Effektiv løsning af differentialligningssystem med "dsolve"; hvor problemet opskrives på vektor/matrix-form

Opgave:

Løs differentialigningssystemet:

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 4 & -1 \\ 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 2 \cdot e^t \\ \sin(t) \\ t^2 + 2 \cdot t \end{bmatrix}$$

Løsning:

restart
with(*LinearAlgebra*) :

$$Xm := \begin{bmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{bmatrix} :$$

$$A := \begin{bmatrix} 1 & -1 & 1 \\ 2 & 4 & -1 \\ 0 & 0 & 3 \end{bmatrix} :$$

$$X := \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} :$$

$$B := \begin{bmatrix} 2 \cdot e^t \\ \sin(t) \\ t^2 + 2 \cdot t \end{bmatrix} :$$

$$Xm = A \cdot X + B = = \begin{bmatrix} D(x_1)(t) \\ D(x_2)(t) \\ D(x_3)(t) \end{bmatrix} = \begin{bmatrix} x_1(t) - x_2(t) + x_3(t) + 2\, e^t \\ 2\, x_1(t) + 4\, x_2(t) - x_3(t) + \sin(t) \\ 3\, x_3(t) + t^2 + 2\, t \end{bmatrix}$$

$$L := dsolve(Xm = A \cdot X + B) =$$

$$\Big\{ x_1(t) = e^{3t}\,\_C2 + e^{2t}\,\_C1 + \frac{1}{6}\, t^2 - \frac{1}{10}\cos(t) - 3\, e^t - \frac{1}{10}\sin(t) + \frac{11}{18}\, t + \frac{49}{108}, x_2(t) =$$

$-2\, e^{3t}\_C2 - e^{2t}\_C1 - \frac{11}{18}\, t - \frac{1}{5}\sin(t) + 2\, e^{t} - \frac{49}{108} - \frac{1}{6}\, t^2 + e^{3t}\_C3, x_3(t) = -\frac{1}{3}\, t^2 - \frac{8}{9}\, t - \frac{8}{27} + e^{3t}\_C3\}$

$x_1 := unapply(rhs(L[1]), t) : x_2 := unapply(rhs(L[2]), t) : x_3 := unapply(rhs(L[3]), t) :$

Den fuldstændige løsning:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} e^{3t}\_C2 + e^{2t}\_C1 + \frac{t^2}{6} - \frac{\cos(t)}{10} - 3\, e^{t} - \frac{\sin(t)}{10} + \frac{11\, t}{18} + \frac{49}{108} \\ -2\, e^{3t}\_C2 - e^{2t}\_C1 - \frac{11\, t}{18} - \frac{\sin(t)}{5} + 2\, e^{t} - \frac{49}{108} - \frac{t^2}{6} + e^{3t}\_C3 \\ -\frac{t^2}{3} - \frac{8\, t}{9} - \frac{8}{27} + e^{3t}\_C3 \end{bmatrix}$$