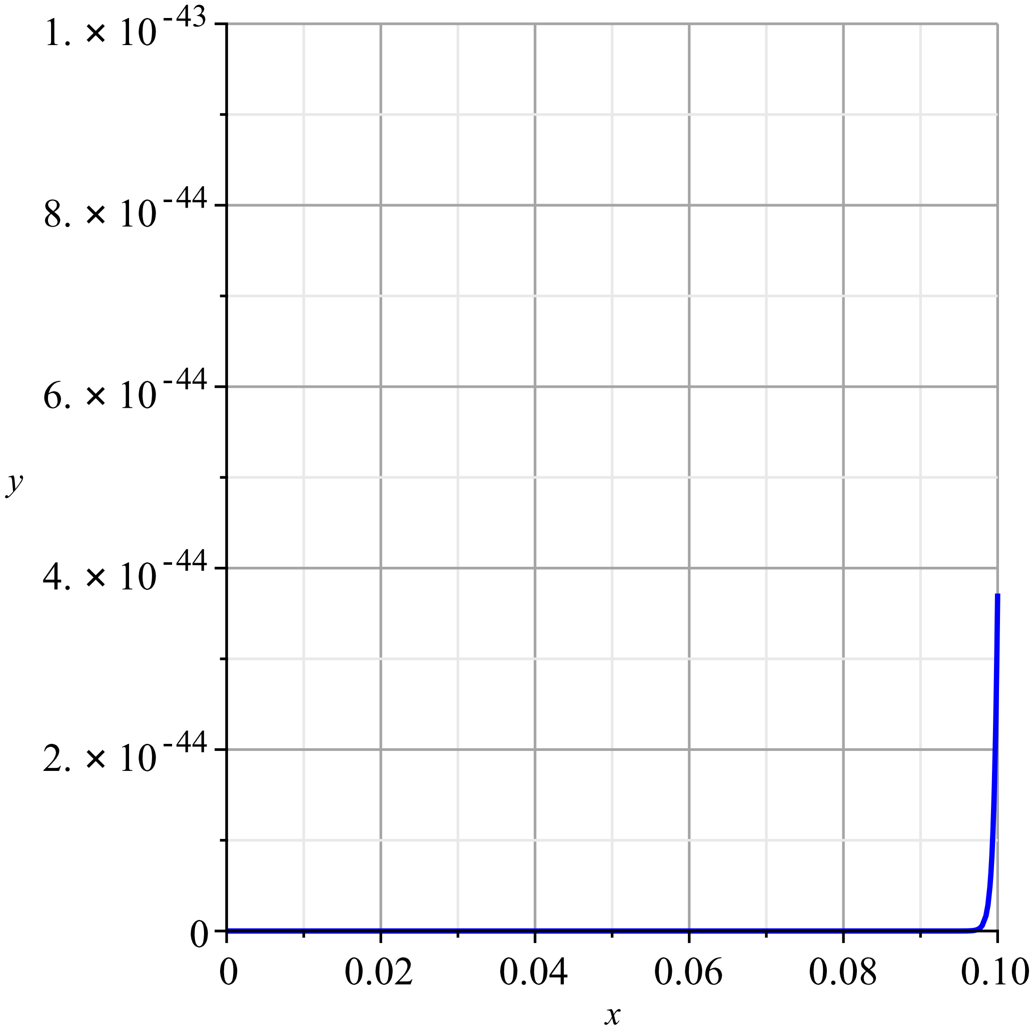
> *display*(*plot1*, *plot2*, *gridlines*)



> $fh(0.1)$

$$3.720075976\ 10^{-44} \quad (3)$$

> $plot\left(fh(x), x = 0..0.1, y = 0..10^{-43}, color = blue, thickness = 2, gridlines\right)$



Funktionen er så flad i $x_0 = 0$, at alle afledede i dette punkt er 0!

Derfor er Taylorrækken ud fra $x_0 = 0$ identisk med nulpolynomiet.
Det betyder, at Taylorrækken ud fra $x_0 = 0$ ALDRIG konvergerer imod $f(x)$ for noget $x > 0$.

> $\lim_{x \to 0+} (fh(x))$

$$0 \qquad (4)$$

Dvs. $f(0) = 0$, og f er kontinuert i 0.

> $f_1 := unapply\left(\frac{d}{dx} fh(x), x\right)$

$$f_1 := x \to \frac{2 e^{-\frac{1}{x^2}}}{x^3} \qquad (5)$$

> $\lim_{x \to 0+} (f_1(x))$

$$0 \qquad (6)$$

> $\lim_{x \to 0+} \left(\frac{fh(x) - 0}{x - 0}\right)$

$$0 \qquad (7)$$

Dvs. $f'(0) = 0$, og f' er kontinuert i 0.

> $f_2 := unapply\left(\frac{d}{dx} f_1(x), x\right)$

$$f_2 := x \rightarrow -\frac{6\,e^{-\frac{1}{x^2}}}{x^4} + \frac{4\,e^{-\frac{1}{x^2}}}{x^6} \tag{8}$$

> $\lim_{x \rightarrow 0+} (f_2(x))$

$$0 \tag{9}$$

> $\lim_{x \rightarrow 0+} \left(\frac{f_1(x) - 0}{x - 0}\right)$

$$0 \tag{10}$$

Dvs. $f''(0) = 0$, og f'' er kontinuert i 0.

Osv.

> **for** n **from** 2 **by** 1 **to** 10 **do** $f_{n+1} := unapply\left(\frac{d}{dx} f_n(x), x\right)$ **end do**

$$f_3 := x \rightarrow \frac{24\,e^{-\frac{1}{x^2}}}{x^5} - \frac{36\,e^{-\frac{1}{x^2}}}{x^7} + \frac{8\,e^{-\frac{1}{x^2}}}{x^9}$$

$$f_4 := x \rightarrow -\frac{120\,e^{-\frac{1}{x^2}}}{x^6} + \frac{300\,e^{-\frac{1}{x^2}}}{x^8} - \frac{144\,e^{-\frac{1}{x^2}}}{x^{10}} + \frac{16\,e^{-\frac{1}{x^2}}}{x^{12}}$$

$$f_5 := x \rightarrow \frac{720\,e^{-\frac{1}{x^2}}}{x^7} - \frac{2640\,e^{-\frac{1}{x^2}}}{x^9} + \frac{2040\,e^{-\frac{1}{x^2}}}{x^{11}} - \frac{480\,e^{-\frac{1}{x^2}}}{x^{13}} + \frac{32\,e^{-\frac{1}{x^2}}}{x^{15}}$$

$$f_6 := x \rightarrow -\frac{5040\,e^{-\frac{1}{x^2}}}{x^8} + \frac{25200\,e^{-\frac{1}{x^2}}}{x^{10}} - \frac{27720\,e^{-\frac{1}{x^2}}}{x^{12}} + \frac{10320\,e^{-\frac{1}{x^2}}}{x^{14}} - \frac{1440\,e^{-\frac{1}{x^2}}}{x^{16}} + \frac{64\,e^{-\frac{1}{x^2}}}{x^{18}}$$

$$f_7 := x \rightarrow \frac{40320\,e^{-\frac{1}{x^2}}}{x^9} - \frac{262080\,e^{-\frac{1}{x^2}}}{x^{11}} + \frac{383040\,e^{-\frac{1}{x^2}}}{x^{13}} - \frac{199920\,e^{-\frac{1}{x^2}}}{x^{15}} + \frac{43680\,e^{-\frac{1}{x^2}}}{x^{17}} - \frac{4032\,e^{-\frac{1}{x^2}}}{x^{19}} + \frac{128\,e^{-\frac{1}{x^2}}}{x^{21}}$$

$$f_8 := x \rightarrow -\frac{362880\,e^{-\frac{1}{x^2}}}{x^{10}} + \frac{2963520\,e^{-\frac{1}{x^2}}}{x^{12}} - \frac{5503680\,e^{-\frac{1}{x^2}}}{x^{14}} + \frac{3764880\,e^{-\frac{1}{x^2}}}{x^{16}}$$

$$-\frac{1142400\,e^{-\frac{1}{x^2}}}{x^{18}}+\frac{163968\,e^{-\frac{1}{x^2}}}{x^{20}}-\frac{10752\,e^{-\frac{1}{x^2}}}{x^{22}}+\frac{256\,e^{-\frac{1}{x^2}}}{x^{24}}$$

$$f_9 := x \rightarrow \frac{3628800\,e^{-\frac{1}{x^2}}}{x^{11}}-\frac{36288000\,e^{-\frac{1}{x^2}}}{x^{13}}+\frac{82978560\,e^{-\frac{1}{x^2}}}{x^{15}}-\frac{71245440\,e^{-\frac{1}{x^2}}}{x^{17}}+\frac{28092960\,e^{-\frac{1}{x^2}}}{x^{19}}-\frac{5564160\,e^{-\frac{1}{x^2}}}{x^{21}}+\frac{564480\,e^{-\frac{1}{x^2}}}{x^{23}}-\frac{27648\,e^{-\frac{1}{x^2}}}{x^{25}}+\frac{512\,e^{-\frac{1}{x^2}}}{x^{27}}$$

$$f_{10} := x \rightarrow -\frac{39916800\,e^{-\frac{1}{x^2}}}{x^{12}}+\frac{479001600\,e^{-\frac{1}{x^2}}}{x^{14}}-\frac{1317254400\,e^{-\frac{1}{x^2}}}{x^{16}}+\frac{1377129600\,e^{-\frac{1}{x^2}}}{x^{18}}-\frac{676257120\,e^{-\frac{1}{x^2}}}{x^{20}}+\frac{173033280\,e^{-\frac{1}{x^2}}}{x^{22}}-\frac{24111360\,e^{-\frac{1}{x^2}}}{x^{24}}+\frac{1820160\,e^{-\frac{1}{x^2}}}{x^{26}}-\frac{69120\,e^{-\frac{1}{x^2}}}{x^{28}}+\frac{1024\,e^{-\frac{1}{x^2}}}{x^{30}}$$

$$f_{11} := x \rightarrow \frac{479001600\,e^{-\frac{1}{x^2}}}{x^{13}}-\frac{6785856000\,e^{-\frac{1}{x^2}}}{x^{15}}+\frac{22034073600\,e^{-\frac{1}{x^2}}}{x^{17}}-\frac{27422841600\,e^{-\frac{1}{x^2}}}{x^{19}}+\frac{16279401600\,e^{-\frac{1}{x^2}}}{x^{21}}-\frac{5159246400\,e^{-\frac{1}{x^2}}}{x^{23}}+\frac{924739200\,e^{-\frac{1}{x^2}}}{x^{25}}-\frac{95546880\,e^{-\frac{1}{x^2}}}{x^{27}}+\frac{5575680\,e^{-\frac{1}{x^2}}}{x^{29}}-\frac{168960\,e^{-\frac{1}{x^2}}}{x^{31}}+\frac{2048\,e^{-\frac{1}{x^2}}}{x^{33}} \quad \textbf{(11)}$$

> **for** n **from** 2 **by** 1 **to** 10 **do** $\lim_{x \rightarrow 0+} \left(f_{n+1}(x)\right)$ **end do**

$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0 \quad \textbf{(12)}$$

> **for** n **from** 2 **by** 1 **to** 10 **do** $\lim_{x \rightarrow 0+} \left(\frac{f_n(x) - 0}{x - 0}\right)$ **end do**

0
0
0
0
0
0
0
0
0 **(13)**