En funktion, hvor $\int_0^1\int_0^1 f(x,y)\,\mathrm{d}x\,\mathrm{d}y \neq \int_0^1\int_0^1 f(x,y)\,\mathrm{d}y\,\mathrm{d}x$

Kilde: **Side 124, eksempel 16 i "Counterexamples in Analysis".**

Funktion er defineret i kvadratet $[0;1]\,x\,[0;1]$:

$$f(x,y) := \begin{cases} y^{-2} & hvis\ 0 < x < y < 1 \\ -x^{-2} & hvis\ 0 < y < x < 1 \\ 0 & ellers \end{cases}$$

NB: Funktionen er ikke kontinuert!

Håndregning:

$$\int_0^1 f(x,y)\,\mathrm{d}x = \int_0^y f(x,y)\,\mathrm{d}x + \int_y^1 f(x,y)\,\mathrm{d}x = \int_0^y y^{-2}\,\mathrm{d}x + \int_y^1 (-x^{-2})\,\mathrm{d}x = [y^{-2}\cdot x]_{x=0}^{x=y} + [x^{-1}]_{x=y}^{x=1} =$$
$$(y^{-2}\cdot y - y^{-2}\cdot 0) + (1^{-1} - y^{-1}) = y^{-1} + 1 - y^{-1} = 1$$

$$\int_0^1\int_0^1 f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int_0^1 1\,\mathrm{d}y = [y]_{y=0}^{y=1} = 1 - 0 = \underline{\underline{1}}$$

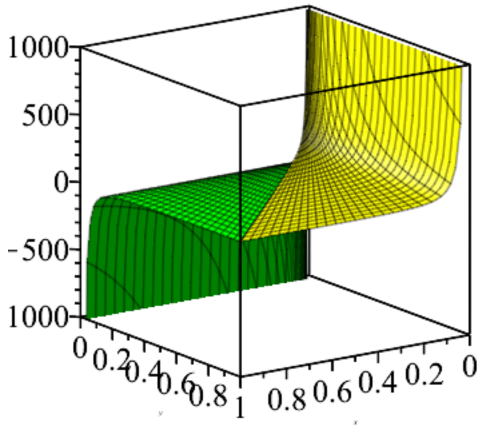
$$\int_0^1 f(x,y)\,\mathrm{d}y = \int_0^x f(x,y)\,\mathrm{d}y + \int_x^1 f(x,y)\,\mathrm{d}y = \int_0^x (-x^{-2})\,\mathrm{d}y + \int_x^1 (y^{-2})\,\mathrm{d}y = [-x^{-2}\cdot y]_{y=0}^{y=x} + [-y^{-1}]_{y=x}^{y=1} =$$
$$(-x^{-2}\cdot x + x^{-2}\cdot 0) + (-1^{-1} + x^{-1}) = -x^{-1} - 1 + x^{-1} = -1$$

$$\int_0^1\int_0^1 f(x,y)\,\mathrm{d}y\,\mathrm{d}x = \int_0^1 -1\,\mathrm{d}x = [-x]_{x=0}^{x=1} = -1 + 0 = \underline{\underline{-1}}$$

Dvs. $\int_0^1\int_0^1 f(x,y)\,\mathrm{d}x\,\mathrm{d}y \neq \int_0^1\int_0^1 f(x,y)\,\mathrm{d}y\,\mathrm{d}x$

Graf:

```
with(plots) :
X := plot3d(⟨u·v, v, (u·v)^(-2)⟩, u = 0 ..1, v = 0 ..1, color = yellow) :
Y := plot3d(⟨u, u·v, -(u·v)^(-2)⟩, u = 0 ..1, v = 0 ..1, color = green) :
display(X, Y, labels = [x, y, z], view = [0 ..1, 0 ..1, -1000 ..1000])
```


1000
500
0
-500
1000
0
0.2
0.4
0.6
0.8
1
0.8
0.6
0.4
0.2
0