Effektiv løsning af differentialligningssystem med "dsolve"; hvor problemet opskrives på <mark>vektor/matrix-form</mark>

Opgave:

Løs differentialigningssystemet:

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 4 & -1 \\ 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 2 \cdot e^t \\ \sin(t) \\ t^2 + 2 \cdot t \end{bmatrix}$$

Løsning:

restart
with(*LinearAlgebra*) :

$$Xm := \begin{bmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{bmatrix} :$$

$$A := \begin{bmatrix} 1 & -1 & 1 \\ 2 & 4 & -1 \\ 0 & 0 & 3 \end{bmatrix} :$$

$$X := \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} :$$

$$B := \begin{bmatrix} 2 \cdot e^t \\ \sin(t) \\ t^2 + 2 \cdot t \end{bmatrix} :$$

$$Xm = A \cdot X + B = \begin{bmatrix} D(x_1)(t) \\ D(x_2)(t) \\ D(x_3)(t) \end{bmatrix} = \begin{bmatrix} x_1(t) - x_2(t) + x_3(t) + 2\,e^t \\ 2\,x_1(t) + 4\,x_2(t) - x_3(t) + \sin(t) \\ 3\,x_3(t) + t^2 + 2\,t \end{bmatrix}$$

$L := dsolve(Xm = A \cdot X + B) =$

$\left\{ x_1(t) = e^{2t} \_C2 + e^{3t} \_C1 + \dfrac{1}{6}\,t^2 - \dfrac{1}{10}\,\sin(t) - 3\,e^t - \dfrac{1}{10}\,\cos(t) + \dfrac{11}{18}\,t + \dfrac{49}{108}, x_2(t) = -e^{2t} \_C2 \right.$

$\left. - 2\,e^{3t} \_C1 - \dfrac{11}{18}\,t + 2\,e^t - \dfrac{1}{5}\,\sin(t) - \dfrac{49}{108} - \dfrac{1}{6}\,t^2 + e^{3t} \_C3, x_3(t) = -\dfrac{1}{3}\,t^2 - \dfrac{8}{9}\,t - \dfrac{8}{27} \right.$

$$+ \mathrm{e}^{3\,t}\_C3 \bigg\}$$

$x_1 := unapply(rhs(L[1]), t) : x_2 := unapply(rhs(L[2]), t) : x_3 := unapply(rhs(L[3]), t) :$

Den fuldstændige løsning:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} \mathrm{e}^{2\,t}\_C2 + \mathrm{e}^{3\,t}\_C1 + \dfrac{t^2}{6} - \dfrac{\sin(t)}{10} - 3\,\mathrm{e}^t - \dfrac{\cos(t)}{10} + \dfrac{11\,t}{18} + \dfrac{49}{108} \\[4mm] -\mathrm{e}^{2\,t}\_C2 - 2\,\mathrm{e}^{3\,t}\_C1 - \dfrac{11\,t}{18} + 2\,\mathrm{e}^t - \dfrac{\sin(t)}{5} - \dfrac{49}{108} - \dfrac{t^2}{6} + \mathrm{e}^{3\,t}\_C3 \\[4mm] -\dfrac{t^2}{3} - \dfrac{8\,t}{9} - \dfrac{8}{27} + \mathrm{e}^{3\,t}\_C3 \end{bmatrix}$$