

## Eksempel på funktion af 2 variable, hvor rækkefølgen af differentiation efter x hhv. y IKKE giver det samme ved ombytning.

$$f(x, y) = \begin{cases} x \cdot y \cdot \frac{x^2 - y^2}{x^2 + y^2} & \text{for } x \neq 0 \wedge y \neq 0 \\ 0 & \text{for } x = 0 \wedge y = 0 \end{cases}$$

Eksemplet er hentet fra bogen "Counterexamples in Analysis":

[http://books.google.com/books?id=cDAMh5n4lkkC&printsec=frontcover&hl=da&source=gbs\\_navlinks\\_s#v=onepage&q=&f=false](http://books.google.com/books?id=cDAMh5n4lkkC&printsec=frontcover&hl=da&source=gbs_navlinks_s#v=onepage&q=&f=false)

> restart

1. ordens afledede:

$\frac{\partial f}{\partial x}$  beregnes (på nær i (0, 0)) :

> diff  $\left( x \cdot y \cdot \frac{x^2 - y^2}{x^2 + y^2}, x \right)$

$$\frac{y(x^2 - y^2)}{x^2 + y^2} + \frac{2x^2y}{x^2 + y^2} - \frac{2x^2y(x^2 - y^2)}{(x^2 + y^2)^2} \quad (1)$$

$\frac{\partial f}{\partial x}(0, y)$  beregnes (på nær i (0, 0)) :

> subs(x=0, (1))

$$-y \quad (2)$$

$\frac{\partial f}{\partial x}(0, 0) = 0$ , da  $|f(x, y) - f(0, 0)| \leq |x| \cdot |y|$

$\frac{\partial f}{\partial y}$  beregnes (på nær i (0, 0)) :

> diff  $\left( x \cdot y \cdot \frac{x^2 - y^2}{x^2 + y^2}, y \right)$

$$\frac{x(x^2 - y^2)}{x^2 + y^2} - \frac{2xy^2}{x^2 + y^2} - \frac{2xy^2(x^2 - y^2)}{(x^2 + y^2)^2} \quad (3)$$

$\frac{\partial f}{\partial y}(x, 0)$  beregnes (på nær i (0, 0)) :

> subs(y=0, (3))

$$x \quad (4)$$

$\frac{\partial f}{\partial y}(0, 0) = 0$ , da  $|f(x, y) - f(0, 0)| \leq |x| \cdot |y|$

2. ordens afledede:

Beregning af  $\frac{\partial^2 f}{\partial y \cdot \partial x}(0, 0)$  :

$$\lim_{y \rightarrow 0} \left( \frac{\frac{\partial f}{\partial x}(0, y) - \frac{\partial f}{\partial x}(0, 0)}{y} \right)$$

$$> \lim_{y \rightarrow 0} \left( \frac{(2) - 0}{y} \right)$$

-1

(5)

Dvs.  $\frac{\partial^2 f}{\partial y \cdot \partial x}(0, 0) = -1$

Beregning af  $\frac{\partial^2 f}{\partial x \cdot \partial y}(0, 0)$  :

$$\lim_{x \rightarrow 0} \left( \frac{\frac{\partial f}{\partial y}(x, 0) - \frac{\partial f}{\partial y}(0, 0)}{x} \right)$$

$$> \lim_{x \rightarrow 0} \left( \frac{(4) - 0}{x} \right)$$

1

(6)

Dvs.  $\frac{\partial^2 f}{\partial x \cdot \partial y}(0, 0) = 1$

Altså er  $\frac{\partial^2 f}{\partial y \cdot \partial x}(0, 0) \neq \frac{\partial^2 f}{\partial x \cdot \partial y}(0, 0)$