

MODELLERING AF REB, SPIRALER, ARMBÅND MAPLE WORKSHOP DTU 2023



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(2009-): ekstern lektor, Matematik 1, DTU Compute

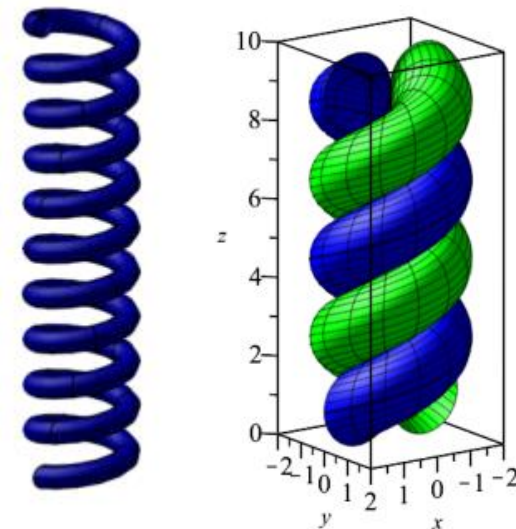
(1979-2018): lektor/pædagogisk faglig koordinator, Helsingør Gymnasium)

EMNER

- REB
- SPIRALER
- ARMBÅND

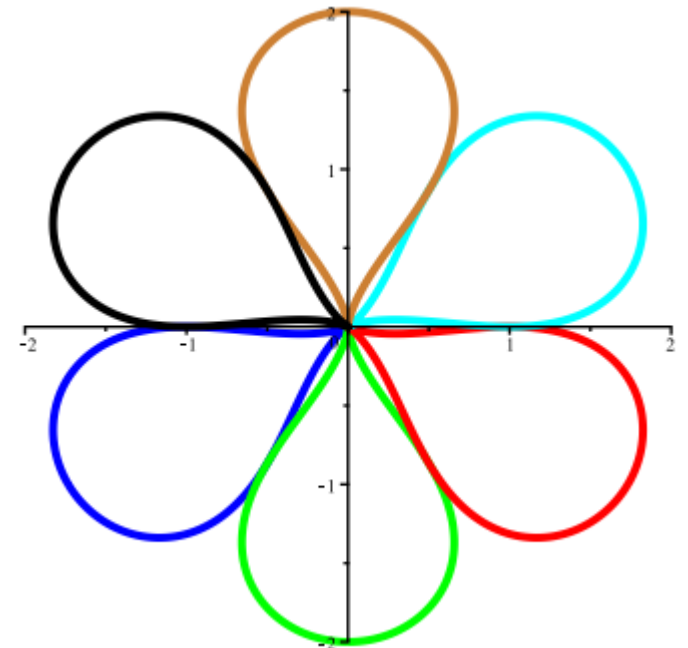
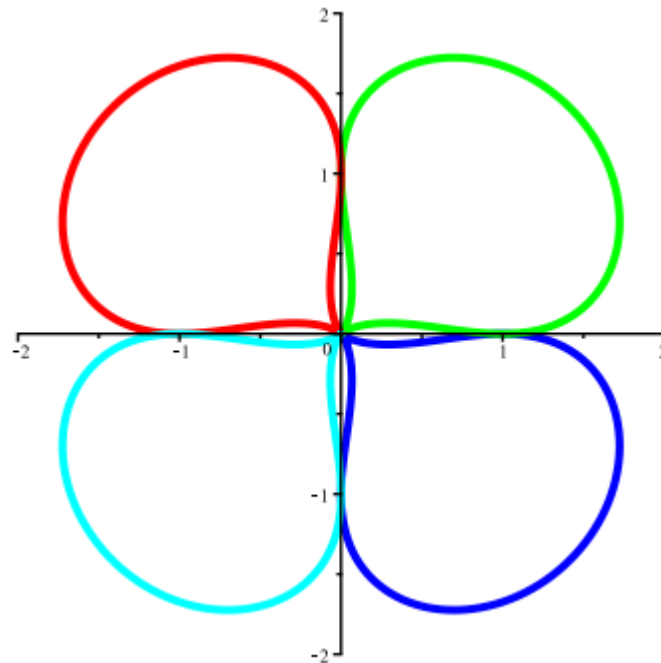
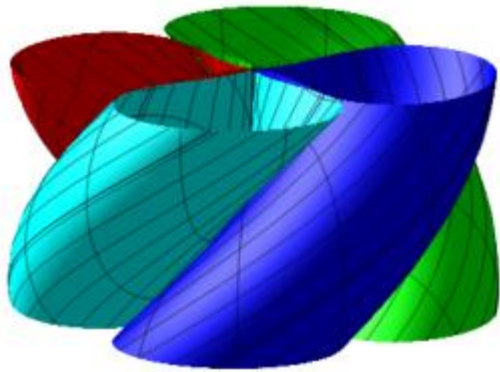
MODELLERING AF REB

- Fortsættelse af 3D-modellering
- ”**Reb**” er inspireret af 3 artikler fra Jakob Bohr (DTU) og Kasper Olsen (KU):
<https://link.springer.com/content/pdf/10.1007/s00214-009-0639-4.pdf>
<https://iopscience.iop.org/article/10.1209/0295-5075/93/60004/pdf>
<https://link.springer.com/content/pdf/10.1007/s00214-010-0761-3.pdf>
- Link til min Maple-modellering af reb:
<https://steen-toft.dk/mat/3d-print/reb.htm>
- 1. forsøg (lodret cirkel, som roteres og løftes løbende):
<https://steen-toft.dk/mat/3d-print/reb/reb1.pdf>



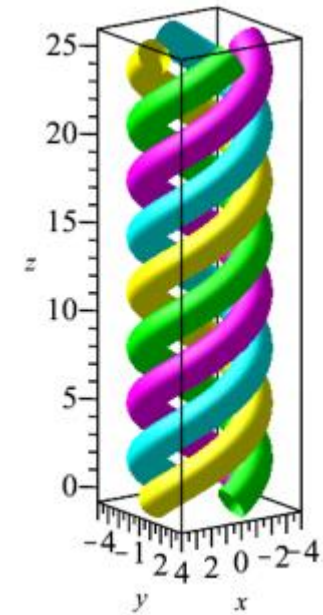
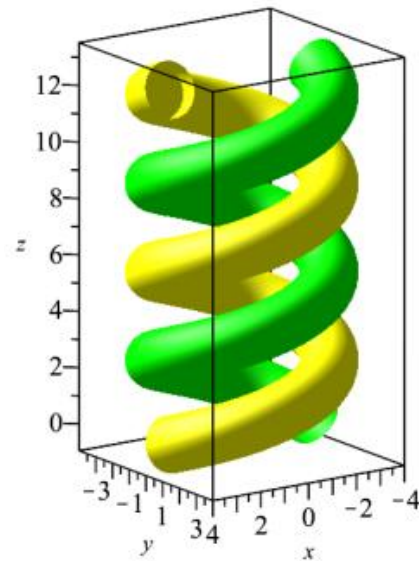
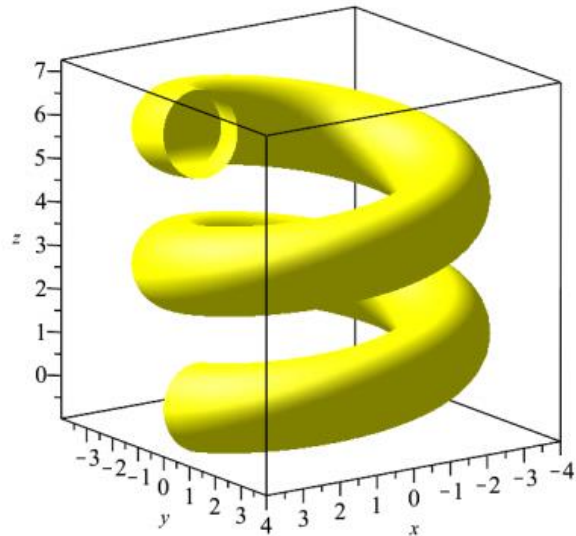
MODELLERING AF REB

- Problem: vandret snit giver ikke cirkler!
<https://steen-toft.dk/mat/3d-print/reb/reb2.pdf>



MODELLERING AF REB

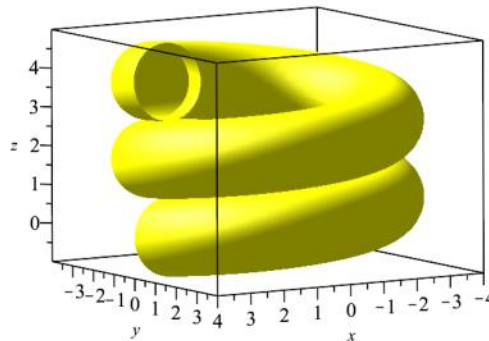
- Udvidelse af helix med en cirkel **vinkelret** på snoningen:
<https://steen-toft.dk/mat/3d-print/reb/reb3.pdf>



MODELLERING AF REB

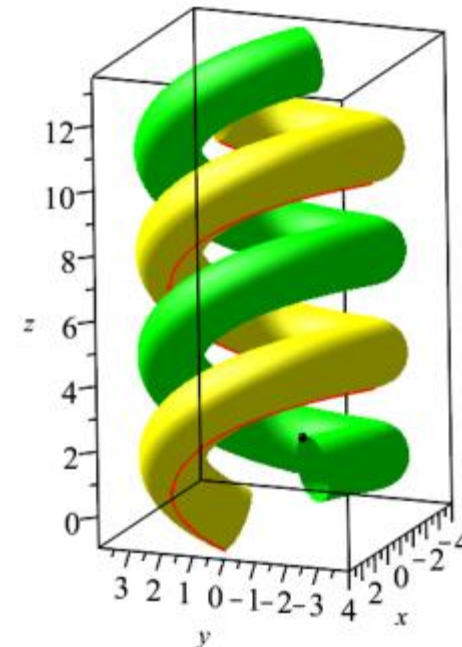
- Nu skal snoningerne ligge **tæt**:
<https://steen-toft.dk/mat/3d-print/reb/reb4.pdf>

- Går let med 1 kordel, idet stigningen er lig med diameteren af cirklen.



- Er der flere kordeler, så kan stigningen pr. omgang ikke beregnes analytisk, men alene **numerisk**.

*Det sorte punkt skal have 0 afstand til den **røde** bundlinje!*



NUMERISK LØSNING

$$\text{Difference}(u, h) = \begin{bmatrix} 3 \cos(u) - \frac{h \sin(u)}{\sqrt{h^2 + 9}} + 3 \\ 3 \sin(u) + \frac{h \cos(u)}{\sqrt{h^2 + 9}} - \frac{h}{\sqrt{h^2 + 9}} \\ hu - \frac{6}{\sqrt{h^2 + 9}} \end{bmatrix}$$

$$L := \text{fsolve}(\{ \text{Difference}(u, h)[1] = 0, \text{Difference}(u, h)[3] = 0 \}, \{ h = 0 .. 2, u = 0 .. 2 \cdot \pi \}) = \{ h = 0.6514057475, u = 3.000367376 \}$$

NB: kan ikke løses eksakt, derfor anvendes den numeriske løser på 2 af ligningerne (ikke 3 - for så findes der ikke en løsning).

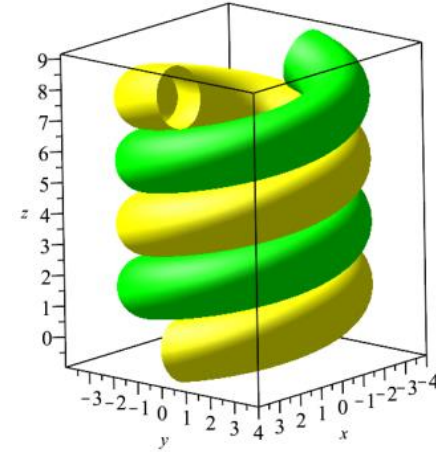
$$U := \text{rhs}(L[2]) : H := \text{rhs}(L[1]) :$$

$$\text{evalf}(\text{Difference}(U, H)) = \begin{bmatrix} 0. \\ -8. \times 10^{-10} \\ 0. \end{bmatrix}$$

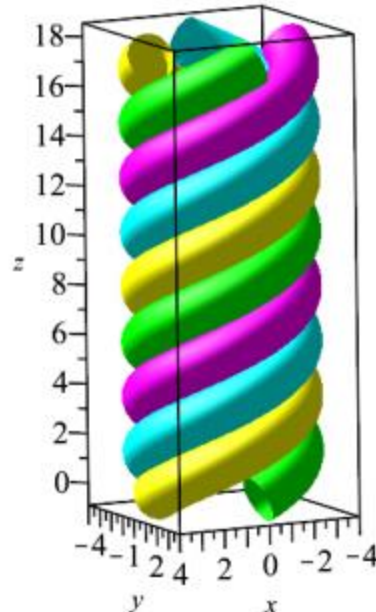
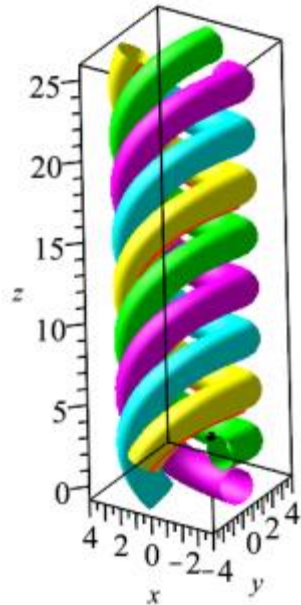
Dvs. med den givne H-værdi vil det sorte punkt ligge på den gule helix.

MODELLERING AF REB

- Når stigningen pr. omgang er beregnet numerisk, så kan der tegnes:

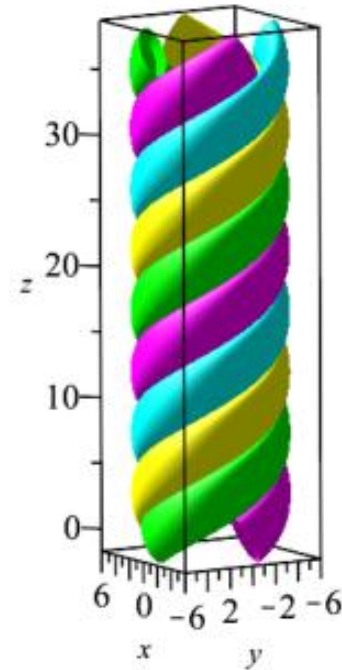
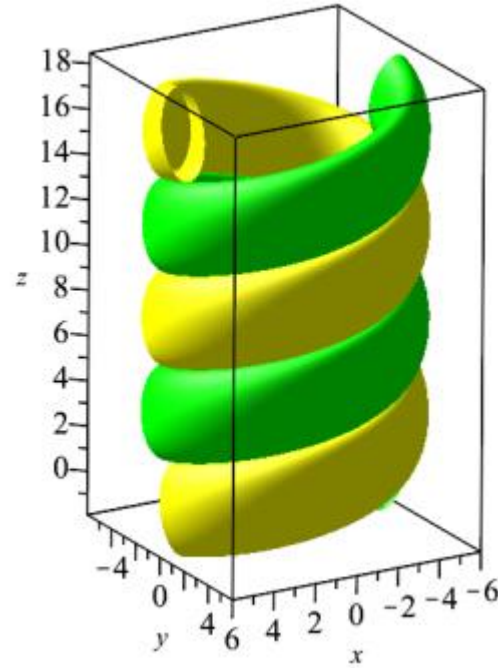
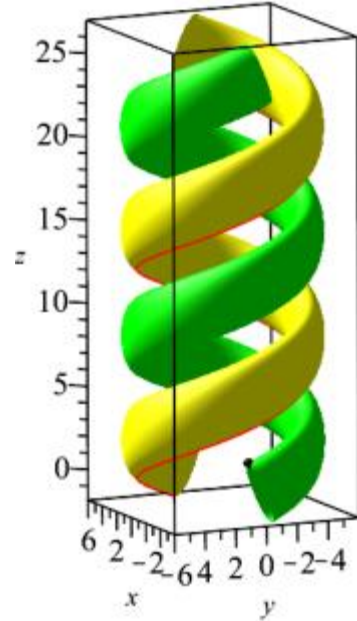
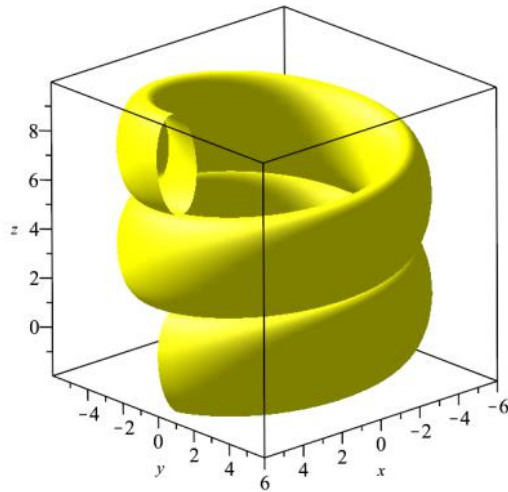


- Med 4 kordeler:



MODELLERING AF REB

- **Ambition:** udvidelsen af helixen er **elliptisk** (frem for cirkulær)!



MODELLERING AF SPIRALER

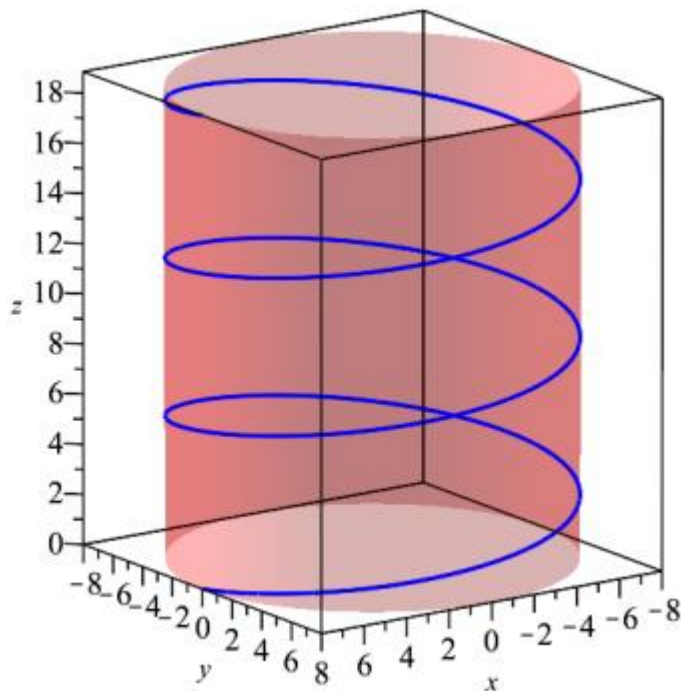
- ”**Spiraler**” er inspireret af denne artikel fundet på internettet:
<http://elib.mi.sanu.ac.rs/files/journals/vm/57/vmn57p2-10.pdf>
- Link til mit Maple-modellering af spiraler:
<https://steen-toft.dk/mat/3d-print/spiraler.htm>
- Handler om at udvide **helix**-begrebet, så man tegner **spiraler** på andet end en cylinder!

MODELLERING AF SPIRALER

Cylindrisk helix

$$r_1(t) := R_z(t) \cdot \langle r, 0, c \cdot t \rangle :$$

$$r_1(t) = \begin{bmatrix} \cos(t) r \\ \sin(t) r \\ c t \end{bmatrix}$$



$$r_{cylinder}(u, v) := R_z(v) \cdot \langle r, 0, c \cdot u \rangle :$$

$$r_{cylinder}(u, v) = \begin{bmatrix} \cos(v) r \\ \sin(v) r \\ c u \end{bmatrix}$$

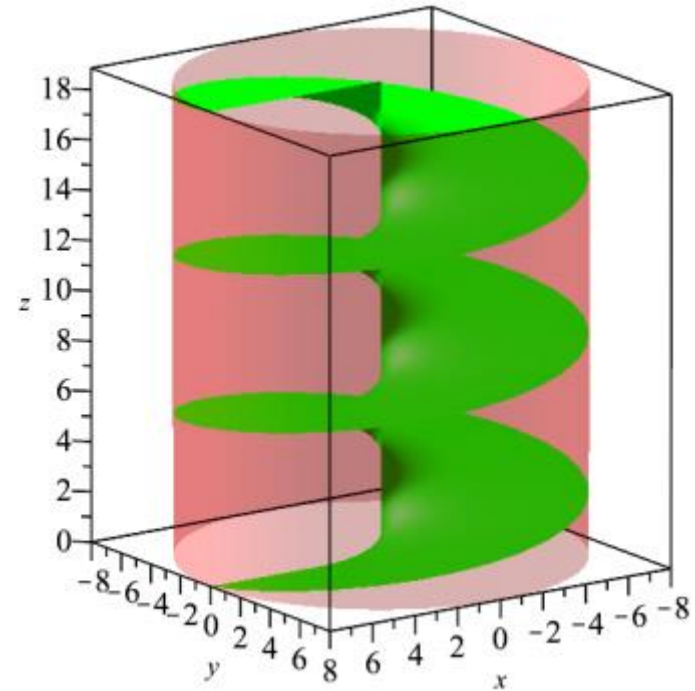
R_z er
rotationsmatricen
om z-aksen

$$R_z(\theta) := \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Cylindrisk helicoide

$$r_2(u, v) := R_z(v) \cdot \langle u \cdot r, 0, c \cdot v \rangle :$$

$$r_2(u, v) = \begin{bmatrix} \cos(v) u r \\ \sin(v) u r \\ c v \end{bmatrix}$$

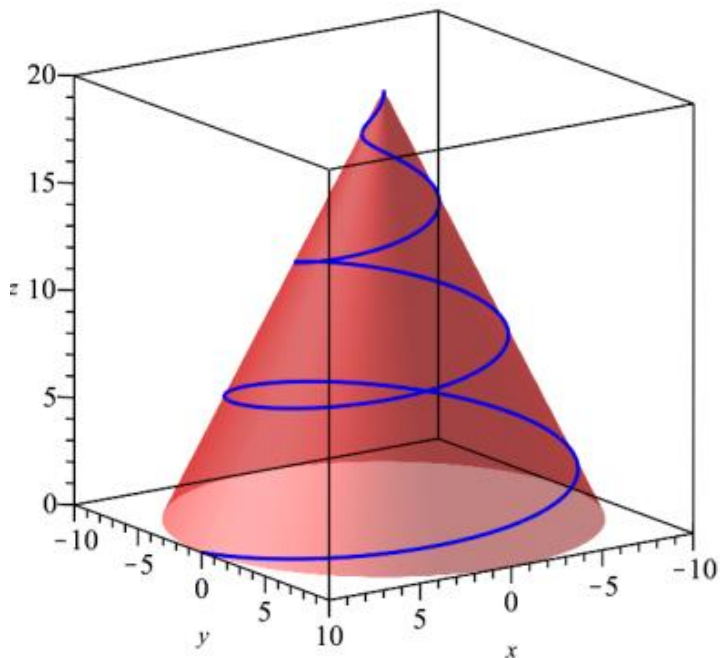


MODELLERING AF SPIRALER

Konisk helix

$$r_3(t) := R_z(t) \cdot \langle (\beta + \alpha \cdot t), 0, c \cdot t \rangle :$$

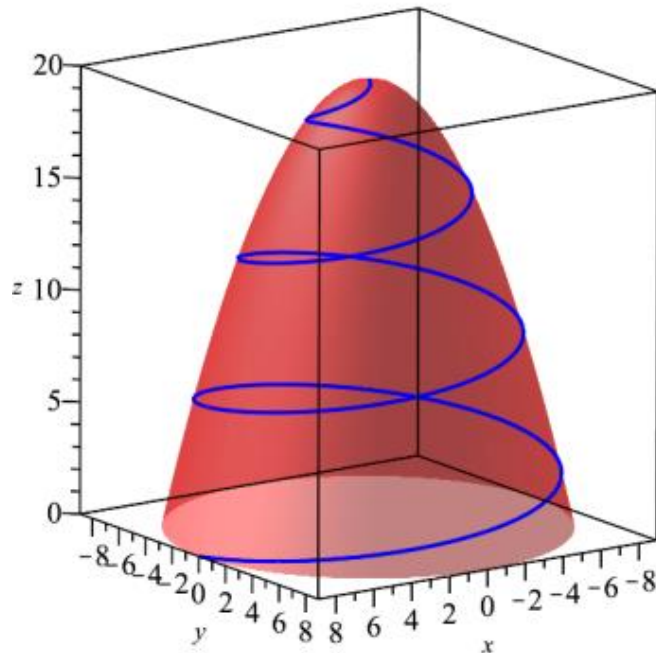
$$r_3(t) = \begin{bmatrix} \cos(t) (\alpha t + \beta) \\ \sin(t) (\alpha t + \beta) \\ c t \end{bmatrix}$$



Parabolsk helix

$$r_5(t) := R_z(t) \cdot \langle \alpha \cdot \sqrt{\beta - t}, 0, c \cdot t \rangle :$$

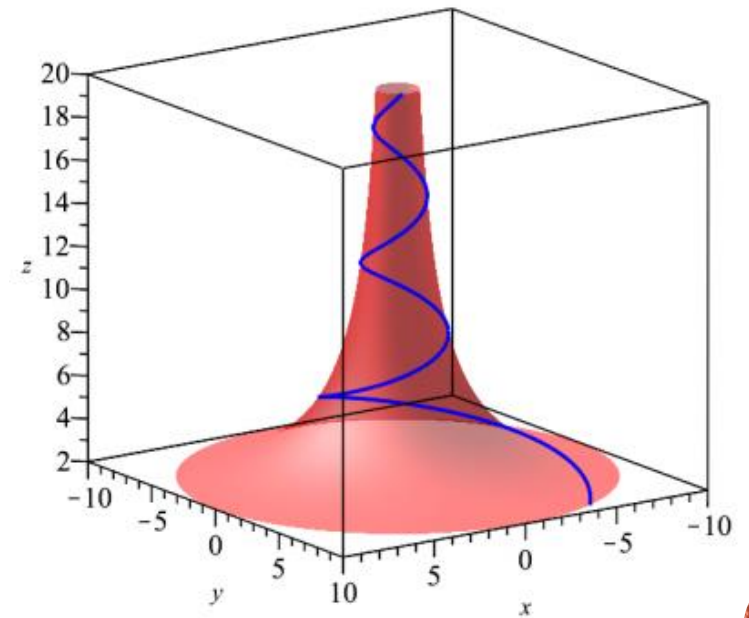
$$r_5(t) = \begin{bmatrix} \cos(t) \alpha \sqrt{\beta - t} \\ \sin(t) \alpha \sqrt{\beta - t} \\ c t \end{bmatrix}$$



Hyperbolsk helix

$$r_7(t) := R_z(t) \cdot \langle \frac{\alpha}{t}, 0, c \cdot t \rangle :$$

$$r_7(t) = \begin{bmatrix} \frac{\cos(t) \alpha}{t} \\ \frac{\sin(t) \alpha}{t} \\ c t \end{bmatrix}$$

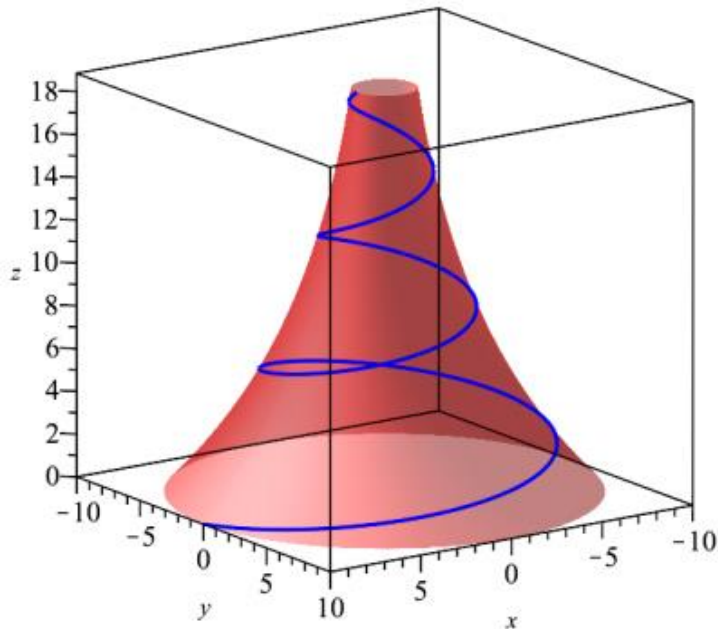


MODELLERING AF SPIRALER

Logaritmisk helix

$$r_9(t) := R_z(t) \cdot \langle \alpha \cdot e^{\beta t}, 0, c \cdot t \rangle :$$

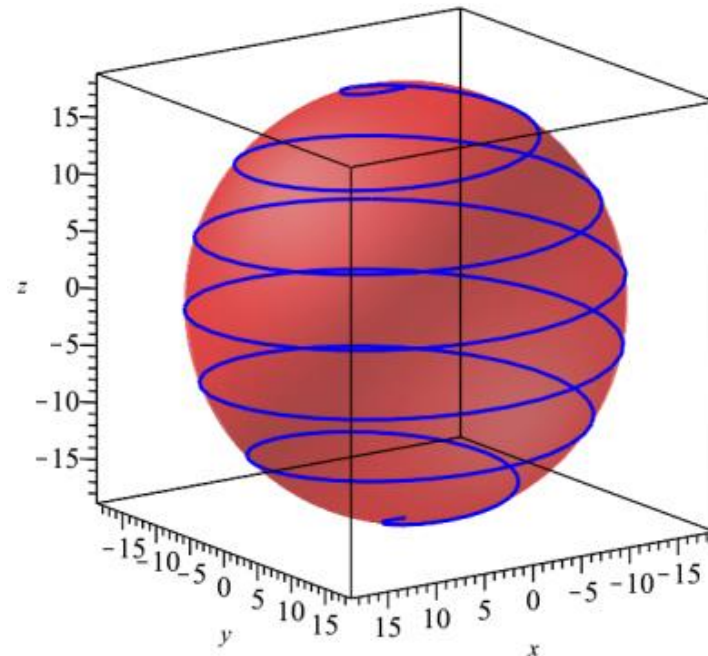
$$r_9(t) = \begin{bmatrix} \cos(t) \alpha e^{\beta t} \\ \sin(t) \alpha e^{\beta t} \\ c t \end{bmatrix}$$



Sfærisk helix

$$r_{11}(t) := R_z\left(\frac{t}{c}\right) \cdot \langle \sqrt{r^2 - t^2}, 0, t \rangle :$$

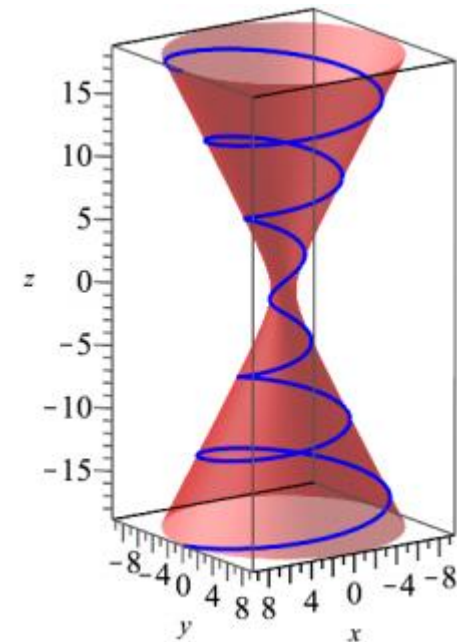
$$r_{11}(t) = \begin{bmatrix} \cos\left(\frac{t}{c}\right) \sqrt{r^2 - t^2} \\ \sin\left(\frac{t}{c}\right) \sqrt{r^2 - t^2} \\ t \end{bmatrix}$$



Hyperbolsk helix

$$r_{13}(t) := R_z\left(\frac{t}{c}\right) \cdot \left\langle p \cdot \sqrt{\frac{t^2}{q^2} + 1}, 0, t \right\rangle :$$

$$r_{13}(t) = \begin{bmatrix} \cos\left(\frac{t}{c}\right) p \sqrt{\frac{t^2}{q^2} + 1} \\ \sin\left(\frac{t}{c}\right) p \sqrt{\frac{t^2}{q^2} + 1} \\ t \end{bmatrix}$$

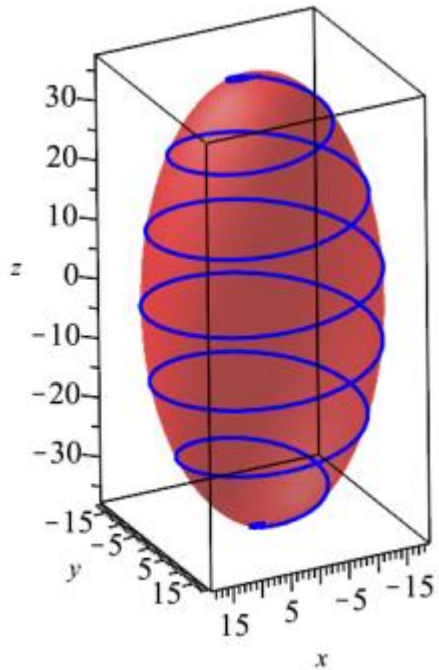


MODELLERING AF SPIRALER

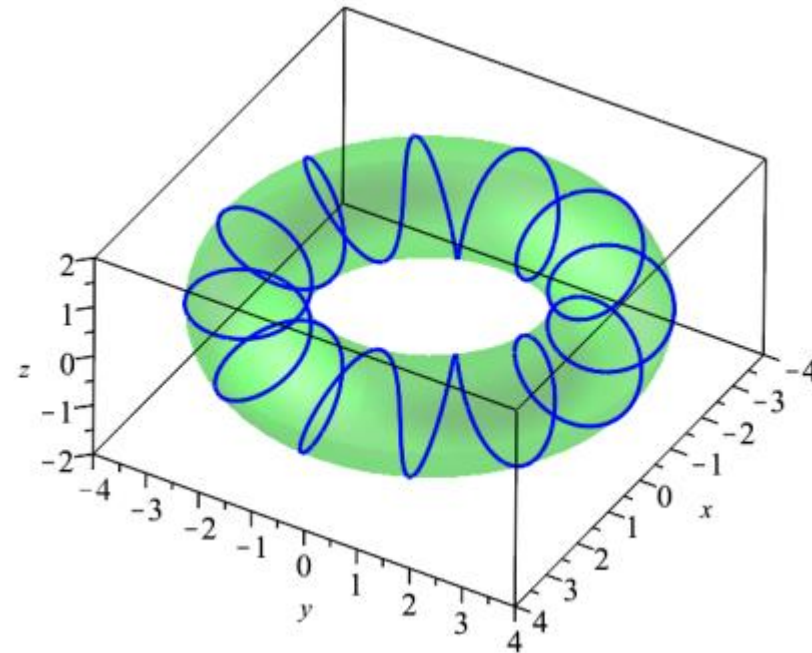
Ellipsoidisk helix

$$r_{17}(t) := R_z\left(\frac{t}{c}\right) \cdot \langle \sqrt{r^2 - t^2}, 0, a \cdot t \rangle :$$

$$r_{17}(t) = \begin{bmatrix} \cos\left(\frac{t}{c}\right) \sqrt{r^2 - t^2} \\ \sin\left(\frac{t}{c}\right) \sqrt{r^2 - t^2} \\ a t \end{bmatrix}$$



Helix på en torus



MODELLERING AF ARMBÅND

- Se mit website: <https://steen-toft.dk/mat/3d-print/armbaand.htm>
- Samlet fil med beregninger og plots (28 sider):
<https://steen-toft.dk/mat/3d-print/armbaand/armbaand.pdf>
 - Parametrisering af cirkel, ellipse og asteroide
 - Anvendelse af rotationsmatricer om akserne
 - Vridning (tvist) og variation af tykkelsen
- Lagring som STL-fil, så armbåndet kan 3D-printes

MODELLERING AF ARMBÅND

