Tegning af 2D-parametriseret område

Anvendelse af Integrator8-pakken

Dobbeltintegraler med ikke-faste grænser

Eksemplerne er fra Maple-demo 24a_Planintegral

Kortfattet oversigt over kommandoerne i Integrator8-pakken kan findes på Steens hjemmeside

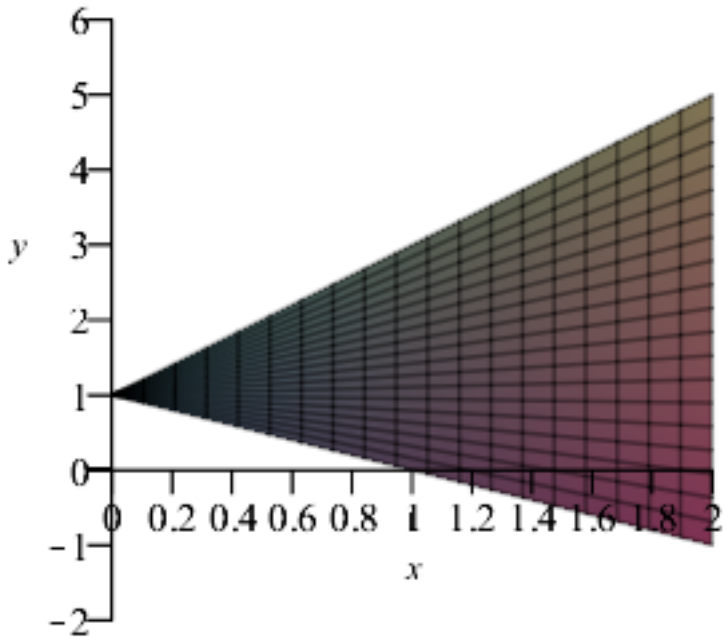
Plot af parametriseret område i planen (2D)

```
> restart
> with(Integrator8) :
```

Eksempel 1 (trekant)

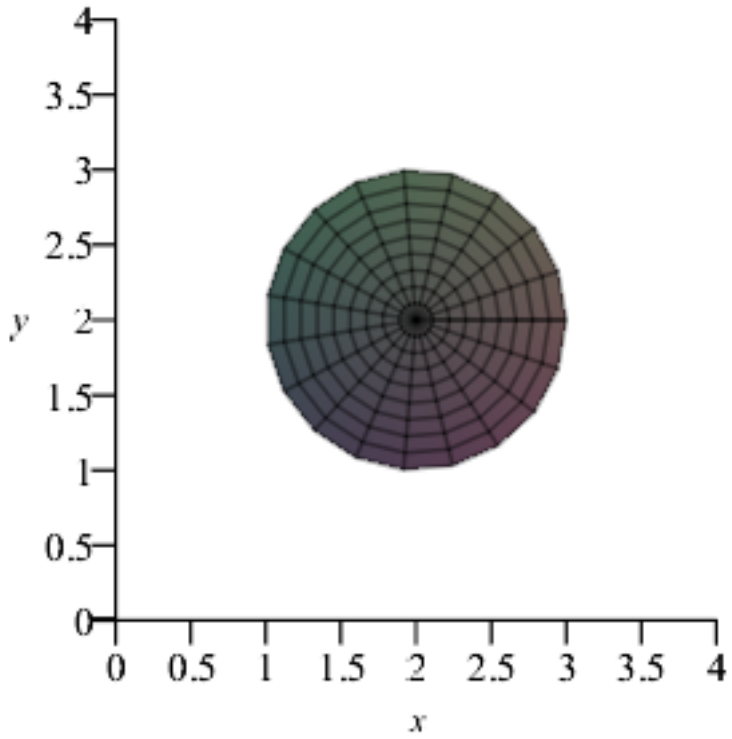
Det parametriserede 2D-område kan - *tilsyneladende* - kun tegnes ved at tegne et 3D-plot, som ses ovenfra!

```
> plot3d(⟨u, 1 − u + 3·v·u, 0⟩, u = 0..2, v = 0..1, labels = [x, y, ""], axes = normal,
      orientation = [−90, 0], view = [0..2, −2..6, −1..1], tickmarks = [10, 10, 10], grid
      = [20, 20])
```



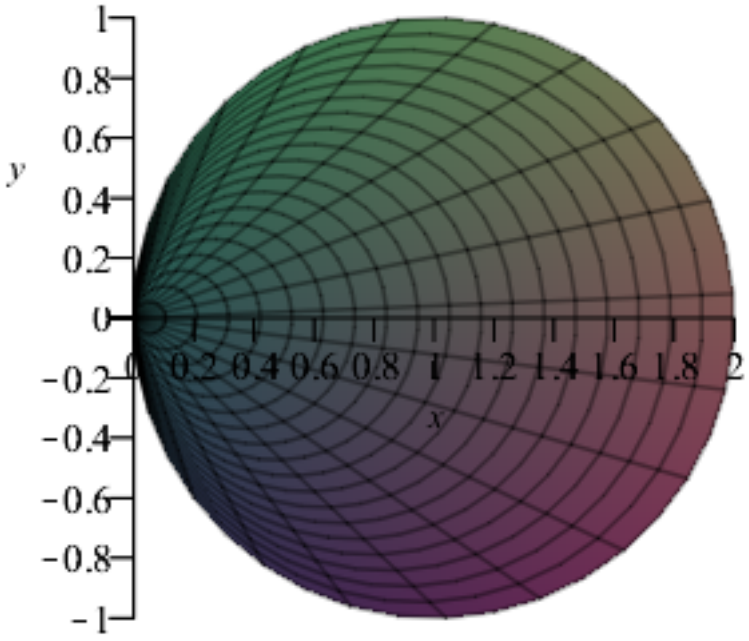
Eksempel 2 (cirkel)

```
> plot3d( ⟨2 + u·cos(v), 2 + u·sin(v), 0⟩, u = 0..1, v = 0..2·π, labels = [x, y, ""], axes
    = normal, orientation = [-90, 0], view = [0..4, 0..4, -1..1], tickmarks = [10, 10,
  10], grid = [10, 20])
```



Eksempel 3 (cirkel)

```
> plot3d( <2·u·cos(v)·cos(v), 2·u·cos(v)·sin(v), 0>, u = 0..1, v = -π/2..π/2, labels
    = [x, y, ""], axes = normal, orientation = [-90, 0], view = [0..2, -1..1, -1..1],
    tickmarks = [10, 10, 10], grid = [20, 40] )
```

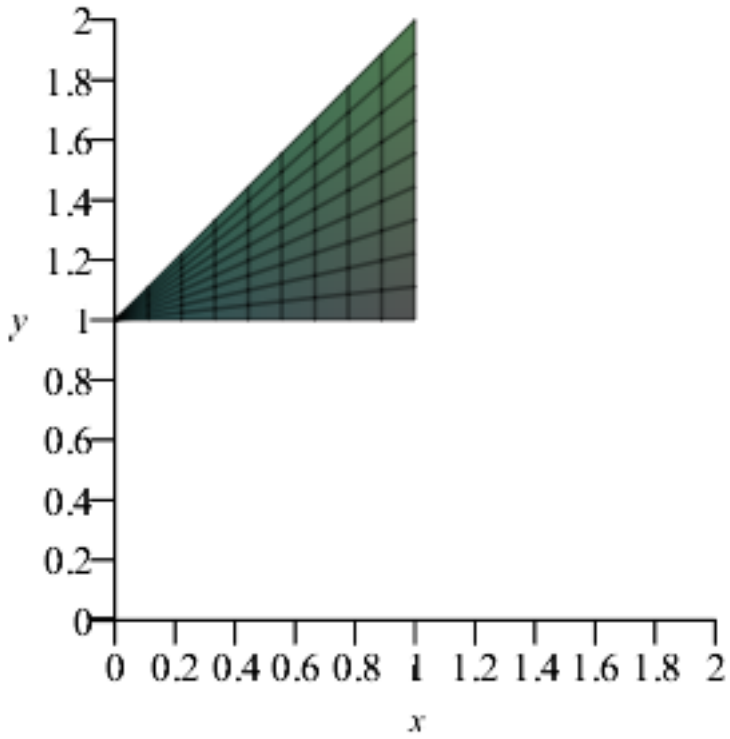


Et planintegral over et trekantet område

```
> r := (u, v) → ⟨u, 1 + v·u⟩ : r(u, v)
```

$$\begin{bmatrix} u \\ v\,u+1 \end{bmatrix} \tag{1.4.1}$$

```
> plot3d(⟨r(u, v)[1], r(u, v)[2], 0⟩, u = 0..1, v = 0..1, labels = [x, y, ""], axes
      = normal, orientation = [-90, 0], view = [0..2, 0..2, -1..1], tickmarks = [10, 10,
      10], grid = [10, 10])
```



a) Beregnet med Integrator8-pakken

> $f := (x, y) \to 2 \cdot x \cdot y : f(x, y)$

$$2\,y\,x \qquad \textbf{(1.4.1.1)}$$

> $B := [0, 1, 0, 1]$

$$B := [0, 1, 0, 1] \qquad \textbf{(1.4.1.2)}$$

> $planIntGo(r, B, f)$

$$\frac{11}{12} \qquad \textbf{(1.4.1.3)}$$

b) Beregnet med integral, hvor grænserne ikke er konstante

Parameteren y løber mellem 1 og x+1 (idet den rette linje $y = x + 1$ begrænser opadtil).
Parameteren x løber mellem 0 og 1.

> $\int_0^1 \int_1^{x+1} f(x, y)\, dy\, dx$

$$\frac{11}{12} \qquad \textbf{(1.4.2.1)}$$

Opdelt i 2 trin:

> $\int_1^{x+1} f(x,y)\ \mathrm{d}y$

$$x\left((x+1)^2-1\right) \qquad \textbf{(1.4.2.2)}$$

> $\int_0^1 \textbf{(1.4.2.2)}\ \mathrm{d}x$

$$\frac{11}{12} \qquad \textbf{(1.4.2.3)}$$

eller

Parameteren x løber mellem $y-1$ og 1 (idet den rette linje $y=x+1$ begrænser opadtil).
Parameteren y løber mellem 1 og 2.

> $\int_1^2\int_{y-1}^1 f(x,y)\ \mathrm{d}x\ \mathrm{d}y$

$$\frac{11}{12} \qquad \textbf{(1.4.2.4)}$$

Opdelt i 2 trin:

> $\int_{y-1}^1 f(x,y)\ \mathrm{d}x$

$$y\left(1-(y-1)^2\right) \qquad \textbf{(1.4.2.5)}$$

> $\int_1^2 \textbf{(1.4.2.5)}\ \mathrm{d}y$

$$\frac{11}{12} \qquad \textbf{(1.4.2.6)}$$

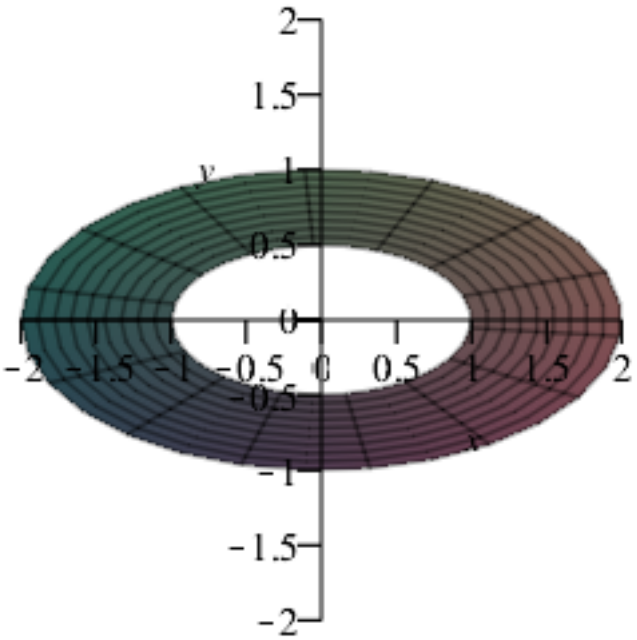
Et planintegral og massemidtpunkt

Planintegralet

> $r := (u,v) \to \left\langle u\cdot\cos(v),\ \frac{1}{2}\cdot u\cdot\sin(v)\right\rangle : r(u,v)$

$$\begin{bmatrix} u\cos(v) \\ \frac{u\sin(v)}{2} \end{bmatrix} \qquad \textbf{(1.5.1.1)}$$

> $plot3d(\langle r(u,v)[1], r(u,v)[2], 0\rangle, u=1..2, v=-\pi..\pi, labels=[x,y,""], axes=normal, orientation=[-90,0], view=[-2..2,-2..2,-1..1], tickmarks=[10,10,10], grid=[10,30])$



> $f := (x, y) \to (x-1)^2 (y+1)^2$

$$f := (x, y) \mapsto (x-1)^2 (y+1)^2 \qquad \textbf{(1.5.1.2)}$$

> $B := [1, 2, -\pi, \pi]$

$$B := [1, 2, -\pi, \pi] \qquad \textbf{(1.5.1.3)}$$

> $planIntGo(r, B, f)$

$$\frac{267\,\pi}{64} \qquad \textbf{(1.5.1.4)}$$

Massemidtpunktet

> $planCmGo(r, B, f)$

$$\left[-\frac{94}{89}, \frac{34}{89}\right] \qquad \textbf{(1.5.2.1)}$$