

Tangentielt kurveintegral (i 3 dimensioner)

restart

with(plots) :

with(Integrator8) :

with(VektorAnalyse4) = [div, grad, kryds, prik, rot, vop]

Parameterkurve (eksempel)

Parameterfremstilling af parablen, hvor $u \in [-1 ..2]$:

$r(u) := \langle \cos(\pi \cdot u), u^2, u \rangle :$

$$r(u) = \begin{bmatrix} \cos(\pi u) \\ u^2 \\ u \end{bmatrix}$$

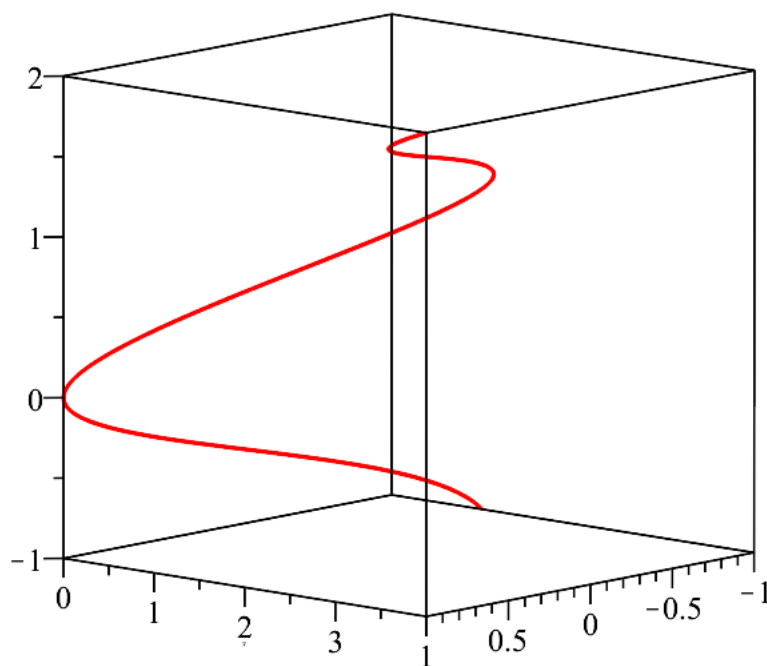
Vektorfelt (eksempel)

$V(x, y, z) := \langle x^2, 1, z \rangle :$

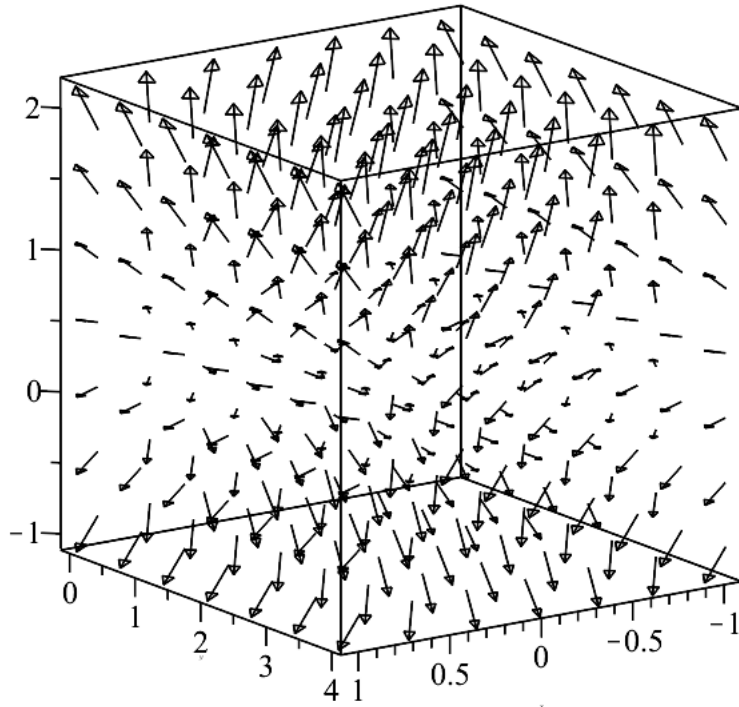
$$V(x, y, z) = \begin{bmatrix} x^2 \\ 1 \\ z \end{bmatrix}$$

Graf for kurven

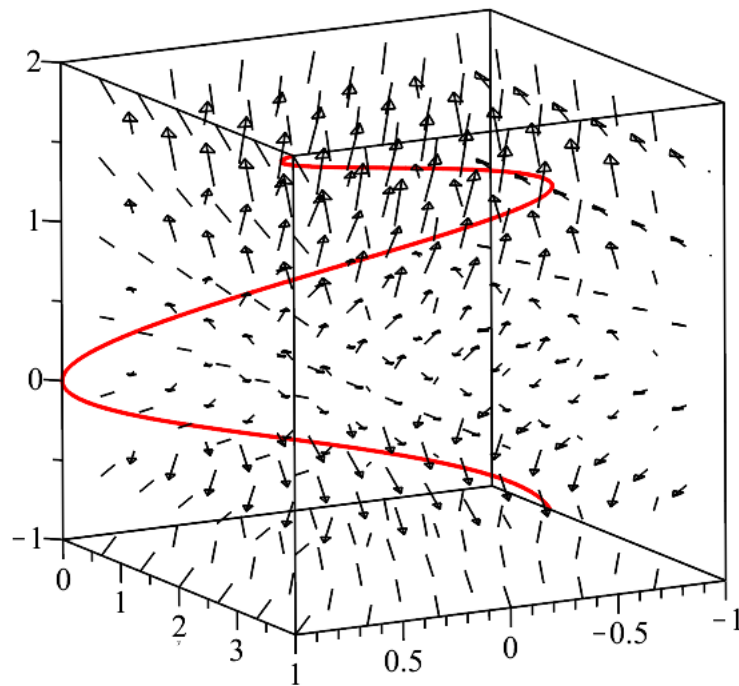
$K := \text{spacecurve}([\text{vop}(r(u)), u = -1 ..2], \text{color} = \text{red}, \text{thickness} = 3, \text{labels} = [x, y, z], \text{view} = [-1 ..1, 0 ..4, -1 ..2])$



$F := \text{fieldplot3d}(V(x, y, z), x = -1 ..1, y = 0 ..4, z = -1 ..2, \text{color} = \text{black}, \text{arrows} = \text{SLIM}, \text{grid} = [7, 7, 7])$



$display(K, F)$



▼ Tangentielt kurveintegral

▼ Trinvis

$rm(u) := diff(r(u), u) :$

$$rm(u) = \begin{bmatrix} -\pi \sin(\pi u) \\ 2u \\ 1 \end{bmatrix}$$

$$V(\text{vop}(r(u))) = \begin{bmatrix} \cos(\pi u)^2 \\ 1 \\ u \end{bmatrix}$$

$$\text{integrand} := \text{prik}(V(\text{vop}(r(u))), rm(u)) = -\cos(\pi u)^2 \pi \sin(\pi u) + 3u$$

$$\int_{-1}^2 \text{integrand} du = \frac{31}{6}$$

Med færdig formel

$$\int_{-1}^2 \text{prik}(V(\text{vop}(r(u))), \text{diff}(r(u), u)) du = \frac{31}{6}$$

Med Integrator8-pakken

$$INT := [-1, 2]:$$

$$\text{tangKurveIntGo}(r, INT, V) = \frac{31}{6}$$

$$\text{evalf}(\%) = 5.166666667$$

Konklusion: det tangentielle kurveintegral er $\frac{31}{6} \approx 5.17$