Eksempel på funktion af 2 variable, hvor rækkefølgen af differentiation efter x hhv. y IKKE giver det samme ved ombytning.

$$f(x,y) = \begin{cases} x \cdot y \cdot \dfrac{x^2 - y^2}{x^2 + y^2} & \text{for } x \neq 0 \wedge y \neq 0 \\ 0 & \text{for } x = 0 \wedge y = 0 \end{cases}$$

Eksemplet er hentet fra bogen "Counterexamples in Analysis":
http://books.google.com/books?id=cDAMh5n4lkkC&printsec=frontcover&hl=da&source=gbs_navlinks_s#v=onepage&q=&f=false

> *restart*

1. ordens afledede:

$\frac{\partial f}{\partial x}$ *beregnes* (*pånær i* (0, 0)) :

> $diff\left(x \cdot y \cdot \frac{x^2 - y^2}{x^2 + y^2}, x\right)$

$$\frac{y\,(x^2 - y^2)}{x^2 + y^2} + \frac{2\,x^2\,y}{x^2 + y^2} - \frac{2\,x^2\,y\,(x^2 - y^2)}{(x^2 + y^2)^2} \quad \textbf{(1)}$$

$\frac{\partial f}{\partial x}(0, y)$ *beregnes* (*pånær i* (0, 0)) :

> $subs(x = 0, \textbf{(1)})$

$$-y \quad \textbf{(2)}$$

$\frac{\partial f}{\partial x}(0, 0) = 0$, da $|f(x, y) - f(0, 0)| \leq |x| \cdot |y|$

$\frac{\partial f}{\partial y}$ *beregnes*(*pånær i* (0, 0)) :

> $diff\left(x \cdot y \cdot \frac{x^2 - y^2}{x^2 + y^2}, y\right)$

$$\frac{x\,(x^2 - y^2)}{x^2 + y^2} - \frac{2\,x\,y^2}{x^2 + y^2} - \frac{2\,x\,y^2\,(x^2 - y^2)}{(x^2 + y^2)^2} \quad \textbf{(3)}$$

$\frac{\partial f}{\partial y}(x, 0)$ *beregnes*(*pånær i* (0, 0)) :

> $subs(y = 0, \textbf{(3)})$

$$x \quad \textbf{(4)}$$

$\frac{\partial f}{\partial y}(0, 0) = 0$, da $|f(x, y) - f(0, 0)| \leq |x| \cdot |y|$

2. ordens afledede:

Beregning af $\frac{\partial^2 f}{\partial y \cdot \partial x}(0,0)$:

$$\lim_{y \to 0} \left(\frac{\frac{\partial f}{\partial x}(0,y) - \frac{\partial f}{\partial x}(0,0)}{y} \right)$$

> $\lim_{y \to 0} \left(\frac{(2) - 0}{y} \right)$

$$-1 \qquad (5)$$

Dvs. $\frac{\partial^2 f}{\partial y \cdot \partial x}(0,0) = -1$

Beregning af $\frac{\partial^2 f}{\partial x \cdot \partial y}(0,0)$:

$$\lim_{x \to 0} \left(\frac{\frac{\partial f}{\partial y}(x,0) - \frac{\partial f}{\partial y}(0,0)}{x} \right)$$

> $\lim_{x \to 0} \left(\frac{(4) - 0}{x} \right)$

$$1 \qquad (6)$$

Dvs. $\frac{\partial^2 f}{\partial x \cdot \partial y}(0,0) = 1$

Altså er $\frac{\partial^2 f}{\partial y \cdot \partial x}(0,0) \neq \frac{\partial^2 f}{\partial x \cdot \partial y}(0,0)$