Løsning af differentialligninger med elasticiteter

restart

Elasticiteten af en funktion $f(x)$ er defineret ved:

$$\frac{x}{f(x)} \cdot f'(x)$$

Elasticiteten defineres i Maple således:

$$El_x(f) := \frac{x}{f} \cdot \frac{d}{dx} f:$$

Differentialligninger kan løses med kommandoen "**dsolve**".

Konstant elasticitet

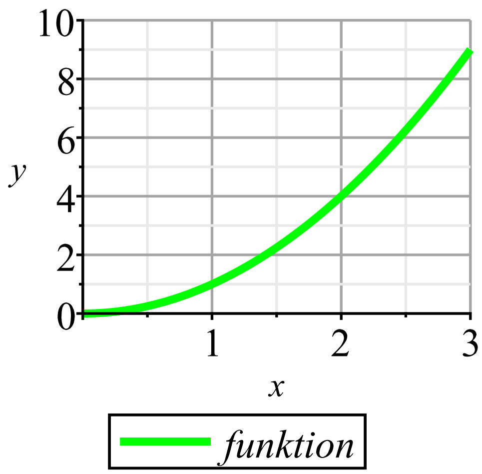
$dsolve\left(El_x(f(x)) = k\right) = f(x) = \_C1\, x^k$

Dvs. potensfunktionen $c \cdot x^k$ har elasticiteten k i alle værdier af x.

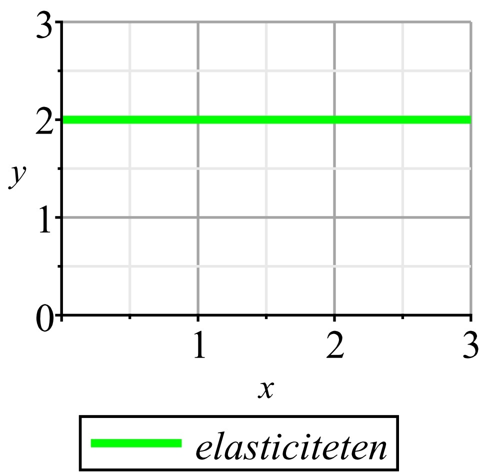
Test med håndregning:

$$El_x\left(c \cdot x^k\right) = \frac{x}{c \cdot x^k} \cdot \left(c \cdot k \cdot x^{k-1}\right) = \frac{c \cdot k \cdot x^k}{c \cdot x^k} = \underline{\underline{k}} \qquad \textbf{OK}$$

$plot\left(x^2, x = 0..3, y = 0..10, gridlines, color = green, thickness = 3, legend = funktion\right)$



$plot\left(El_x\left(x^2\right), x = 0..3, y = 0..3, gridlines, color = green, thickness = 3, legend = elasticiteten\right)$



Lineær elasticitet

$dsolve\left(El_x(f(x)) = a \cdot x + b\right) = f(x) = \_C1\, e^{a\,x}\, x^b$

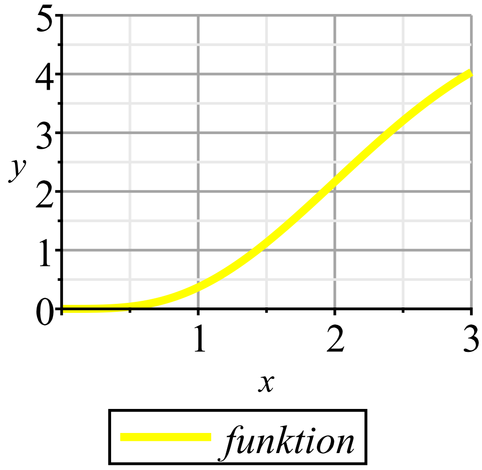
Dvs. funktionen $c \cdot e^{a \cdot x} \cdot x^b$ har elasticiteten $a \cdot x + b$ i alle værdier af x.

Test med håndregning:

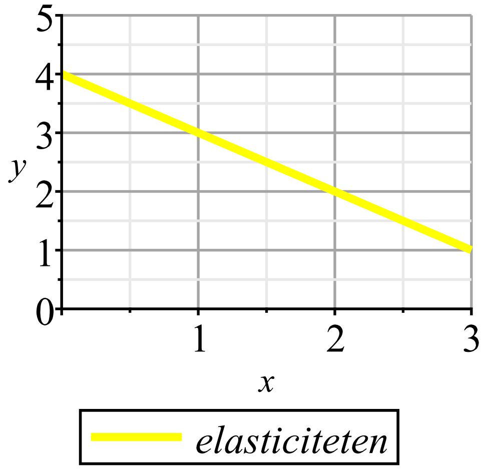
$$El_x\left(c \cdot e^{a \cdot x} \cdot x^b\right) = \frac{x}{c \cdot e^{a \cdot x} \cdot x^b} \cdot \left(c \cdot e^{a \cdot x} \cdot a \cdot x^b + c \cdot e^{a \cdot x} \cdot b \cdot x^{b-1}\right) = \frac{c \cdot e^{a \cdot x} \cdot x^b \cdot (a \cdot x + b)}{c \cdot e^{a \cdot x} \cdot x^b} = \underline{\underline{a \cdot x + b}}$$

OK

$plot\left(e^{-x} \cdot x^4, x = 0..3, y = 0..5, gridlines, color = yellow, thickness = 3, legend = funktion\right)$



$plot\left(El_x\left(e^{-x} \cdot x^4\right), x = 0..3, y = 0..5, gridlines, color = yellow, thickness = 3, legend = elasticiteten\right)$



Potens elasticitet

$dsolve\left(El_x(f(x)) = x^n\right) = f(x) = \_C1\, e^{\frac{x^n}{n}}$

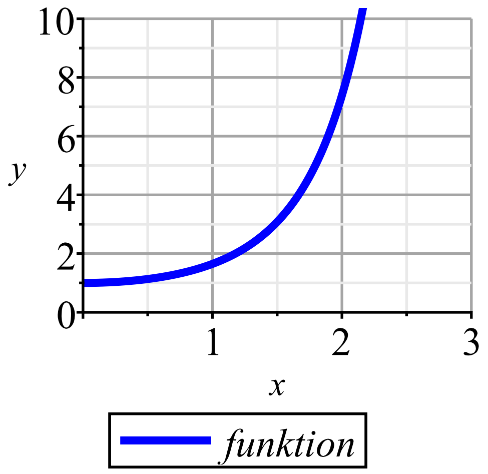
Dvs. funktionen $c \cdot e^{\frac{x^n}{n}}$ har elasticiteten x^n i alle værdier af x.

Test med håndregning:

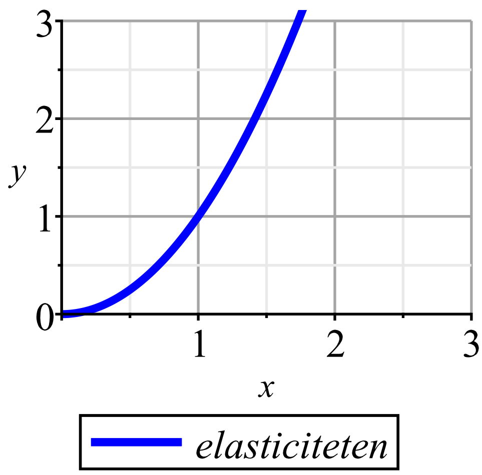
$$El_x\left(c \cdot e^{\frac{x^n}{n}}\right) = \frac{x}{c \cdot e^{\frac{x^n}{n}}} \cdot \left(c \cdot e^{\frac{x^n}{n}} \cdot \frac{1}{n} \cdot n \cdot x^{n-1}\right) = \frac{x^n \cdot c \cdot e^{\frac{x^n}{n}}}{c \cdot e^{\frac{x^n}{n}}} = \underline{\underline{x^n}} \quad \textbf{OK}$$

$plot\left(e^{\frac{x^2}{2}}, x = 0..3, y = 0..10, gridlines, color = blue, thickness = 3, legend = funktion\right)$

$plot\left(El_x\left(e^{\frac{x^2}{2}}\right), x = 0..3, y = 0..3, gridlines,\right.$



$color = blue, thickness = 3, legend = elasticiteten\Big)$



Hyperbel elasticitet

$dsolve\left(El_x(f(x)) = \frac{a}{x}\right) = f(x) = \_C1\, e^{-\frac{a}{x}}$

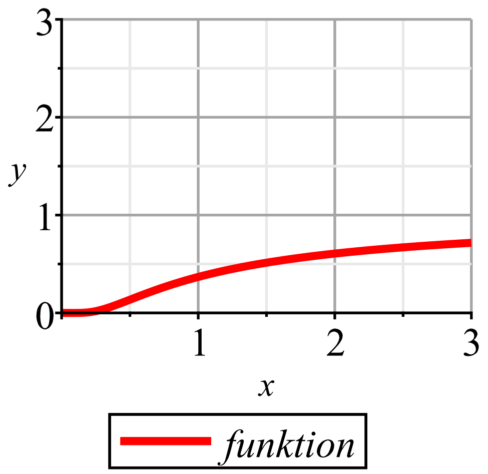
Dvs. funktionen $c \cdot e^{-\frac{a}{x}}$ har elasticiteten $\frac{a}{x}$ i alle værdier af x.

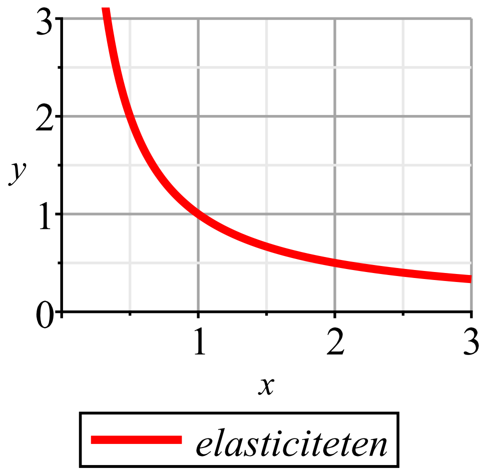
<u>Test med håndregning</u>:

$El_x\left(c\cdot e^{-\frac{a}{x}}\right) = \frac{x}{c\cdot e^{-\frac{a}{x}}}\cdot\left(c\cdot e^{-\frac{a}{x}}\cdot(-a)\cdot(-1\cdot x^{-2})\right) = \frac{a\cdot x^{-1}\cdot c\cdot e^{-\frac{a}{x}}}{c\cdot e^{-\frac{a}{x}}} = a\cdot x^{-1} = \underline{\underline{\frac{a}{x}}}$ **OK**

$plot\left(e^{-\frac{1}{x}}, x = 0..3, y = 0..3, gridlines, color = red, thickness = 3, legend = funktion\right)$

$plot\left(El_x\left(e^{-\frac{1}{x}}\right), x = 0..3, y = 0..3, gridlines, color = red, thickness = 3, legend = elasticiteten\right)$


3
2
y
1
0
1
2
3
x
funktion


3
2
y
1
0
1
2
3
x
elasticiteten