Tangentielt kurveintegral (i 3 dimensioner)

$restart$
$with(plots):$
$with(Integrator8):$
$with(VektorAnalyse4) = [div, grad, kryds, prik, rot, vop]$

Parameterkurve (eksempel)

Parameterfremstilling af parablen, hvor $u \in [-1..2]$:

$r(u) := \langle \cos(\pi \cdot u), u^2, u \rangle :$

$$r(u) = \begin{bmatrix} \cos(\pi u) \\ u^2 \\ u \end{bmatrix}$$

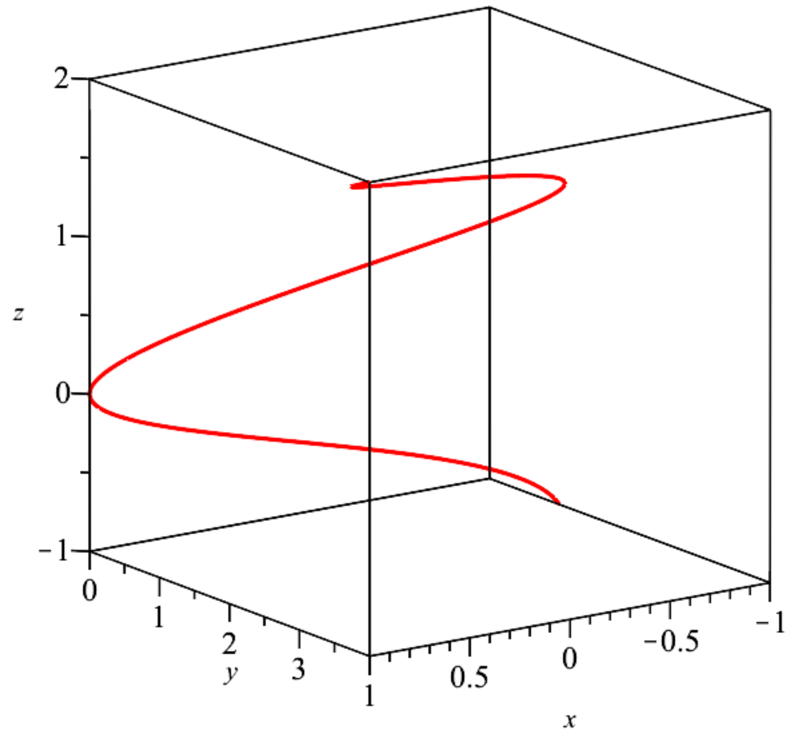
Vektorfelt (eksempel)

$V(x, y, z) := \langle x^2, 1, z \rangle :$

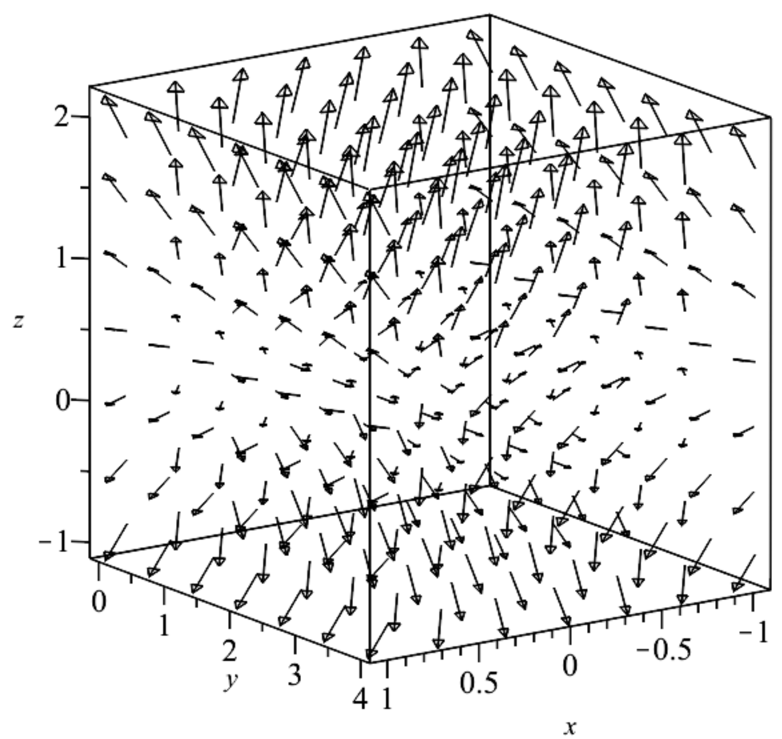
$$V(x, y, z) = \begin{bmatrix} x^2 \\ 1 \\ z \end{bmatrix}$$

Graf for kurven

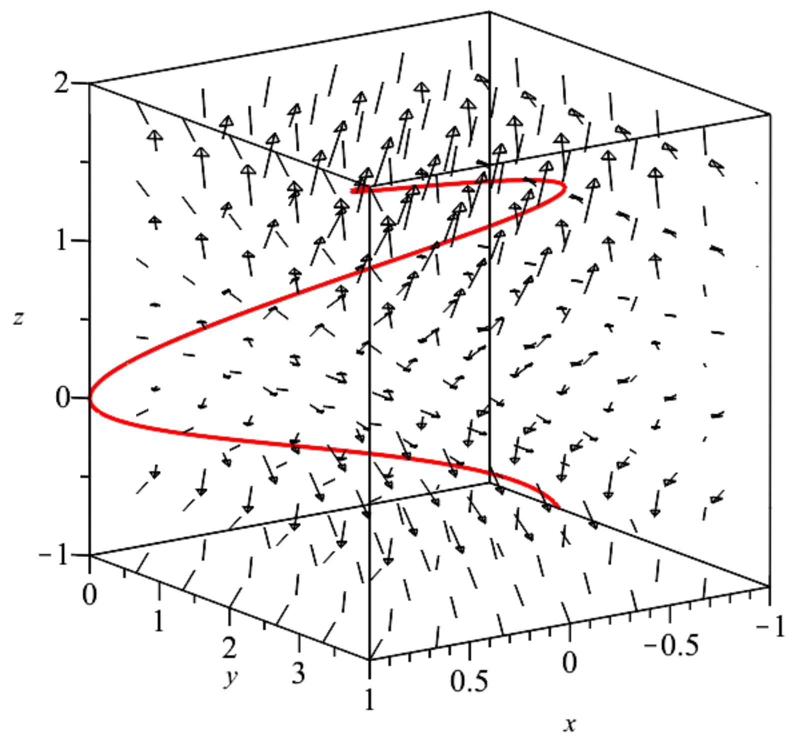
$K := spacecurve([vop(r(u)), u = -1..2], color = red, thickness = 3, labels = [x, y, z], view = [-1..1, 0..4, -1..2])$



$F := fieldplot3d(V(x, y, z), x = -1..1, y = 0..4, z = -1..2, color = black, arrows = SLIM, grid = [7, 7, 7])$



$display(K, F)$



Tangentielt kurveintegral

Trinvis

$rm(u) := diff(r(u), u) :$

$$rm(u) = \begin{bmatrix} -\pi \sin(\pi u) \\ 2u \\ 1 \end{bmatrix}$$

$$V(vop(r(u))) = \begin{bmatrix} \cos(\pi u)^2 \\ 1 \\ u \end{bmatrix}$$

$$integrand := prik(V(vop(r(u))), rm(u)) = -\cos(\pi u)^2 \pi \sin(\pi u) + 3u$$

$$\int_{-1}^{2} integrand \, \mathrm{d}u = \frac{31}{6}$$

Med færdig formel

$$\int_{-1}^{2} prik(V(vop(r(u))), diff(r(u), u)) \, \mathrm{d}u = \frac{31}{6}$$

Med Integrator8-pakken

$INT := [-1, 2]:$

$tangKurveIntGo(r, INT, V) = \frac{31}{6}$

$evalf(\%) = 5.166666667$

Konklusion: det tangentielle kurveintegral er $\frac{31}{6} \approx 5.17$