

Huskemåde for divergens og rotation

Divergensen og rotationen, som anvendes i Maple på Matematik 1, ligger i Steens Maple-pakke "VektorAnalyse4".

Anvendelse: især i Maxwells ligninger i fysik, som beskriver elektriske (E) og magnetiske (B) felters udbredelse:

https://en.wikipedia.org/wiki/Maxwell%27s_equations

Differential equations
$\nabla \cdot \mathbf{E} = 4\pi\rho$
$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B} = \frac{1}{c} \left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} \right)$

Nabla:

$$\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

Vektorfunktion:

$$V = \langle V_x, V_y, V_z \rangle$$

▼ Divergensen (divergence)

NB: divergensen af et vektorfelt er en skalar!

Divergensen kan opfattes som skalarproduktet mellem ∇ og V :

$$\text{div}(V) = \nabla \cdot V = \frac{\partial}{\partial x} V_x + \frac{\partial}{\partial y} V_y + \frac{\partial}{\partial z} V_z$$

▼ Eksempel

restart

with(VektorAnalyse4) :

$$V(x, y, z) := \langle x^2 \cdot y, x + y + z, z^3 \rangle :$$

$$\text{div}(V)(x, y, z) = 2xy + 3z^2 + 1$$

$$\text{div}(V)(1, 2, 3) = 32$$

NB: $\text{div}(V)$ giver en **function**.

Rotationen (curl)

NB: rotationen af vektorfeltet er en vektor!

Rotationen kan opfattes som krydsproduktet af ∇ og V :

$$\begin{aligned}
 \text{rot}(V) = \nabla \times V &= \begin{bmatrix} \frac{\partial}{\partial x} V_x - \frac{\partial}{\partial z} V_z \\ \frac{\partial}{\partial y} V_y - \frac{\partial}{\partial z} V_z \\ \frac{\partial}{\partial x} V_x - \frac{\partial}{\partial y} V_y \end{bmatrix} \\
 &= \left(\frac{\partial}{\partial y} V_z - \frac{\partial}{\partial z} V_y \right) \cdot \vec{i} - \left(\frac{\partial}{\partial x} V_z - \frac{\partial}{\partial z} V_x \right) \cdot \vec{j} + \left(\frac{\partial}{\partial x} V_y - \frac{\partial}{\partial y} V_x \right) \cdot \vec{k} \\
 &= \begin{bmatrix} \frac{\partial}{\partial y} V_z - \frac{\partial}{\partial z} V_y \\ - \left(\frac{\partial}{\partial x} V_z - \frac{\partial}{\partial z} V_x \right) \\ \frac{\partial}{\partial x} V_y - \frac{\partial}{\partial y} V_x \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial y} V_z - \frac{\partial}{\partial z} V_y \\ \frac{\partial}{\partial z} V_x - \frac{\partial}{\partial x} V_z \\ \frac{\partial}{\partial x} V_y - \frac{\partial}{\partial y} V_x \end{bmatrix} \\
 &= \left\langle \frac{\partial}{\partial y} V_z - \frac{\partial}{\partial z} V_y, \frac{\partial}{\partial z} V_x - \frac{\partial}{\partial x} V_z, \frac{\partial}{\partial x} V_y - \frac{\partial}{\partial y} V_x \right\rangle
 \end{aligned}$$

Eksempel

restart

with(VektorAnalyse4) :

$$V(x, y, z) := \langle x^2 \cdot y, x + y + z, z^3 \rangle :$$

$$\text{rot}(V)(x, y, z) = \begin{bmatrix} -1 \\ 0 \\ -x^2 + 1 \end{bmatrix}$$

$$\text{rot}(V)(1, 2, 3) = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

NB: $\text{rot}(V)$ giver en **function**.