

Parametrisering og plots af Tycho Brahe Planetarium

Fakta om Planetarium:

<http://www.planetariet.dk/bes%C3%B8g-os/om-planetariet/historien-om-planetariet/byggefasen>

Højde: 38 m
Diameter: 26 m
Taghældning: 30°

NB: Højeste punkt på Tycho Brahe Planetarium placeres i (0, 0, 38).

> *restart*

> *with(plots) :*

▼ Randen i bunden (cirkel)

Parametrisering med 1 parameter i 3D, hvor $u \in [0; 2 \cdot \pi]$:

> $r1 := u \rightarrow \langle a \cdot \cos(u) + a, a \cdot \sin(u), 0 \rangle :$
' $r1(u)$ '= $r1(u)$

$$r1(u) = \begin{bmatrix} a \cos(u) + a \\ a \sin(u) \\ 0 \end{bmatrix} \quad (1.1)$$

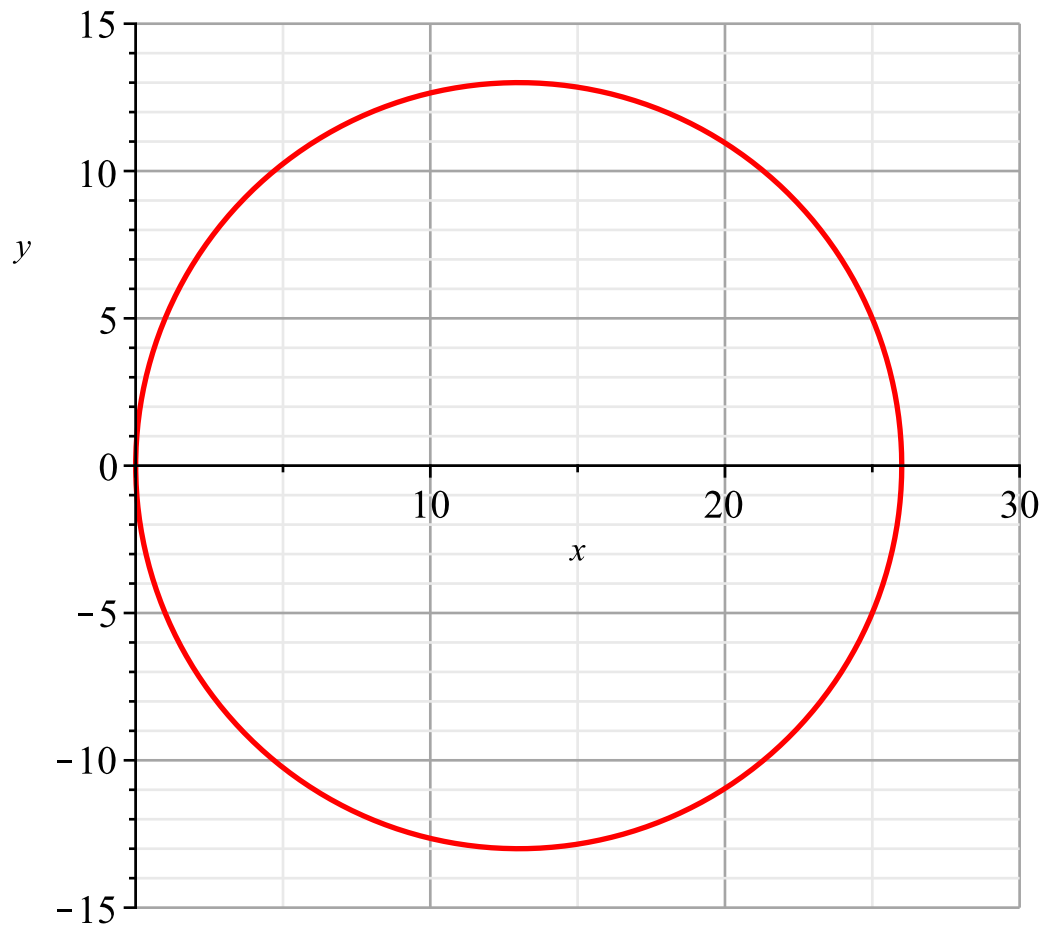
For at plotte vælges en værdi for a (radius i cirklen):

> $a := 13$

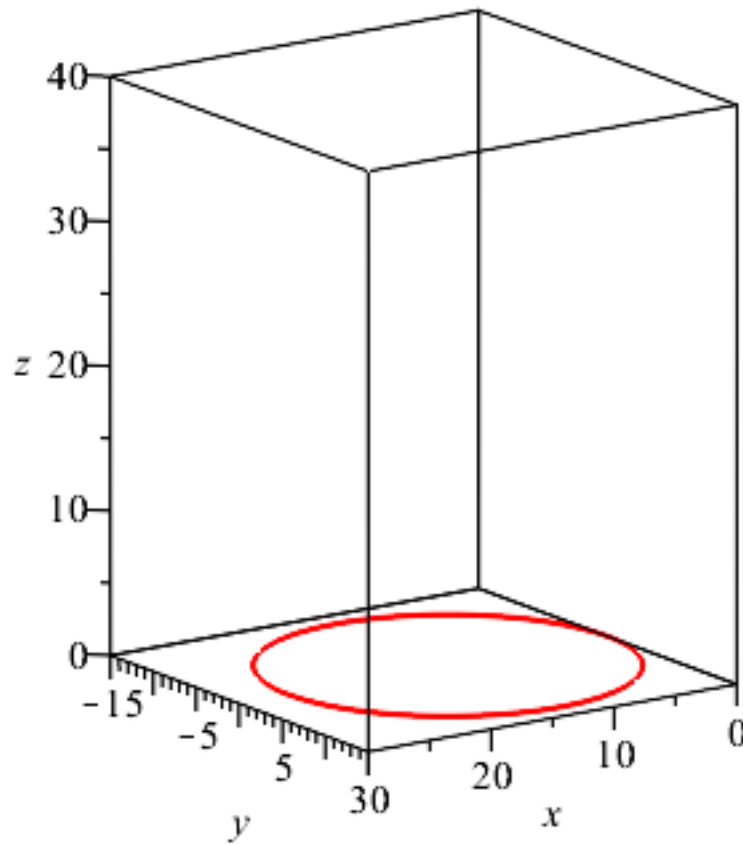
$a := 13$ (1.2)

Plot i 2D:

> $plot([r1(u)_1, r1(u)_2, u = 0 .. 2 \cdot \pi], labels = [x, y], view = [0 .. 30, -15 .. 15], thickness = 2, color = red, gridlines, scaling = constrained);$

**Plot i 3D:**

```
> spacecurve(r1(u), u = 0 .. 2 * pi, axes = box, labels = [x, y, z], view = [0 .. 30, -15 .. 15, 0 .. 40],  
color = red, thickness = 2, scaling = constrained)
```



▼ Bunden (cirkelskive)

```
> unassign('a')
```

Parametrisering med 2 parametre i 3D, hvor $u \in [0; 2 \cdot \pi]$ og $v \in [0; 1]$:

```
> r2 := (u, v) → ⟨v·a·cos(u) + a, v·a·sin(u), 0⟩ :
'r2(u, v)' = r2(u, v)
```

$$r2(u, v) = \begin{bmatrix} v a \cos(u) + a \\ v a \sin(u) \\ 0 \end{bmatrix} \quad (2.1)$$

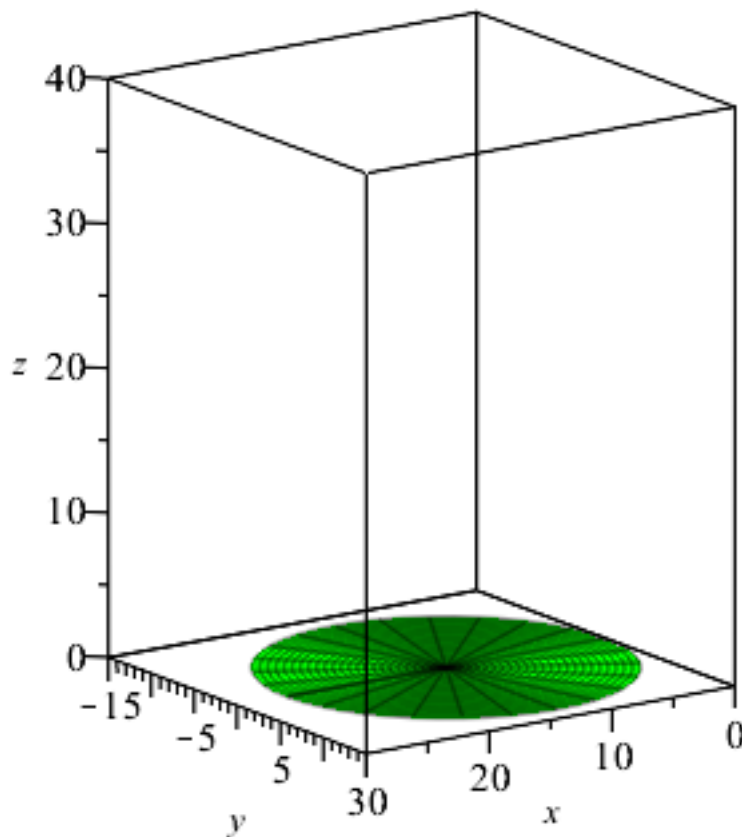
For at plotte vælges en værdi for a (radius i cirklen):

```
> a := 13
```

$$a := 13 \quad (2.2)$$

Plot i 3D:

```
> plot3d(r2(u, v), u = 0 .. 2 · π, v = 0 .. 1, labels = [x, y, z], axes = box, view = [0 .. 30, -15 .. 15, 0 .. 40], scaling = constrained, color = green)
```



▼ Kanten langs toppen (løftet cirkel)

> `unassign('a')`

Hældningskoefficienten for tagfladen, når vinklen med vandret er 30° :

> $\alpha := \tan\left(\frac{-30}{180} \cdot \pi\right)$

$$\alpha := -\frac{1}{3} \sqrt{3}$$

(3.1)

Ligning for plan, som indeholder tagfladen:

$$z = \alpha \cdot x + h = -\frac{\sqrt{3}}{3} \cdot x + 38$$

Parametrisering med 1 parameter i 3D, hvor $u \in [0; 2 \cdot \pi]$:

> $r3 := u \rightarrow \langle a \cdot \cos(u) + a, a \cdot \sin(u), \alpha \cdot (a \cdot \cos(u) + a) + h \rangle$:
 $'r3(u)' = r3(u)$

$$r3(u) = \begin{bmatrix} a \cos(u) + a \\ a \sin(u) \\ -\frac{1}{3} \sqrt{3} (a \cos(u) + a) + h \end{bmatrix} \quad (3.2)$$

For at plote vælges en værdi for a (radius i cirklen på jorden) og h (højden):

```
> a := 13;
   h := 38
```

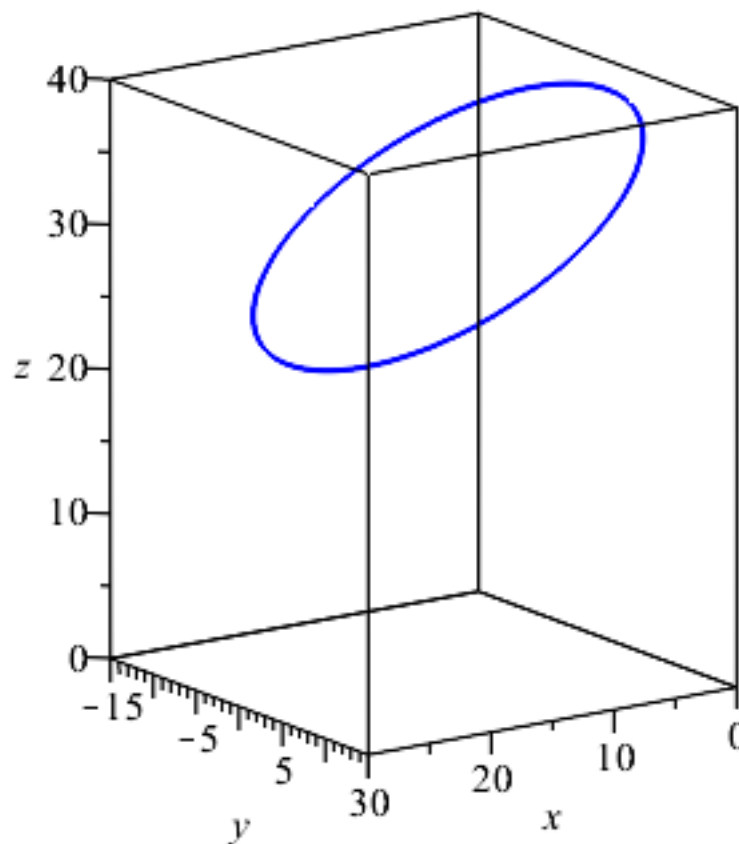
```
a := 13
```

```
h := 38
```

(3.3)

Plot i 3D:

```
> spacecurve(r3(u), u = 0 .. 2 * pi, axes = box, labels = [x, y, z], view = [0 .. 30, -15 .. 15, 0 .. 40],
   color = blue, thickness = 2, scaling = constrained)
```



▼ Skrå tagflade (løftet cirkelskive)

```
> unassign('a');
   unassign('h')
```

Parametrisering med 2 parametre i 3D, hvor $u \in [0; 2 \cdot \pi]$ og $v \in [0; 1]$:

> $r4 := (u, v) \rightarrow \langle v \cdot a \cdot \cos(u) + a, v \cdot a \cdot \sin(u), \alpha \cdot (v \cdot a \cdot \cos(u) + a) + h \rangle$:
 $r4(u, v) = r4(u, v)$

$$r4(u, v) = \begin{bmatrix} v a \cos(u) + a \\ v a \sin(u) \\ -\frac{1}{3} \sqrt{3} (v a \cos(u) + a) + h \end{bmatrix} \quad (4.1)$$

For at plote vælges en værdi for a (radius i cirklen på jorden) og h (højden):

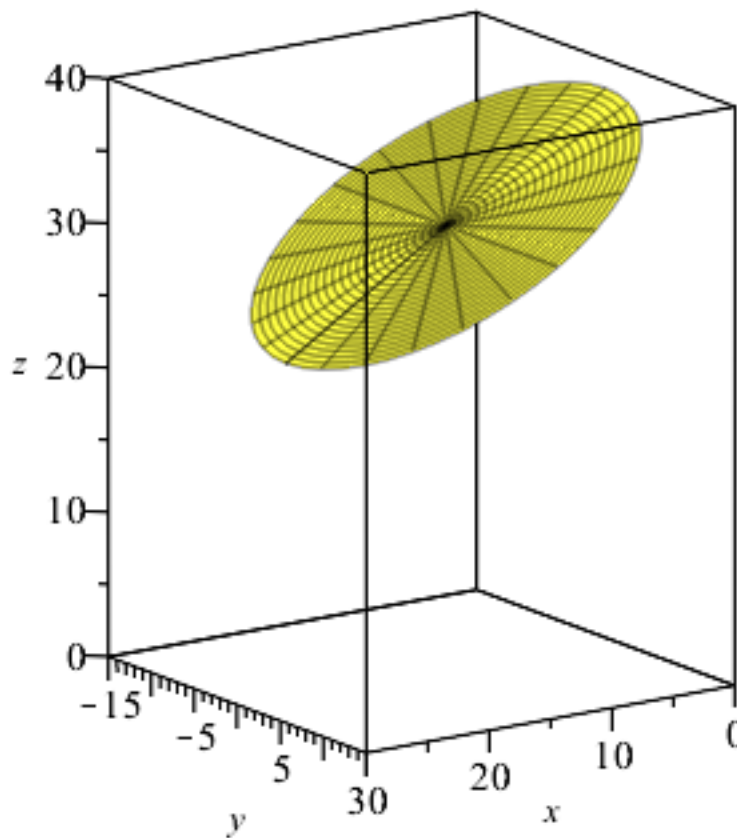
> $a := 13$;
 $h := 38$

$a := 13$
 $h := 38$

(4.2)

Plot i 3D:

> $plot3d(r4(u, v), u = 0 .. 2 \cdot \pi, v = 0 .. 1, labels = [x, y, z], axes = box, view = [0 .. 30, -15 .. 15, 0 .. 40], scaling = constrained, color = yellow)$



▼ **Vægfladen (afskåret cylinder)**

```
> unassign('a');
unassign('h')
```

Parametrisering med 2 parametre i 3D, hvor $u \in [0; 2 \cdot \pi]$ og $v \in [0; 1]$:

```
> r5 := u → ⟨ a · cos(u) + a, a · sin(u), v · (α · (a · cos(u) + a) + h) ⟩ :
'r5(u, v)' = r5(u, v)
```

$$r5(u, v) = \begin{bmatrix} a \cos(u) + a \\ a \sin(u) \\ v \left(-\frac{1}{3} \sqrt{3} (a \cos(u) + a) + h \right) \end{bmatrix} \quad (5.1)$$

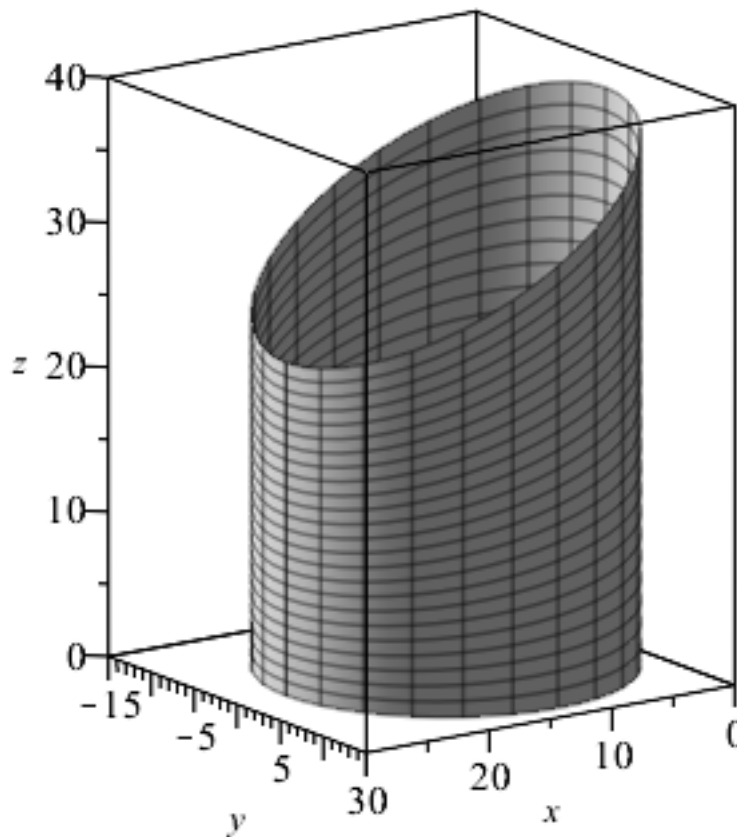
For at plotte vælges en værdi for a (radius i cirklen på jorden) og h (højden):

```
> a := 13;
h := 38
```

$$\begin{aligned} a &:= 13 \\ h &:= 38 \end{aligned} \quad (5.2)$$

Plot i 3D:

```
> plot3d(r5(u, v), u = 0 .. 2 · π, v = 0 .. 1, labels = [x, y, z], axes = box, view = [0 .. 30, -15 .. 15, 0 .. 40], scaling = constrained, color = gray)
```

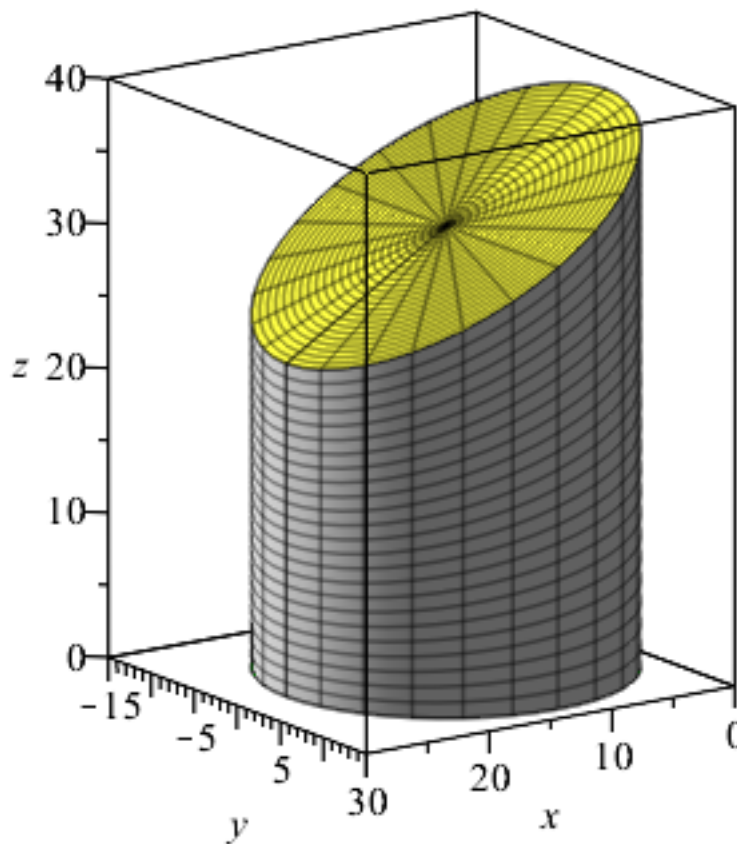


Planetarium (3 stk. 2D-flader)

Bruger ovenstående parametriseringer med 2 parametre i 3D.

Plot i 3D:

```
> bund := plot3d(r2(u, v), u = 0 .. 2 * pi, v = 0 .. 1, labels = [x, y, z], axes = box, view = [0 .. 30, -15 .. 15, 0 .. 40], scaling = constrained, color = green) :
tag := plot3d(r4(u, v), u = 0 .. 2 * pi, v = 0 .. 1, labels = [x, y, z], axes = box, view = [0 .. 30, -15 .. 15, 0 .. 40], scaling = constrained, color = yellow) :
sideflade := plot3d(r5(u, v), u = 0 .. 2 * pi, v = 0 .. 1, labels = [x, y, z], axes = box, view = [0 .. 30, -15 .. 15, 0 .. 40], scaling = constrained, color = gray) :
display(bund, tag, sideflade)
```



Planetarium (3D-rum)

```
> unassign('a');
unassign('h')
```

Parametrisering med 3 parametre i 3D, hvor $u \in [0; 2 \cdot \pi]$ og $v \in [0; 1]$ og $w \in [0; 1]$:

```
> r6 := (u, v, w) → ⟨ w · a · cos(u) + a, w · a · sin(u), v · (α · (w · a · cos(u) + a) + h) ⟩ :
```


' $r6(u, v, w)$ ' = $r6(u, v, w)$

$$r6(u, v, w) = \begin{bmatrix} w a \cos(u) + a \\ w a \sin(u) \\ v \left(-\frac{1}{3} \sqrt{3} (w a \cos(u) + a) + h \right) \end{bmatrix} \quad (7.1)$$

For at plotte vælges en værdi for a (radius i cirklen på jorden) og h (højden):

> $a := 13;$
 > $h := 38$

$a := 13$

$h := 38$

(7.2)

> $with(Integrator8) :$

> $B := [0, 2 \cdot \pi, 0, 1, 0, 1]$

$B := [0, 2 \pi, 0, 1, 0, 1]$

(7.3)

> $net := [10, 10, 10]$

$net := [10, 10, 10]$

(7.4)

> $bygning := Integrator8[sideFlader](r6, B, net) :$

Plot i 3D:

> $display(bygning, axes = box, labels = [x, y, z])$

