

Kurveintegral (i 2 eller 3 dimensioner)

restart

with(plots) :

with(Integrator8) :

with(VektorAnalyse3) = [div, grad, kryds, prik, rot, vop]

\mathbb{R}^2 (parabelstykke)

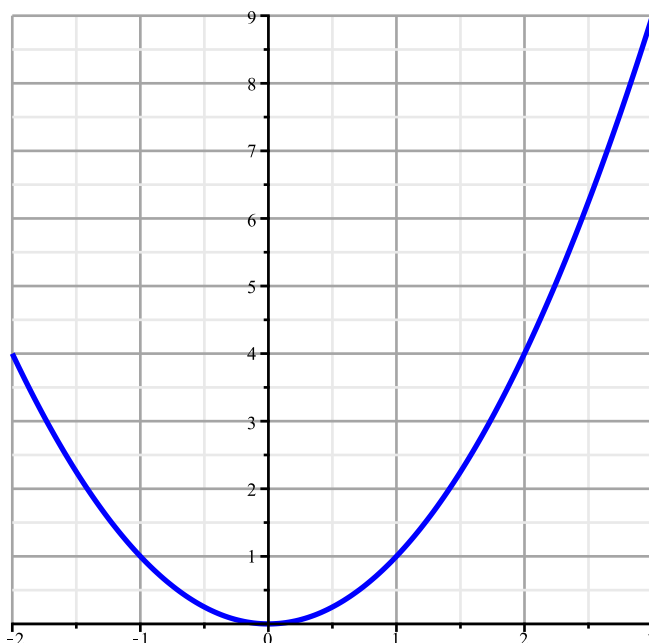
Parameterfremstilling af parabelen, hvor $u \in [-2..3]$:

$$r(u) := \langle u, u^2 \rangle :$$

$$r(u) = \begin{bmatrix} u \\ u^2 \end{bmatrix}$$

Graf

plot([vop(r(u)), u=-2..3], gridlines, color=blue, thickness=2)



Kurvelængde

Trinvis

$$rm(u) := \text{diff}(r(u), u) :$$

$$rm(u) = \begin{bmatrix} 1 \\ 2u \end{bmatrix}$$

$$\text{Jacobi} := \sqrt{\text{prik}(rm(u), rm(u))} = \sqrt{4u^2 + 1}$$

$$\int_{-2}^3 1 \cdot \text{Jacobi} \, du = \sqrt{17} + \frac{\text{arcsinh}(4)}{4} + \frac{3\sqrt{37}}{2} + \frac{\text{arcsinh}(6)}{4}$$

Med færdig formel

$$\int_{-2}^3 1 \cdot \text{LinearAlgebra}[\text{Norm}](\text{diff}(r(u), u), 2) \, du = \sqrt{17} + \frac{\text{arcsinh}(4)}{4} + \frac{3\sqrt{37}}{2} + \frac{\text{arcsinh}(6)}{4}$$

eller

$$\int_{-2}^3 1 \cdot \sqrt{\text{prik}(\text{diff}(r(u), u), \text{diff}(r(u), u))} \, du = \sqrt{17} + \frac{\text{arcsinh}(4)}{4} + \frac{3\sqrt{37}}{2} + \frac{\text{arcsinh}(6)}{4}$$

Med Integrator8-pakken

$$INT := [-2, 3]:$$

$$\text{kurveIntGo}(r, INT, 1) = \sqrt{17} + \frac{\operatorname{arcsinh}(4)}{4} + \frac{3\sqrt{37}}{2} + \frac{\operatorname{arcsinh}(6)}{4}$$

$$\text{evalf}(\%) = 14.39387252$$

Konklusion: kurvelængden er ca. 14.4

Masse af kurve

Hvis man antager, at kurven har en massetæthed (masse pr. længdeenhed) kaldet f , så kan man beregne kurvens samlede masse.

I dette eksempel afhænger massetætheden alene af x :

$$f(x, y) := 2 \cdot x + 1:$$

Trinvis

$$rm(u) := \text{diff}(r(u), u):$$

$$rm(u) = \begin{bmatrix} 1 \\ 2u \end{bmatrix}$$

$$\text{Jacobi} := \sqrt{\text{prik}(rm(u), rm(u))} = \sqrt{4u^2 + 1}$$

$$\int_{-2}^3 f(\text{vop}(r(u))) \cdot \text{Jacobi} \, du = -\frac{11\sqrt{17}}{6} + \frac{\operatorname{arcsinh}(4)}{4} + \frac{23\sqrt{37}}{3} + \frac{\operatorname{arcsinh}(6)}{4}$$

Med færdig formel

$$\int_{-2}^3 f(\text{vop}(r(u))) \cdot \text{LinearAlgebra}[\text{Norm}](\text{diff}(r(u), u), 2) \, du =$$

$$-\frac{11\sqrt{17}}{6} + \frac{\operatorname{arcsinh}(4)}{4} + \frac{23\sqrt{37}}{3} + \frac{\operatorname{arcsinh}(6)}{4}$$

eller

$$\int_{-2}^3 f(\text{vop}(r(u))) \cdot \sqrt{\text{prik}(\text{diff}(r(u), u), \text{diff}(r(u), u))} \, du =$$

$$-\frac{11\sqrt{17}}{6} + \frac{\operatorname{arcsinh}(4)}{4} + \frac{23\sqrt{37}}{3} + \frac{\operatorname{arcsinh}(6)}{4}$$

Med Integrator8-pakken

$$INT := [-2, 3]:$$

$$\text{kurveIntGo}(r, INT, f) = -\frac{11\sqrt{17}}{6} + \frac{\operatorname{arcsinh}(4)}{4} + \frac{23\sqrt{37}}{3} + \frac{\operatorname{arcsinh}(6)}{4}$$

$$\text{evalf}(\%) = 40.22210885$$

Konklusion: kurven masse er ca. 40.2

 \mathbb{R}^3 (spiral, helix)

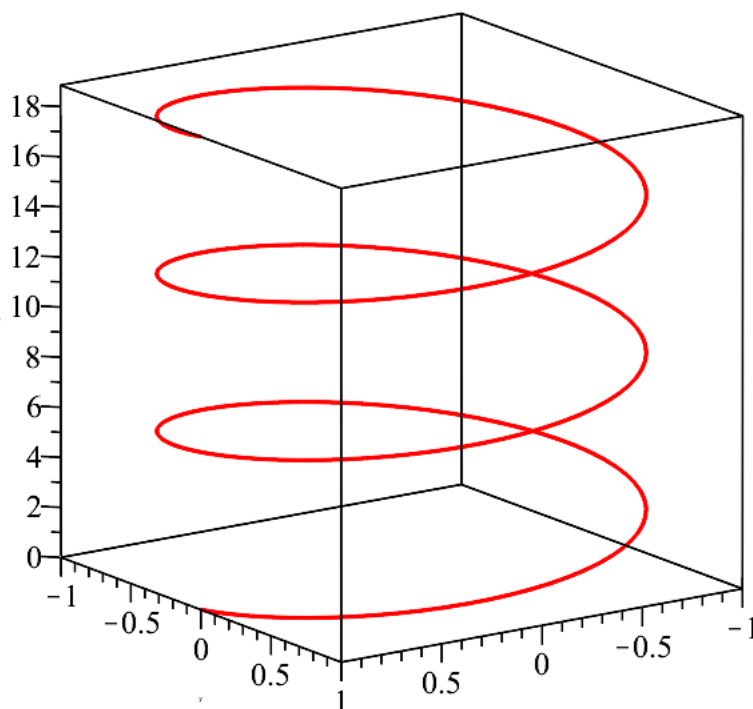
Parameterfremstilling af spiralen, hvor $u \in [0; 6 \cdot \pi]$:

$$r(u) := \langle \cos(u), \sin(u), u \rangle:$$

$$r(u) = \begin{bmatrix} \cos(u) \\ \sin(u) \\ u \end{bmatrix}$$

Graf

`spacecurve([vop(r(u)), u=0..6·π], color=red, thickness=3, labels=[x,y,z])`



Kurvelængde

Trinvis

$rm(u) := \text{diff}(r(u), u) :$

$$rm(u) = \begin{bmatrix} -\sin(u) \\ \cos(u) \\ 1 \end{bmatrix}$$

$$\text{Jacobi} := \sqrt{\text{prik}(rm(u), rm(u))} = \sqrt{1 + \sin(u)^2 + \cos(u)^2}$$

$$\int_0^{6 \cdot \pi} 1 \cdot \text{Jacobi} \, du = 6\sqrt{2} \pi$$

Med færdig formel

$$\int_0^{6 \cdot \pi} 1 \cdot \text{LinearAlgebra}[\text{Norm}](\text{diff}(r(u), u), 2) \, du = 6\sqrt{2} \pi$$

eller

$$\int_0^{6 \cdot \pi} 1 \cdot \sqrt{\text{prik}(\text{diff}(r(u), u), \text{diff}(r(u), u))} \, du = 6\sqrt{2} \pi$$

Med Integrator8-pakken

$INT := [0, 6 \cdot \pi] :$

$$\text{kurveIntGo}(r, INT, 1) = 6\sqrt{2} \pi$$

`evalf(%) = 26.65729763`

Konklusion: kurvelængden er

$$\underline{\underline{6 \cdot \sqrt{2} \cdot \pi \approx 26.7}}$$

▼ Masse af kurve

Hvis man antager, at kurven har en massetæthed (masse pr. længdeenhed) kaldet f , så kan man beregne kurvens samlede masse.

I dette eksempel afhænger massetætheden alene af z :

$$f(x, y, z) := e^{\frac{z}{20}} :$$

▼ Trinvis

$$rm(u) := \text{diff}(r(u), u) :$$

$$rm(u) = \begin{bmatrix} -\sin(u) \\ \cos(u) \\ 1 \end{bmatrix}$$

$$Jacobi := \sqrt{\text{prik}(rm(u), rm(u))} = \sqrt{1 + \sin(u)^2 + \cos(u)^2}$$

$$\int_0^{6 \cdot \pi} f(\text{vop}(r(u))) \cdot Jacobi \, du = -20 \sqrt{2} + 20 e^{\frac{3\pi}{10}} \sqrt{2}$$

▼ Med færdig formel

$$\int_0^{6 \cdot \pi} f(\text{vop}(r(u))) \cdot \text{LinearAlgebra}[\text{Norm}](\text{diff}(r(u), u), 2) \, du = -20 \sqrt{2} + 20 e^{\frac{3\pi}{10}} \sqrt{2}$$

eller

$$\int_0^{6 \cdot \pi} f(\text{vop}(r(u))) \cdot \sqrt{\text{prik}(\text{diff}(r(u), u), \text{diff}(r(u), u))} \, du = -20 \sqrt{2} + 20 e^{\frac{3\pi}{10}} \sqrt{2}$$

▼ Med Integrator8-pakken

$$INT := [0, 6 \cdot \pi] :$$

$$\text{kurveIntGo}(r, INT, f) = 20 \sqrt{2} \left(e^{\frac{3\pi}{10}} - 1 \right)$$

$$\text{evalf}(\%) = 44.30257034$$

Konklusion: kurven masse er ca. 44.3