

En funktion, hvor $\int_0^1 \int_0^1 f(x, y) \, dx \, dy \neq \int_0^1 \int_0^1 f(x, y) \, dy \, dx$

Kilde: Side 124, eksempel 16 i "Counterexamples in Analysis".

Funktion er defineret i kvadratet $[0; 1] \times [0; 1]$:

$$f(x, y) := \begin{cases} y^{-2} & \text{hvis } 0 < x < y < 1 \\ -x^{-2} & \text{hvis } 0 < y < x < 1 \\ 0 & \text{ellers} \end{cases}$$

NB: Funktionen er ikke kontinuert!

Håndregning:

$$\int_0^1 f(x, y) \, dx = \int_0^y f(x, y) \, dx + \int_y^1 f(x, y) \, dx = \int_0^y y^{-2} \, dx + \int_y^1 (-x^{-2}) \, dx = [y^{-2} \cdot x]_{x=0}^{x=y} + [x^{-1}]_{x=y}^{x=1} = (y^{-2} \cdot y - y^{-2} \cdot 0) + (1^{-1} - y^{-1}) = y^{-1} + 1 - y^{-1} = 1$$

$$\int_0^1 \int_0^1 f(x, y) \, dx \, dy = \int_0^1 1 \, dy = [y]_{y=0}^{y=1} = 1 - 0 = \underline{\underline{1}}$$

$$\int_0^1 f(x, y) \, dy = \int_0^x f(x, y) \, dy + \int_x^1 f(x, y) \, dy = \int_0^x (-x^{-2}) \, dy + \int_x^1 (y^{-2}) \, dy = [-x^{-2} \cdot y]_{y=0}^{y=x} + [-y^{-1}]_{y=x}^{y=1} = (-x^{-2} \cdot x + x^{-2} \cdot 0) + (-1^{-1} + x^{-1}) = -x^{-1} - 1 + x^{-1} = -1$$

$$\int_0^1 \int_0^1 f(x, y) \, dy \, dx = \int_0^1 -1 \, dx = [-x]_{x=0}^{x=1} = -1 + 0 = \underline{\underline{-1}}$$

Dvs. $\int_0^1 \int_0^1 f(x, y) \, dx \, dy \neq \int_0^1 \int_0^1 f(x, y) \, dy \, dx$

Graf:

with(plots) :

$X := \text{plot3d}(\langle u \cdot v, v, (u \cdot v)^{-2} \rangle, u = 0 \dots 1, v = 0 \dots 1, \text{color} = \text{yellow}) :$

$Y := \text{plot3d}(\langle u, u \cdot v, -(u \cdot v)^{-2} \rangle, u = 0 \dots 1, v = 0 \dots 1, \text{color} = \text{green}) :$

$\text{display}(X, Y, \text{labels} = [x, y, z], \text{view} = [0 \dots 1, 0 \dots 1, -1000 \dots 1000])$

