

EP-valg (beregning på antal mandater pr. land)

restart

with(ArrayTools) : with(plots) :

▼ EP-data

Henter tallene i regnearket og konverterer tallen til en liste.

▼ Befolkninger

```
Befolkninger := [83203320, 67842582, 59607184, 47432805, 37654247, 19038098, 17734036, 11631136, 10603810, 10545457, 10440000, 10352042, 9689010, 8967500, 6838937, 5864667, 5541241, 5434712, 5060004, 3862305, 2805998, 2107180, 1875757, 1331796, 904700, 643648, 520971]
```

Befolkninger := convert(% , list) :

Befolkninger =

```
[83203320, 67842582, 59607184, 47432805, 37654247, 19038098, 17734036, 11631136, 10603810, 10545457, 10440000, 10352042, 9689010, 8967500, 6838937, 5864667, 5541241, 5434712, 5060004, 3862305, 2805998, 2107180, 1875757, 1331796, 904700, 643648, 520971]
```

NumElems(Befolkninger) = 27

Mandater

```
Mandater := [ 96, 79, 76, 61, 52, 33, 31, 21, 21, 21, 21, 21, 20, 17, 15, 15, 15, 14, 12, 11, 9, 9, 7, 6, 6 ]
```

Mandater := convert(% , list) :

Mandater = [96, 79, 76, 61, 52, 33, 31, 21, 21, 21, 21, 21, 21, 20, 17, 15, 15, 15, 14, 12, 11, 9, 9, 7, 6, 6]

NumElems(Mandater) = 27

add(Mandater[i], i = 1 .. 27) = 716

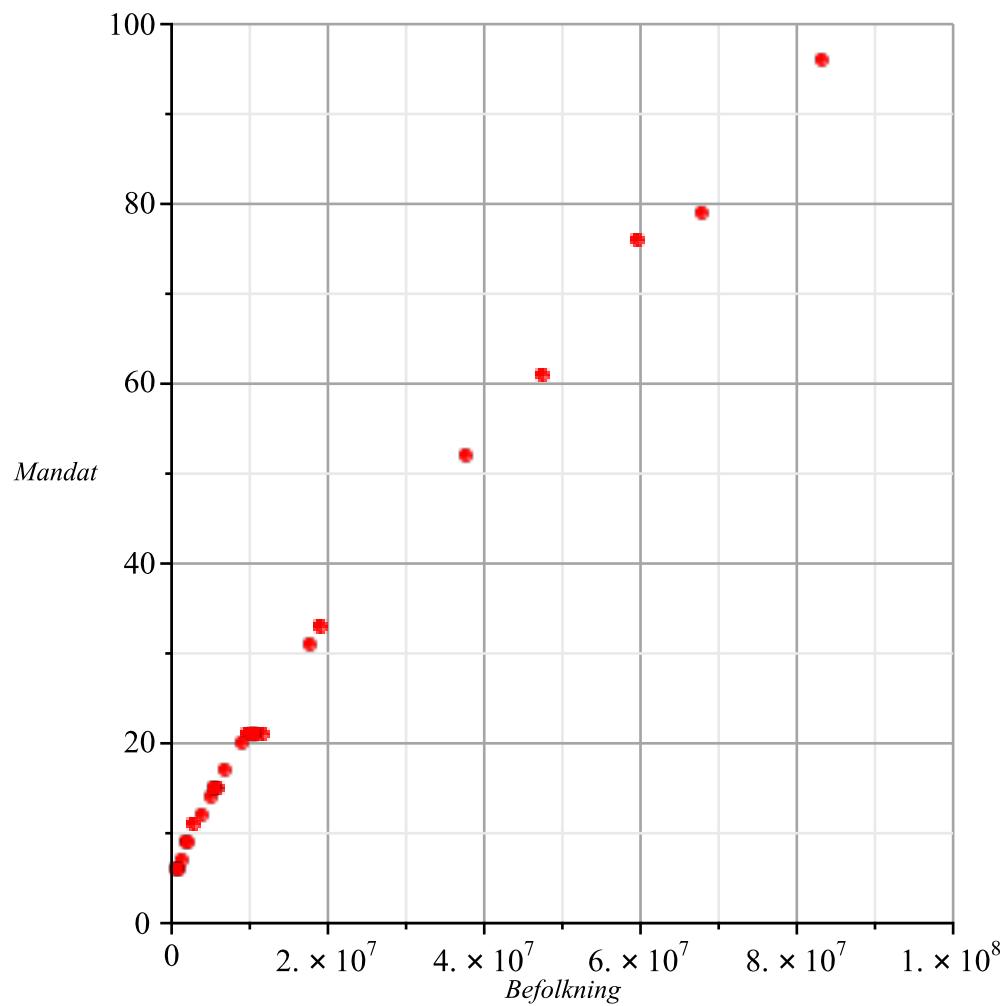
OK: 27 EU-lande, 716 mandater i EU-parlamentet.

Graf

Punkter := [seq([Befolkninger[i], Mandater[i]], i = 1 .. 27)]

*Punkter := [[83203320, 96], [67842582, 79], [59607184, 76], [47432805, 61], [37654247, 52], (1.3.1)
[19038098, 33], [17734036, 31], [11631136, 21], [10603810, 21], [10545457, 21], [10440000,
21], [10352042, 21], [9689010, 21], [8967500, 20], [6838937, 17], [5864667, 15], [5541241,
15], [5434712, 15], [5060004, 14], [3862305, 12], [2805998, 11], [2107180, 9], [1875757, 9],
[1331796, 7], [904700, 6], [643648, 6], [520971, 6]]*

EUP := pointplot(Punkter, gridlines, view = [0 .. 1000000000, 0 .. 100], labels = [Befolkning, Mandat], color = red, symbol = solidcircle, symbolsize = 10)



Dette er grafen, som viser antal mandater ud fra befolkningstallet.

Lagrange polynomiet

Formel

Lagrange polynomiet er af grad 1 mindre end antal punkter, dvs. 26 her. **Det går eksakt igennem alle punkter.**

Prøver med interpolation (Lagranges polynomium):

https://en.wikipedia.org/wiki/Polynomial_interpolation

Consider the Lagrange basis functions given by

$$L_{n,j}(x) = \prod_{k \neq j} \frac{x - x_k}{x_j - x_k}.$$

Notice that $L_{n,j}$ is a polynomial of degree n . Furthermore,

$$p(x) = \sum_{j=0}^n y_j L_{n,j}(x)$$

is an interpolating polynomial of degree n .

https://en.wikipedia.org/wiki/Lagrange_polynomial

$$\begin{aligned}
p(x) &= \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}y_1 + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}y_n \\
&= \sum_{i=0}^n \left(\prod_{\substack{0 \leq j \leq n \\ j \neq i}} \frac{x-x_j}{x_i-x_j} \right) y_i
\end{aligned}$$

NB: Hvis $x=x_0$, så bliver 1. brøk 1. Dvs. 1. led bliver y_0 . De øvrige brøker bliver alle 0, derfor bliver de øvrige led alle 0. Samlede sum bliver så y_0 .

Derfor vil polynomiet opfylde $p(x_0) = y_0$. Tilsvarende for de øvrige x'er.

Interpolationsformlen fra Wikipedia implementeres således i Maple:

NB: listerne med datapunkterne har indeks fra 1 til 27, derfor må summationsvariablene i og j opdateres med 1.

piecewise gør, at når man kommer til at dividere med 0, så erstattes udtrykket med faktoren 1, som er uskyldig!

unassign('i') : unassign('j') :

De 2 lister omdøbes:

$L_x := \text{Befolkninger}$:

$L_y := \text{Mandater}$:

Formlen implementeres:

$$P_{\text{Lagrange}} := \text{unapply} \left(\sum_{i=1}^{27} \left(\left(\prod_{j=1}^{27} \text{piecewise} \left(i=j, 1, i \neq j, \frac{x - L_x[j]}{L_x[i] - L_x[j]} \right) \right) \cdot L_y[i] \right), x \right) :$$

Lagrange-polynomium bliver af 26. grad !!

$P_{\text{Lagrange}}(x)$

$$\begin{aligned}
& 79 \left(-\frac{x}{15360738} + \frac{13867220}{2560123} \right) \left(\frac{x}{8235398} - \frac{29803592}{4117699} \right) \left(\frac{x}{20409777} \right. \\
& \quad \left. - \frac{15810935}{6803259} \right) \left(\frac{x}{30188335} - \frac{37654247}{30188335} \right) \left(\frac{x}{48804484} - \frac{9519049}{24402242} \right) \left(\frac{x}{50108546} \right. \\
& \quad \left. - \frac{8867018}{25054273} \right) \left(\frac{x}{56211446} - \frac{5815568}{28105723} \right) \left(\frac{x}{57238772} - \frac{5301905}{28619386} \right) \left(\frac{x}{57297125} \right. \\
& \quad \left. - \frac{10545457}{57297125} \right) \left(\frac{x}{57402582} - \frac{1740000}{9567097} \right) \left(\frac{x}{57490540} - \frac{5176021}{28745270} \right) \left(\frac{x}{58153572} \right. \\
& \quad \left. - \frac{1614835}{9692262} \right) \left(\frac{x}{58875082} - \frac{4483750}{29437541} \right) \left(\frac{x}{61003645} - \frac{6838937}{61003645} \right) \left(\frac{x}{61977915} \right. \\
& \quad \left. - \frac{1954889}{20659305} \right) \left(\frac{x}{62301341} - \frac{5541241}{62301341} \right) \left(\frac{x}{62407870} - \frac{2717356}{31203935} \right) \left(\frac{x}{62782578} \right. \\
& \quad \left. - \frac{843334}{10463763} \right) \left(\frac{x}{63980277} - \frac{1287435}{21326759} \right) \left(\frac{x}{65036584} - \frac{1402999}{32518292} \right) \left(\frac{x}{65735402} \right. \\
& \quad \left. - \frac{1053590}{32867701} \right) \left(\frac{x}{65966825} - \frac{1875757}{65966825} \right) \left(\frac{x}{66510786} - \frac{221966}{11085131} \right) \left(\frac{x}{66937882} \right. \\
& \quad \left. - \frac{452350}{33468941} \right) \left(\frac{x}{67198934} - \frac{321824}{33599467} \right) \left(\frac{x}{67321611} - \frac{173657}{22440537} \right) + 76 \left(-\frac{x}{23596136} \right. \\
& \quad \left. + \frac{10400415}{2949517} \right) \left(-\frac{x}{8235398} + \frac{33921291}{4117699} \right) \left(\frac{x}{12174379} - \frac{6776115}{1739197} \right) \left(\frac{x}{21952937} \right)
\end{aligned} \tag{2.1.1}$$

$$\begin{aligned}
& - \frac{37654247}{21952937} \left(\frac{x}{40569086} - \frac{9519049}{20284543} \right) \left(\frac{x}{41873148} - \frac{4433509}{10468287} \right) \left(\frac{x}{47976048} \right. \\
& - \frac{726946}{2998503} \left(\frac{x}{49003374} - \frac{757415}{3500241} \right) \left(\frac{x}{49061727} - \frac{811189}{3773979} \right) \left(\frac{x}{49167184} \right. \\
& - \frac{652500}{3072949} \left(\frac{x}{49255142} - \frac{5176021}{24627571} \right) \left(\frac{x}{49918174} - \frac{4844505}{24959087} \right) \left(\frac{x}{50639684} \right. \\
& - \frac{2241875}{12659921} \left(\frac{x}{52768247} - \frac{976991}{7538321} \right) \left(\frac{x}{53742517} - \frac{5864667}{53742517} \right) \left(\frac{x}{54065943} \right. \\
& - \frac{5541241}{54065943} \left(\frac{x}{54172472} - \frac{679339}{6771559} \right) \left(\frac{x}{54547180} - \frac{1265001}{13636795} \right) \left(\frac{x}{55744879} \right. \\
& - \frac{3862305}{55744879} \left(\frac{x}{56801186} - \frac{107923}{2184661} \right) \left(\frac{x}{57500004} - \frac{526795}{14375001} \right) \left(\frac{x}{57731427} \right. \\
& - \frac{144289}{4440879} \left(\frac{x}{58275388} - \frac{332949}{14568847} \right) \left(\frac{x}{58702484} - \frac{226175}{14675621} \right) \left(\frac{x}{58963536} \right. \\
& - \frac{40228}{3685221} \left(\frac{x}{59086213} - \frac{520971}{59086213} \right) + 61 \left(-\frac{x}{35770515} + \frac{5546888}{2384701} \right) \left(-\frac{x}{20409777} \right. \\
& + \frac{22614194}{6803259} \left(-\frac{x}{12174379} + \frac{8515312}{1739197} \right) \left(\frac{x}{9778558} - \frac{37654247}{9778558} \right) \left(\frac{x}{28394707} \right. \\
& - \frac{19038098}{28394707} \left(\frac{x}{29698769} - \frac{17734036}{29698769} \right) \left(\frac{x}{35801669} - \frac{11631136}{35801669} \right) \left(\frac{x}{36828995} \right. \\
& - \frac{302966}{1052257} \left(\frac{x}{36887348} - \frac{620321}{2169844} \right) \left(\frac{x}{36992805} - \frac{696000}{2466187} \right) \left(\frac{x}{37080763} \right. \\
& - \frac{10352042}{37080763} \left(\frac{x}{37743795} - \frac{645934}{2516253} \right) \left(\frac{x}{38465305} - \frac{105500}{452533} \right) \left(\frac{x}{40593868} \right. \\
& - \frac{976991}{5799124} \left(\frac{x}{41568138} - \frac{1954889}{13856046} \right) \left(\frac{x}{41891564} - \frac{5541241}{41891564} \right) \left(\frac{x}{41998093} \right. \\
& - \frac{5434712}{41998093} \left(\frac{x}{42372801} - \frac{1686668}{14124267} \right) \left(\frac{x}{43570500} - \frac{257487}{2904700} \right) \left(\frac{x}{44626807} \right. \\
& - \frac{2805998}{44626807} \left(\frac{x}{45325625} - \frac{421436}{9065125} \right) \left(\frac{x}{45557048} - \frac{1875757}{45557048} \right) \left(\frac{x}{46101009} \right. \\
& - \frac{443932}{15367003} \left(\frac{x}{46528105} - \frac{180940}{9305621} \right) \left(\frac{x}{46789157} - \frac{643648}{46789157} \right) \left(\frac{x}{46911834} \right. \\
& - \frac{173657}{15637278} \left. \right) + 52 \left(-\frac{x}{45549073} + \frac{83203320}{45549073} \right) \left(-\frac{x}{30188335} + \frac{67842582}{30188335} \right) \left(\right. \\
& - \frac{x}{21952937} + \frac{59607184}{21952937} \left. \right) \left(-\frac{x}{9778558} + \frac{47432805}{9778558} \right) \left(\frac{x}{18616149} \right. \\
& - \frac{19038098}{18616149} \left(\frac{x}{19920211} - \frac{17734036}{19920211} \right) \left(\frac{x}{26023111} - \frac{11631136}{26023111} \right) \left(\frac{x}{27050437} \right. \\
& - \frac{10603810}{27050437} \left(\frac{x}{27108790} - \frac{10545457}{27108790} \right) \left(\frac{x}{27214247} - \frac{10440000}{27214247} \right) \left(\frac{x}{27302205} \right. \\
& - \frac{10352042}{27302205} \left(\frac{x}{27965237} - \frac{9689010}{27965237} \right) \left(\frac{x}{28686747} - \frac{8967500}{28686747} \right) \left(\frac{x}{30815310} \right. \\
& - \frac{6838937}{30815310} \left(\frac{x}{31789580} - \frac{5864667}{31789580} \right) \left(\frac{x}{32113006} - \frac{5541241}{32113006} \right) \left(\frac{x}{32219535} \right. \\
& - \frac{5434712}{32219535} \left(\frac{x}{32594243} - \frac{5060004}{32594243} \right) \left(\frac{x}{33791942} - \frac{3862305}{33791942} \right) \left(\frac{x}{34848249} \right)
\end{aligned}$$

$$\begin{aligned}
& - \frac{2805998}{34848249} \left(\frac{x}{35547067} - \frac{2107180}{35547067} \right) \left(\frac{x}{35778490} - \frac{1875757}{35778490} \right) \left(\frac{x}{36322451} \right. \\
& - \frac{1331796}{36322451} \left(\frac{x}{36749547} - \frac{904700}{36749547} \right) \left(\frac{x}{37010599} - \frac{643648}{37010599} \right) \left(\frac{x}{37133276} \right. \\
& - \frac{520971}{37133276} \Big) + 33 \left(-\frac{x}{64165222} + \frac{41601660}{32082611} \right) \left(-\frac{x}{48804484} + \frac{33921291}{24402242} \right) \Big(\\
& -\frac{x}{40569086} + \frac{29803592}{20284543} \Big) \left(-\frac{x}{28394707} + \frac{47432805}{28394707} \right) \left(-\frac{x}{18616149} \right. \\
& + \frac{37654247}{18616149} \left(\frac{x}{1304062} - \frac{8867018}{652031} \right) \left(\frac{x}{7406962} - \frac{5815568}{3703481} \right) \left(\frac{x}{8434288} \right. \\
& - \frac{5301905}{4217144} \left(\frac{x}{8492641} - \frac{10545457}{8492641} \right) \left(\frac{x}{8598098} - \frac{5220000}{4299049} \right) \left(\frac{x}{8686056} \right. \\
& - \frac{5176021}{4343028} \left(\frac{x}{9349088} - \frac{4844505}{4674544} \right) \left(\frac{x}{10070598} - \frac{4483750}{5035299} \right) \left(\frac{x}{12199161} \right. \\
& - \frac{6838937}{12199161} \left(\frac{x}{13173431} - \frac{5864667}{13173431} \right) \left(\frac{x}{13496857} - \frac{5541241}{13496857} \right) \Big(\frac{x}{13603386} \\
& - \frac{2717356}{6801693} \left(\frac{x}{13978094} - \frac{2530002}{6989047} \right) \left(\frac{x}{15175793} - \frac{3862305}{15175793} \right) \Big(\frac{x}{16232100} \\
& - \frac{1402999}{8116050} \left(\frac{x}{16930918} - \frac{1053590}{8465459} \right) \left(\frac{x}{17162341} - \frac{1875757}{17162341} \right) \Big(\frac{x}{17706302} \\
& - \frac{665898}{8853151} \left(\frac{x}{18133398} - \frac{452350}{9066699} \right) \left(\frac{x}{18394450} - \frac{321824}{9197225} \right) \Big(\frac{x}{18517127} \\
& - \frac{520971}{18517127} \Big) + 31 \left(-\frac{x}{65469284} + \frac{20800830}{16367321} \right) \left(-\frac{x}{50108546} + \frac{33921291}{25054273} \right) \Big(\\
& -\frac{x}{41873148} + \frac{14901796}{10468287} \Big) \left(-\frac{x}{29698769} + \frac{47432805}{29698769} \right) \left(-\frac{x}{19920211} + \frac{37654247}{19920211} \right) \Big(\\
& -\frac{x}{1304062} + \frac{9519049}{652031} \Big) \left(\frac{x}{6102900} - \frac{2907784}{1525725} \right) \left(\frac{x}{7130226} - \frac{5301905}{3565113} \right) \Big(\frac{x}{7188579} \\
& - \frac{10545457}{7188579} \left(\frac{x}{7294036} - \frac{2610000}{1823509} \right) \left(\frac{x}{7381994} - \frac{5176021}{3690997} \right) \Big(\frac{x}{8045026} \\
& - \frac{4844505}{4022513} \left(\frac{x}{8766536} - \frac{2241875}{2191634} \right) \left(\frac{x}{10895099} - \frac{6838937}{10895099} \right) \Big(\frac{x}{11869369} \\
& - \frac{5864667}{11869369} \left(\frac{x}{12192795} - \frac{5541241}{12192795} \right) \left(\frac{x}{12299324} - \frac{1358678}{3074831} \right) \Big(\frac{x}{12674032} \\
& - \frac{1265001}{3168508} \left(\frac{x}{13871731} - \frac{3862305}{13871731} \right) \left(\frac{x}{14928038} - \frac{1402999}{7464019} \right) \Big(\frac{x}{15626856} \\
& - \frac{526795}{3906714} \left(\frac{x}{15858279} - \frac{1875757}{15858279} \right) \left(\frac{x}{16402240} - \frac{332949}{4100560} \right) \Big(\frac{x}{16829336} \\
& - \frac{226175}{4207334} \left(\frac{x}{17090388} - \frac{160912}{4272597} \right) \left(\frac{x}{17213065} - \frac{520971}{17213065} \right) + 96 \left(\frac{x}{15360738} \right. \\
& - \frac{11307097}{2560123} \left(\frac{x}{23596136} - \frac{7450898}{2949517} \right) \left(\frac{x}{35770515} - \frac{3162187}{2384701} \right) \Big(\frac{x}{45549073} \\
& - \frac{37654247}{45549073} \left(\frac{x}{64165222} - \frac{9519049}{32082611} \right) \left(\frac{x}{65469284} - \frac{4433509}{16367321} \right) \Big(\frac{x}{71572184} \\
& - \frac{1453892}{8946523} \left(\frac{x}{72599510} - \frac{1060381}{7259951} \right) \left(\frac{x}{72657863} - \frac{10545457}{72657863} \right) \Big(\frac{x}{72763320}
\end{aligned}$$

$$\begin{aligned}
& - \frac{3000}{20909} \left(\frac{x}{72851278} - \frac{5176021}{36425639} \right) \left(\frac{x}{73514310} - \frac{322967}{2450477} \right) \left(\frac{x}{74235820} \right. \\
& - \frac{448375}{3711791} \left(\frac{x}{76364383} - \frac{6838937}{76364383} \right) \left(\frac{x}{77338653} - \frac{1954889}{25779551} \right) \left(\frac{x}{77662079} \right. \\
& - \frac{5541241}{77662079} \left(\frac{x}{77768608} - \frac{679339}{9721076} \right) \left(\frac{x}{78143316} - \frac{421667}{6511943} \right) \left(\frac{x}{79341015} \right. \\
& - \frac{257487}{5289401} \left(\frac{x}{80397322} - \frac{1402999}{40198661} \right) \left(\frac{x}{81096140} - \frac{105359}{4054807} \right) \left(\frac{x}{81327563} \right. \\
& - \frac{1875757}{81327563} \left(\frac{x}{81871524} - \frac{3827}{235263} \right) \left(\frac{x}{82298620} - \frac{45235}{4114931} \right) \left(\frac{x}{82559672} \right. \\
& - \frac{80456}{10319959} \left(\frac{x}{82682349} - \frac{173657}{27560783} \right) + 21 \left(- \frac{x}{71572184} + \frac{10400415}{8946523} \right) \left(\right. \\
& - \frac{x}{56211446} + \frac{33921291}{28105723} \left(- \frac{x}{47976048} + \frac{3725449}{2998503} \right) \left(- \frac{x}{35801669} + \frac{47432805}{35801669} \right) \left(\right. \\
& - \frac{x}{26023111} + \frac{37654247}{26023111} \left(- \frac{x}{7406962} + \frac{9519049}{3703481} \right) \left(- \frac{x}{6102900} \right. \\
& + \frac{4433509}{1525725} \left(\frac{x}{1027326} - \frac{5301905}{513663} \right) \left(\frac{x}{1085679} - \frac{10545457}{1085679} \right) \left(\frac{x}{1191136} \right. \\
& - \frac{326250}{37223} \left(\frac{x}{1279094} - \frac{5176021}{639547} \right) \left(\frac{x}{1942126} - \frac{4844505}{971063} \right) \left(\frac{x}{2663636} \right. \\
& - \frac{2241875}{665909} \left(\frac{x}{4792199} - \frac{6838937}{4792199} \right) \left(\frac{x}{5766469} - \frac{5864667}{5766469} \right) \left(\frac{x}{6089895} \right. \\
& - \frac{5541241}{6089895} \left(\frac{x}{6196424} - \frac{679339}{774553} \right) \left(\frac{x}{6571132} - \frac{1265001}{1642783} \right) \left(\frac{x}{7768831} \right. \\
& - \frac{3862305}{7768831} \left(\frac{x}{8825138} - \frac{1402999}{4412569} \right) \left(\frac{x}{9523956} - \frac{526795}{2380989} \right) \left(\frac{x}{9755379} \right. \\
& - \frac{1875757}{9755379} \left(\frac{x}{10299340} - \frac{332949}{2574835} \right) \left(\frac{x}{10726436} - \frac{226175}{2681609} \right) \left(\frac{x}{10987488} \right. \\
& - \frac{20114}{343359} \left(\frac{x}{11110165} - \frac{47361}{1010015} \right) + 21 \left(- \frac{x}{72599510} + \frac{8320332}{7259951} \right) \left(- \frac{x}{57238772} \right. \\
& + \frac{33921291}{28619386} \left(- \frac{x}{49003374} + \frac{4257656}{3500241} \right) \left(- \frac{x}{36828995} + \frac{1355223}{1052257} \right) \left(- \frac{x}{27050437} \right. \\
& + \frac{37654247}{27050437} \left(- \frac{x}{8434288} + \frac{9519049}{4217144} \right) \left(- \frac{x}{7130226} + \frac{8867018}{3565113} \right) \left(- \frac{x}{1027326} \right. \\
& + \frac{5815568}{513663} \left(\frac{x}{58353} - \frac{10545457}{58353} \right) \left(\frac{x}{163810} - \frac{1044000}{16381} \right) \left(\frac{x}{251768} \right. \\
& - \frac{5176021}{125884} \left(\frac{x}{914800} - \frac{968901}{91480} \right) \left(\frac{x}{1636310} - \frac{896750}{163631} \right) \left(\frac{x}{3764873} \right. \\
& - \frac{976991}{537839} \left(\frac{x}{4739143} - \frac{5864667}{4739143} \right) \left(\frac{x}{5062569} - \frac{5541241}{5062569} \right) \left(\frac{x}{5169098} \right. \\
& - \frac{2717356}{2584549} \left(\frac{x}{5543806} - \frac{2530002}{2771903} \right) \left(\frac{x}{6741505} - \frac{772461}{1348301} \right) \left(\frac{x}{7797812} \right. \\
& - \frac{1402999}{3898906} \left(\frac{x}{8496630} - \frac{210718}{849663} \right) \left(\frac{x}{8728053} - \frac{1875757}{8728053} \right) \left(\frac{x}{9272014} \right. \\
& - \frac{665898}{4636007} \left(\frac{x}{9699110} - \frac{90470}{969911} \right) \left(\frac{x}{9960162} - \frac{321824}{4980081} \right) \left(\frac{x}{10082839} \right)
\end{aligned}$$

$$\begin{aligned}
& - \frac{520971}{10082839} \Big) + 21 \left(-\frac{x}{72657863} + \frac{83203320}{72657863} \right) \left(-\frac{x}{57297125} + \frac{67842582}{57297125} \right) \Big(\\
& - \frac{x}{49061727} + \frac{4585168}{3773979} \Big) \left(-\frac{x}{36887348} + \frac{2790165}{2169844} \right) \left(-\frac{x}{27108790} + \frac{37654247}{27108790} \right) \Big(\\
& - \frac{x}{8492641} + \frac{19038098}{8492641} \Big) \left(-\frac{x}{7188579} + \frac{17734036}{7188579} \right) \left(-\frac{x}{1085679} + \frac{11631136}{1085679} \right) \Big(\\
& - \frac{x}{58353} + \frac{10603810}{58353} \Big) \left(\frac{x}{105457} - \frac{10440000}{105457} \right) \left(\frac{x}{193415} - \frac{10352042}{193415} \right) \left(\frac{x}{856447} \right. \\
& \left. - \frac{9689010}{856447} \right) \left(\frac{x}{1577957} - \frac{527500}{92821} \right) \left(\frac{x}{3706520} - \frac{6838937}{3706520} \right) \left(\frac{x}{4680790} \right. \\
& \left. - \frac{5864667}{4680790} \right) \left(\frac{x}{5004216} - \frac{5541241}{5004216} \right) \left(\frac{x}{5110745} - \frac{5434712}{5110745} \right) \left(\frac{x}{5485453} \right. \\
& \left. - \frac{5060004}{5485453} \right) \left(\frac{x}{6683152} - \frac{3862305}{6683152} \right) \left(\frac{x}{7739459} - \frac{215846}{595343} \right) \left(\frac{x}{8438277} \right. \\
& \left. - \frac{2107180}{8438277} \right) \left(\frac{x}{8669700} - \frac{144289}{666900} \right) \left(\frac{x}{9213661} - \frac{1331796}{9213661} \right) \left(\frac{x}{9640757} \right. \\
& \left. - \frac{904700}{9640757} \right) \left(\frac{x}{9901809} - \frac{643648}{9901809} \right) \left(\frac{x}{10024486} - \frac{520971}{10024486} \right) + 21 \left(-\frac{x}{72763320} \right. \\
& \left. + \frac{23909}{20909} \right) \left(-\frac{x}{57402582} + \frac{11307097}{9567097} \right) \left(-\frac{x}{49167184} + \frac{3725449}{3072949} \right) \left(-\frac{x}{36992805} \right. \\
& \left. + \frac{3162187}{2466187} \right) \left(-\frac{x}{27214247} + \frac{37654247}{27214247} \right) \left(-\frac{x}{8598098} + \frac{9519049}{4299049} \right) \left(-\frac{x}{7294036} \right. \\
& \left. + \frac{4433509}{1823509} \right) \left(-\frac{x}{1191136} + \frac{363473}{37223} \right) \left(-\frac{x}{163810} + \frac{1060381}{16381} \right) \left(-\frac{x}{105457} \right. \\
& \left. + \frac{10545457}{105457} \right) \left(\frac{x}{87958} - \frac{5176021}{43979} \right) \left(\frac{x}{750990} - \frac{322967}{25033} \right) \left(\frac{x}{1472500} \right. \\
& \left. - \frac{3587}{589} \right) \left(\frac{x}{3601063} - \frac{6838937}{3601063} \right) \left(\frac{x}{4575333} - \frac{1954889}{1525111} \right) \left(\frac{x}{4898759} \right. \\
& \left. - \frac{5541241}{4898759} \right) \left(\frac{x}{5005288} - \frac{679339}{625661} \right) \left(\frac{x}{5379996} - \frac{421667}{448333} \right) \left(\frac{x}{6577695} \right. \\
& \left. - \frac{85829}{146171} \right) \left(\frac{x}{7634002} - \frac{1402999}{3817001} \right) \left(\frac{x}{8332820} - \frac{105359}{416641} \right) \left(\frac{x}{8564243} \right. \\
& \left. - \frac{1875757}{8564243} \right) \left(\frac{x}{9108204} - \frac{3827}{26173} \right) \left(\frac{x}{9535300} - \frac{9047}{95353} \right) \left(\frac{x}{9796352} \right. \\
& \left. - \frac{10057}{153068} \right) \left(\frac{x}{9919029} - \frac{173657}{3306343} \right) + 21 \left(-\frac{x}{72851278} + \frac{41601660}{36425639} \right) \left(-\frac{x}{57490540} \right. \\
& \left. + \frac{33921291}{28745270} \right) \left(-\frac{x}{49255142} + \frac{29803592}{24627571} \right) \left(-\frac{x}{37080763} + \frac{47432805}{37080763} \right) \left(-\frac{x}{27302205} \right. \\
& \left. + \frac{37654247}{27302205} \right) \left(-\frac{x}{8686056} + \frac{9519049}{4343028} \right) \left(-\frac{x}{7381994} + \frac{8867018}{3690997} \right) \left(-\frac{x}{1279094} \right. \\
& \left. + \frac{5815568}{639547} \right) \left(-\frac{x}{251768} + \frac{5301905}{125884} \right) \left(-\frac{x}{193415} + \frac{10545457}{193415} \right) \left(-\frac{x}{87958} \right. \\
& \left. + \frac{5220000}{43979} \right) \left(\frac{x}{663032} - \frac{4844505}{331516} \right) \left(\frac{x}{1384542} - \frac{4483750}{692271} \right) \left(\frac{x}{3513105} \right. \\
& \left. - \frac{6838937}{3513105} \right) \left(\frac{x}{4487375} - \frac{5864667}{4487375} \right) \left(\frac{x}{4810801} - \frac{5541241}{4810801} \right) \left(\frac{x}{4917330} \right)
\end{aligned}$$

$$\begin{aligned}
& - \frac{2717356}{2458665} \left(\frac{x}{5292038} - \frac{2530002}{2646019} \right) \left(\frac{x}{6489737} - \frac{3862305}{6489737} \right) \left(\frac{x}{7546044} \right. \\
& - \frac{1402999}{3773022} \left(\frac{x}{8244862} - \frac{1053590}{4122431} \right) \left(\frac{x}{8476285} - \frac{1875757}{8476285} \right) \left(\frac{x}{9020246} \right. \\
& - \frac{665898}{4510123} \left(\frac{x}{9447342} - \frac{452350}{4723671} \right) \left(\frac{x}{9708394} - \frac{321824}{4854197} \right) \left(\frac{x}{9831071} - \frac{520971}{9831071} \right) \\
& + 21 \left(-\frac{x}{73514310} + \frac{2773444}{2450477} \right) \left(-\frac{x}{58153572} + \frac{11307097}{9692262} \right) \left(-\frac{x}{49918174} \right. \\
& + \frac{29803592}{24959087} \left(-\frac{x}{37743795} + \frac{3162187}{2516253} \right) \left(-\frac{x}{27965237} + \frac{37654247}{27965237} \right) \left(-\frac{x}{9349088} \right. \\
& + \frac{9519049}{4674544} \left(-\frac{x}{8045026} + \frac{8867018}{4022513} \right) \left(-\frac{x}{1942126} + \frac{5815568}{971063} \right) \left(-\frac{x}{914800} \right. \\
& + \frac{1060381}{91480} \left(-\frac{x}{856447} + \frac{10545457}{856447} \right) \left(-\frac{x}{750990} + \frac{348000}{25033} \right) \left(-\frac{x}{663032} \right. \\
& + \frac{5176021}{331516} \left(\frac{x}{721510} - \frac{896750}{72151} \right) \left(\frac{x}{2850073} - \frac{6838937}{2850073} \right) \left(\frac{x}{3824343} \right. \\
& - \frac{1954889}{1274781} \left(\frac{x}{4147769} - \frac{5541241}{4147769} \right) \left(\frac{x}{4254298} - \frac{2717356}{2127149} \right) \left(\frac{x}{4629006} \right. \\
& - \frac{843334}{771501} \left(\frac{x}{5826705} - \frac{257487}{388447} \right) \left(\frac{x}{6883012} - \frac{1402999}{3441506} \right) \left(\frac{x}{7581830} \right. \\
& - \frac{210718}{758183} \left(\frac{x}{7813253} - \frac{1875757}{7813253} \right) \left(\frac{x}{8357214} - \frac{221966}{1392869} \right) \left(\frac{x}{8784310} \right. \\
& - \frac{830}{8059} \left(\frac{x}{9045362} - \frac{321824}{4522681} \right) \left(\frac{x}{9168039} - \frac{173657}{3056013} \right) + 20 \left(-\frac{x}{74235820} \right. \\
& + \frac{4160166}{3711791} \left(-\frac{x}{58875082} + \frac{33921291}{29437541} \right) \left(-\frac{x}{50639684} + \frac{14901796}{12659921} \right) \left(-\frac{x}{38465305} \right. \\
& + \frac{558033}{452533} \left(-\frac{x}{28686747} + \frac{37654247}{28686747} \right) \left(-\frac{x}{10070598} + \frac{9519049}{5035299} \right) \left(-\frac{x}{8766536} \right. \\
& + \frac{4433509}{2191634} \left(-\frac{x}{2663636} + \frac{2907784}{665909} \right) \left(-\frac{x}{1636310} + \frac{1060381}{163631} \right) \left(-\frac{x}{1577957} \right. \\
& + \frac{620321}{92821} \left(-\frac{x}{1472500} + \frac{4176}{589} \right) \left(-\frac{x}{1384542} + \frac{5176021}{692271} \right) \left(-\frac{x}{721510} \right. \\
& + \frac{968901}{72151} \left(\frac{x}{2128563} - \frac{6838937}{2128563} \right) \left(\frac{x}{3102833} - \frac{5864667}{3102833} \right) \left(\frac{x}{3426259} \right. \\
& - \frac{5541241}{3426259} \left(\frac{x}{3532788} - \frac{1358678}{883197} \right) \left(\frac{x}{3907496} - \frac{1265001}{976874} \right) \left(\frac{x}{5105195} \right. \\
& - \frac{772461}{1021039} \left(\frac{x}{6161502} - \frac{1402999}{3080751} \right) \left(\frac{x}{6860320} - \frac{105359}{343016} \right) \left(\frac{x}{7091743} \right. \\
& - \frac{1875757}{7091743} \left(\frac{x}{7635704} - \frac{332949}{1908926} \right) \left(\frac{x}{8062800} - \frac{9047}{80628} \right) \left(\frac{x}{8323852} \right. \\
& - \frac{160912}{2080963} \left(\frac{x}{8446529} - \frac{520971}{8446529} \right) + 17 \left(-\frac{x}{76364383} + \frac{83203320}{76364383} \right) \left(-\frac{x}{61003645} \right. \\
& + \frac{67842582}{61003645} \left(-\frac{x}{52768247} + \frac{8515312}{7538321} \right) \left(-\frac{x}{40593868} + \frac{6776115}{5799124} \right) \left(-\frac{x}{30815310} \right. \\
& + \frac{37654247}{30815310} \left(-\frac{x}{12199161} + \frac{19038098}{12199161} \right) \left(-\frac{x}{10895099} + \frac{17734036}{10895099} \right) \left(-\frac{x}{4792199} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{11631136}{4792199} \left(-\frac{x}{3764873} + \frac{1514830}{537839} \right) \left(-\frac{x}{3706520} + \frac{10545457}{3706520} \right) \left(-\frac{x}{3601063} \right. \\
& + \frac{10440000}{3601063} \left(-\frac{x}{3513105} + \frac{10352042}{3513105} \right) \left(-\frac{x}{2850073} + \frac{9689010}{2850073} \right) \left(-\frac{x}{2128563} \right. \\
& + \frac{8967500}{2128563} \left(\frac{x}{974270} - \frac{5864667}{974270} \right) \left(\frac{x}{1297696} - \frac{5541241}{1297696} \right) \left(\frac{x}{1404225} \right. \\
& - \frac{5434712}{1404225} \left(\frac{x}{1778933} - \frac{5060004}{1778933} \right) \left(\frac{x}{2976632} - \frac{3862305}{2976632} \right) \left(\frac{x}{4032939} \right. \\
& - \frac{2805998}{4032939} \left(\frac{x}{4731757} - \frac{2107180}{4731757} \right) \left(\frac{x}{4963180} - \frac{1875757}{4963180} \right) \left(\frac{x}{5507141} \right. \\
& - \frac{1331796}{5507141} \left(\frac{x}{5934237} - \frac{904700}{5934237} \right) \left(\frac{x}{6195289} - \frac{643648}{6195289} \right) \left(\frac{x}{6317966} - \frac{520971}{6317966} \right) \\
& + 15 \left(-\frac{x}{77338653} + \frac{27734440}{25779551} \right) \left(-\frac{x}{61977915} + \frac{22614194}{20659305} \right) \left(-\frac{x}{53742517} \right. \\
& + \frac{59607184}{53742517} \left(-\frac{x}{41568138} + \frac{15810935}{13856046} \right) \left(-\frac{x}{31789580} + \frac{37654247}{31789580} \right) \left(-\frac{x}{13173431} \right. \\
& + \frac{19038098}{13173431} \left(-\frac{x}{11869369} + \frac{17734036}{11869369} \right) \left(-\frac{x}{5766469} + \frac{11631136}{5766469} \right) \left(-\frac{x}{4739143} \right. \\
& + \frac{10603810}{4739143} \left(-\frac{x}{4680790} + \frac{10545457}{4680790} \right) \left(-\frac{x}{4575333} + \frac{3480000}{1525111} \right) \left(-\frac{x}{4487375} \right. \\
& + \frac{10352042}{4487375} \left(-\frac{x}{3824343} + \frac{3229670}{1274781} \right) \left(-\frac{x}{3102833} + \frac{8967500}{3102833} \right) \left(-\frac{x}{974270} \right. \\
& + \frac{6838937}{974270} \left(\frac{x}{323426} - \frac{5541241}{323426} \right) \left(\frac{x}{429955} - \frac{5434712}{429955} \right) \left(\frac{x}{804663} \right. \\
& - \frac{1686668}{268221} \left(\frac{x}{2002362} - \frac{1287435}{667454} \right) \left(\frac{x}{3058669} - \frac{2805998}{3058669} \right) \left(\frac{x}{3757487} \right. \\
& - \frac{2107180}{3757487} \left(\frac{x}{3988910} - \frac{1875757}{3988910} \right) \left(\frac{x}{4532871} - \frac{443932}{1510957} \right) \left(\frac{x}{4959967} \right. \\
& - \frac{904700}{4959967} \left(\frac{x}{5221019} - \frac{643648}{5221019} \right) \left(\frac{x}{5343696} - \frac{173657}{1781232} \right) + 15 \left(-\frac{x}{77662079} \right. \\
& + \frac{83203320}{77662079} \left(-\frac{x}{62301341} + \frac{67842582}{62301341} \right) \left(-\frac{x}{54065943} + \frac{59607184}{54065943} \right) \left(-\frac{x}{41891564} \right. \\
& + \frac{47432805}{41891564} \left(-\frac{x}{32113006} + \frac{37654247}{32113006} \right) \left(-\frac{x}{13496857} + \frac{19038098}{13496857} \right) \left(-\frac{x}{12192795} \right. \\
& + \frac{17734036}{12192795} \left(-\frac{x}{6089895} + \frac{11631136}{6089895} \right) \left(-\frac{x}{5062569} + \frac{10603810}{5062569} \right) \left(-\frac{x}{5004216} \right. \\
& + \frac{10545457}{5004216} \left(-\frac{x}{4898759} + \frac{10440000}{4898759} \right) \left(-\frac{x}{4810801} + \frac{10352042}{4810801} \right) \left(-\frac{x}{4147769} \right. \\
& + \frac{9689010}{4147769} \left(-\frac{x}{3426259} + \frac{8967500}{3426259} \right) \left(-\frac{x}{1297696} + \frac{6838937}{1297696} \right) \left(-\frac{x}{323426} \right. \\
& + \frac{5864667}{323426} \left(\frac{x}{106529} - \frac{5434712}{106529} \right) \left(\frac{x}{481237} - \frac{5060004}{481237} \right) \left(\frac{x}{1678936} \right. \\
& - \frac{3862305}{1678936} \left(\frac{x}{2735243} - \frac{2805998}{2735243} \right) \left(\frac{x}{3434061} - \frac{2107180}{3434061} \right) \left(\frac{x}{3665484} \right. \\
& - \frac{1875757}{3665484} \left(\frac{x}{4209445} - \frac{1331796}{4209445} \right) \left(\frac{x}{4636541} - \frac{904700}{4636541} \right) \left(\frac{x}{4897593} \right)
\end{aligned}$$

$$\begin{aligned}
& - \frac{643648}{4897593} \left(\frac{x}{5020270} - \frac{520971}{5020270} \right) + 15 \left(-\frac{x}{77768608} + \frac{10400415}{9721076} \right) \left(-\frac{x}{62407870} \right. \\
& + \frac{33921291}{31203935} \left(-\frac{x}{54172472} + \frac{7450898}{6771559} \right) \left(-\frac{x}{41998093} + \frac{47432805}{41998093} \right) \left(-\frac{x}{32219535} \right. \\
& + \frac{37654247}{32219535} \left(-\frac{x}{13603386} + \frac{9519049}{6801693} \right) \left(-\frac{x}{12299324} + \frac{4433509}{3074831} \right) \left(-\frac{x}{6196424} \right. \\
& + \frac{1453892}{774553} \left(-\frac{x}{5169098} + \frac{5301905}{2584549} \right) \left(-\frac{x}{5110745} + \frac{10545457}{5110745} \right) \left(-\frac{x}{5005288} \right. \\
& + \frac{1305000}{625661} \left(-\frac{x}{4917330} + \frac{5176021}{2458665} \right) \left(-\frac{x}{4254298} + \frac{4844505}{2127149} \right) \left(-\frac{x}{3532788} \right. \\
& + \frac{2241875}{883197} \left(-\frac{x}{1404225} + \frac{6838937}{1404225} \right) \left(-\frac{x}{429955} + \frac{5864667}{429955} \right) \left(-\frac{x}{106529} \right. \\
& + \frac{5541241}{106529} \left(\frac{x}{374708} - \frac{1265001}{93677} \right) \left(\frac{x}{1572407} - \frac{3862305}{1572407} \right) \left(\frac{x}{2628714} \right. \\
& - \frac{1402999}{1314357} \left(\frac{x}{3327532} - \frac{526795}{831883} \right) \left(\frac{x}{3558955} - \frac{1875757}{3558955} \right) \left(\frac{x}{4102916} \right. \\
& - \frac{332949}{1025729} \left(\frac{x}{4530012} - \frac{226175}{1132503} \right) \left(\frac{x}{4791064} - \frac{80456}{598883} \right) \left(\frac{x}{4913741} - \frac{520971}{4913741} \right) \\
& + 14 \left(-\frac{x}{78143316} + \frac{6933610}{6511943} \right) \left(-\frac{x}{62782578} + \frac{11307097}{10463763} \right) \left(-\frac{x}{54547180} \right. \\
& + \frac{14901796}{13636795} \left(-\frac{x}{42372801} + \frac{15810935}{14124267} \right) \left(-\frac{x}{32594243} + \frac{37654247}{32594243} \right) \left(-\frac{x}{13978094} \right. \\
& + \frac{9519049}{6989047} \left(-\frac{x}{12674032} + \frac{4433509}{3168508} \right) \left(-\frac{x}{6571132} + \frac{2907784}{1642783} \right) \left(-\frac{x}{5543806} \right. \\
& + \frac{5301905}{2771903} \left(-\frac{x}{5485453} + \frac{10545457}{5485453} \right) \left(-\frac{x}{5379996} + \frac{870000}{448333} \right) \left(-\frac{x}{5292038} \right. \\
& + \frac{5176021}{2646019} \left(-\frac{x}{4629006} + \frac{1614835}{771501} \right) \left(-\frac{x}{3907496} + \frac{2241875}{976874} \right) \left(-\frac{x}{1778933} \right. \\
& + \frac{6838937}{1778933} \left(-\frac{x}{804663} + \frac{1954889}{268221} \right) \left(-\frac{x}{481237} + \frac{5541241}{481237} \right) \left(-\frac{x}{374708} \right. \\
& + \frac{1358678}{93677} \left(\frac{x}{1197699} - \frac{1287435}{399233} \right) \left(\frac{x}{2254006} - \frac{1402999}{1127003} \right) \left(\frac{x}{2952824} \right. \\
& - \frac{526795}{738206} \left(\frac{x}{3184247} - \frac{1875757}{3184247} \right) \left(\frac{x}{3728208} - \frac{110983}{310684} \right) \left(\frac{x}{4155304} \right. \\
& - \frac{226175}{1038826} \left(\frac{x}{4416356} - \frac{160912}{1104089} \right) \left(\frac{x}{4539033} - \frac{173657}{1513011} \right) + 12 \left(-\frac{x}{79341015} \right. \\
& + \frac{5546888}{5289401} \left(-\frac{x}{63980277} + \frac{22614194}{21326759} \right) \left(-\frac{x}{55744879} + \frac{59607184}{55744879} \right) \left(-\frac{x}{43570500} \right. \\
& + \frac{3162187}{2904700} \left(-\frac{x}{33791942} + \frac{37654247}{33791942} \right) \left(-\frac{x}{15175793} + \frac{19038098}{15175793} \right) \left(-\frac{x}{13871731} \right. \\
& + \frac{17734036}{13871731} \left(-\frac{x}{7768831} + \frac{11631136}{7768831} \right) \left(-\frac{x}{6741505} + \frac{2120762}{1348301} \right) \left(-\frac{x}{6683152} \right. \\
& + \frac{10545457}{6683152} \left(-\frac{x}{6577695} + \frac{232000}{146171} \right) \left(-\frac{x}{6489737} + \frac{10352042}{6489737} \right) \left(-\frac{x}{5826705} \right. \\
& + \frac{645934}{388447} \left(-\frac{x}{5105195} + \frac{1793500}{1021039} \right) \left(-\frac{x}{2976632} + \frac{6838937}{2976632} \right) \left(-\frac{x}{2002362} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1954889}{667454} \left(-\frac{x}{1678936} + \frac{5541241}{1678936} \right) \left(-\frac{x}{1572407} + \frac{5434712}{1572407} \right) \left(-\frac{x}{1197699} \right. \\
& + \frac{1686668}{399233} \left(\frac{x}{1056307} - \frac{2805998}{1056307} \right) \left(\frac{x}{1755125} - \frac{421436}{351025} \right) \left(\frac{x}{1986548} \right. \\
& - \frac{1875757}{1986548} \left(\frac{x}{2530509} - \frac{443932}{843503} \right) \left(\frac{x}{2957605} - \frac{180940}{591521} \right) \left(\frac{x}{3218657} \right. \\
& - \frac{643648}{3218657} \left(\frac{x}{3341334} - \frac{173657}{1113778} \right) + 11 \left(-\frac{x}{80397322} + \frac{41601660}{40198661} \right) \left(-\frac{x}{65036584} \right. \\
& + \frac{33921291}{32518292} \left(-\frac{x}{56801186} + \frac{2292584}{2184661} \right) \left(-\frac{x}{44626807} + \frac{47432805}{44626807} \right) \left(-\frac{x}{34848249} \right. \\
& + \frac{37654247}{34848249} \left(-\frac{x}{16232100} + \frac{9519049}{8116050} \right) \left(-\frac{x}{14928038} + \frac{8867018}{7464019} \right) \left(-\frac{x}{8825138} \right. \\
& + \frac{5815568}{4412569} \left(-\frac{x}{7797812} + \frac{5301905}{3898906} \right) \left(-\frac{x}{7739459} + \frac{811189}{595343} \right) \left(-\frac{x}{7634002} \right. \\
& + \frac{5220000}{3817001} \left(-\frac{x}{7546044} + \frac{5176021}{3773022} \right) \left(-\frac{x}{6883012} + \frac{4844505}{3441506} \right) \left(-\frac{x}{6161502} \right. \\
& + \frac{4483750}{3080751} \left(-\frac{x}{4032939} + \frac{6838937}{4032939} \right) \left(-\frac{x}{3058669} + \frac{5864667}{3058669} \right) \left(-\frac{x}{2735243} \right. \\
& + \frac{5541241}{2735243} \left(-\frac{x}{2628714} + \frac{2717356}{1314357} \right) \left(-\frac{x}{2254006} + \frac{2530002}{1127003} \right) \left(-\frac{x}{1056307} \right. \\
& + \frac{3862305}{1056307} \left(\frac{x}{698818} - \frac{1053590}{349409} \right) \left(\frac{x}{930241} - \frac{144289}{71557} \right) \left(\frac{x}{1474202} \right. \\
& - \frac{665898}{737101} \left(\frac{x}{1901298} - \frac{452350}{950649} \right) \left(\frac{x}{2162350} - \frac{321824}{1081175} \right) \left(\frac{x}{2285027} - \frac{520971}{2285027} \right) \\
& + 9 \left(-\frac{x}{81096140} + \frac{4160166}{4054807} \right) \left(-\frac{x}{65735402} + \frac{33921291}{32867701} \right) \left(-\frac{x}{57500004} \right. \\
& + \frac{14901796}{14375001} \left(-\frac{x}{45325625} + \frac{9486561}{9065125} \right) \left(-\frac{x}{35547067} + \frac{37654247}{35547067} \right) \left(-\frac{x}{16930918} \right. \\
& + \frac{9519049}{8465459} \left(-\frac{x}{15626856} + \frac{4433509}{3906714} \right) \left(-\frac{x}{9523956} + \frac{2907784}{2380989} \right) \left(-\frac{x}{8496630} \right. \\
& + \frac{1060381}{849663} \left(-\frac{x}{8438277} + \frac{10545457}{8438277} \right) \left(-\frac{x}{8332820} + \frac{522000}{416641} \right) \left(-\frac{x}{8244862} \right. \\
& + \frac{5176021}{4122431} \left(-\frac{x}{7581830} + \frac{968901}{758183} \right) \left(-\frac{x}{6860320} + \frac{448375}{343016} \right) \left(-\frac{x}{4731757} \right. \\
& + \frac{6838937}{4731757} \left(-\frac{x}{3757487} + \frac{5864667}{3757487} \right) \left(-\frac{x}{3434061} + \frac{5541241}{3434061} \right) \left(-\frac{x}{3327532} \right. \\
& + \frac{1358678}{831883} \left(-\frac{x}{2952824} + \frac{1265001}{738206} \right) \left(-\frac{x}{1755125} + \frac{772461}{351025} \right) \left(-\frac{x}{698818} \right. \\
& + \frac{1402999}{349409} \left(\frac{x}{231423} - \frac{1875757}{231423} \right) \left(\frac{x}{775384} - \frac{332949}{193846} \right) \left(\frac{x}{1202480} \right. \\
& - \frac{45235}{60124} \left(\frac{x}{1463532} - \frac{160912}{365883} \right) \left(\frac{x}{1586209} - \frac{520971}{1586209} \right) + 9 \left(-\frac{x}{81327563} \right. \\
& + \frac{83203320}{81327563} \left(-\frac{x}{65966825} + \frac{67842582}{65966825} \right) \left(-\frac{x}{57731427} + \frac{4585168}{4440879} \right) \left(-\frac{x}{45557048} \right. \\
& + \frac{47432805}{45557048} \left(-\frac{x}{35778490} + \frac{37654247}{35778490} \right) \left(-\frac{x}{17162341} + \frac{19038098}{17162341} \right) \left(-\frac{x}{15858279} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{17734036}{15858279} \left(-\frac{x}{9755379} + \frac{11631136}{9755379} \right) \left(-\frac{x}{8728053} + \frac{10603810}{8728053} \right) \left(-\frac{x}{8669700} \right. \\
& + \frac{811189}{666900} \left. \right) \left(-\frac{x}{8564243} + \frac{10440000}{8564243} \right) \left(-\frac{x}{8476285} + \frac{10352042}{8476285} \right) \left(-\frac{x}{7813253} \right. \\
& + \frac{9689010}{7813253} \left. \right) \left(-\frac{x}{7091743} + \frac{8967500}{7091743} \right) \left(-\frac{x}{4963180} + \frac{6838937}{4963180} \right) \left(-\frac{x}{3988910} \right. \\
& + \frac{5864667}{3988910} \left. \right) \left(-\frac{x}{3665484} + \frac{5541241}{3665484} \right) \left(-\frac{x}{3558955} + \frac{5434712}{3558955} \right) \left(-\frac{x}{3184247} \right. \\
& + \frac{5060004}{3184247} \left. \right) \left(-\frac{x}{1986548} + \frac{3862305}{1986548} \right) \left(-\frac{x}{930241} + \frac{215846}{71557} \right) \left(-\frac{x}{231423} \right. \\
& + \frac{2107180}{231423} \left. \right) \left(\frac{x}{543961} - \frac{1331796}{543961} \right) \left(\frac{x}{971057} - \frac{904700}{971057} \right) \left(\frac{x}{1232109} \right. \\
& - \frac{643648}{1232109} \left. \right) \left(\frac{x}{1354786} - \frac{520971}{1354786} \right) + 7 \left(-\frac{x}{81871524} + \frac{239090}{235263} \right) \left(-\frac{x}{66510786} \right. \\
& + \frac{11307097}{11085131} \left. \right) \left(-\frac{x}{58275388} + \frac{14901796}{14568847} \right) \left(-\frac{x}{46101009} + \frac{15810935}{15367003} \right) \left(-\frac{x}{36322451} \right. \\
& + \frac{37654247}{36322451} \left. \right) \left(-\frac{x}{17706302} + \frac{9519049}{8853151} \right) \left(-\frac{x}{16402240} + \frac{4433509}{4100560} \right) \left(-\frac{x}{10299340} \right. \\
& + \frac{2907784}{2574835} \left. \right) \left(-\frac{x}{9272014} + \frac{5301905}{4636007} \right) \left(-\frac{x}{9213661} + \frac{10545457}{9213661} \right) \left(-\frac{x}{9108204} \right. \\
& + \frac{30000}{26173} \left. \right) \left(-\frac{x}{9020246} + \frac{5176021}{4510123} \right) \left(-\frac{x}{8357214} + \frac{1614835}{1392869} \right) \left(-\frac{x}{7635704} \right. \\
& + \frac{2241875}{1908926} \left. \right) \left(-\frac{x}{5507141} + \frac{6838937}{5507141} \right) \left(-\frac{x}{4532871} + \frac{1954889}{1510957} \right) \left(-\frac{x}{4209445} \right. \\
& + \frac{5541241}{4209445} \left. \right) \left(-\frac{x}{4102916} + \frac{1358678}{1025729} \right) \left(-\frac{x}{3728208} + \frac{421667}{310684} \right) \left(-\frac{x}{2530509} \right. \\
& + \frac{1287435}{843503} \left. \right) \left(-\frac{x}{1474202} + \frac{1402999}{737101} \right) \left(-\frac{x}{775384} + \frac{526795}{193846} \right) \left(-\frac{x}{543961} \right. \\
& + \frac{1875757}{543961} \left. \right) \left(\frac{x}{427096} - \frac{226175}{106774} \right) \left(\frac{x}{688148} - \frac{1808}{1933} \right) \left(\frac{x}{810825} - \frac{173657}{270275} \right) + 6 \left(\right. \\
& - \frac{x}{82298620} + \frac{4160166}{4114931} \left. \right) \left(-\frac{x}{66937882} + \frac{33921291}{33468941} \right) \left(-\frac{x}{58702484} + \frac{14901796}{14675621} \right) \left(\right. \\
& - \frac{x}{46528105} + \frac{9486561}{9305621} \left. \right) \left(-\frac{x}{36749547} + \frac{37654247}{36749547} \right) \left(-\frac{x}{18133398} + \frac{9519049}{9066699} \right) \left(\right. \\
& - \frac{x}{16829336} + \frac{4433509}{4207334} \left. \right) \left(-\frac{x}{10726436} + \frac{2907784}{2681609} \right) \left(-\frac{x}{9699110} + \frac{1060381}{969911} \right) \left(\right. \\
& - \frac{x}{9640757} + \frac{10545457}{9640757} \left. \right) \left(-\frac{x}{9535300} + \frac{104400}{95353} \right) \left(-\frac{x}{9447342} + \frac{5176021}{4723671} \right) \left(\right. \\
& - \frac{x}{8784310} + \frac{8889}{8059} \left. \right) \left(-\frac{x}{8062800} + \frac{89675}{80628} \right) \left(-\frac{x}{5934237} + \frac{6838937}{5934237} \right) \left(-\frac{x}{4959967} \right. \\
& + \frac{5864667}{4959967} \left. \right) \left(-\frac{x}{4636541} + \frac{5541241}{4636541} \right) \left(-\frac{x}{4530012} + \frac{1358678}{1132503} \right) \left(-\frac{x}{4155304} \right. \\
& + \frac{1265001}{1038826} \left. \right) \left(-\frac{x}{2957605} + \frac{772461}{591521} \right) \left(-\frac{x}{1901298} + \frac{1402999}{950649} \right) \left(-\frac{x}{1202480} \right. \\
& + \frac{105359}{60124} \left. \right) \left(-\frac{x}{971057} + \frac{1875757}{971057} \right) \left(-\frac{x}{427096} + \frac{332949}{106774} \right) \left(\frac{x}{261052} \right)
\end{aligned}$$

$$\begin{aligned}
& - \frac{160912}{65263} \left(\frac{x}{383729} - \frac{520971}{383729} \right) + 6 \left(-\frac{x}{82559672} + \frac{10400415}{10319959} \right) \left(-\frac{x}{67198934} \right. \\
& + \frac{33921291}{33599467} \left(-\frac{x}{58963536} + \frac{3725449}{3685221} \right) \left(-\frac{x}{46789157} + \frac{47432805}{46789157} \right) \left(-\frac{x}{37010599} \right. \\
& + \frac{37654247}{37010599} \left(-\frac{x}{18394450} + \frac{9519049}{9197225} \right) \left(-\frac{x}{17090388} + \frac{4433509}{4272597} \right) \left(-\frac{x}{10987488} \right. \\
& + \frac{363473}{343359} \left(-\frac{x}{9960162} + \frac{5301905}{4980081} \right) \left(-\frac{x}{9901809} + \frac{10545457}{9901809} \right) \left(-\frac{x}{9796352} \right. \\
& + \frac{163125}{153068} \left(-\frac{x}{9708394} + \frac{5176021}{4854197} \right) \left(-\frac{x}{9045362} + \frac{4844505}{4522681} \right) \left(-\frac{x}{8323852} \right. \\
& + \frac{2241875}{2080963} \left(-\frac{x}{6195289} + \frac{6838937}{6195289} \right) \left(-\frac{x}{5221019} + \frac{5864667}{5221019} \right) \left(-\frac{x}{4897593} \right. \\
& + \frac{5541241}{4897593} \left(-\frac{x}{4791064} + \frac{679339}{598883} \right) \left(-\frac{x}{4416356} + \frac{1265001}{1104089} \right) \left(-\frac{x}{3218657} \right. \\
& + \frac{3862305}{3218657} \left(-\frac{x}{2162350} + \frac{1402999}{1081175} \right) \left(-\frac{x}{1463532} + \frac{526795}{365883} \right) \left(-\frac{x}{1232109} \right. \\
& + \frac{1875757}{1232109} \left(-\frac{x}{688148} + \frac{3741}{1933} \right) \left(-\frac{x}{261052} + \frac{226175}{65263} \right) \left(\frac{x}{122677} - \frac{520971}{122677} \right) \\
& + 6 \left(-\frac{x}{82682349} + \frac{27734440}{27560783} \right) \left(-\frac{x}{67321611} + \frac{22614194}{22440537} \right) \left(-\frac{x}{59086213} \right. \\
& + \frac{59607184}{59086213} \left(-\frac{x}{46911834} + \frac{15810935}{15637278} \right) \left(-\frac{x}{37133276} + \frac{37654247}{37133276} \right) \left(-\frac{x}{18517127} \right. \\
& + \frac{19038098}{18517127} \left(-\frac{x}{17213065} + \frac{17734036}{17213065} \right) \left(-\frac{x}{11110165} + \frac{1057376}{1010015} \right) \left(-\frac{x}{10082839} \right. \\
& + \frac{10603810}{10082839} \left(-\frac{x}{10024486} + \frac{10545457}{10024486} \right) \left(-\frac{x}{9919029} + \frac{3480000}{3306343} \right) \left(-\frac{x}{9831071} \right. \\
& + \frac{10352042}{9831071} \left(-\frac{x}{9168039} + \frac{3229670}{3056013} \right) \left(-\frac{x}{8446529} + \frac{8967500}{8446529} \right) \left(-\frac{x}{6317966} \right. \\
& + \frac{6838937}{6317966} \left(-\frac{x}{5343696} + \frac{1954889}{1781232} \right) \left(-\frac{x}{5020270} + \frac{5541241}{5020270} \right) \left(-\frac{x}{4913741} \right. \\
& + \frac{5434712}{4913741} \left(-\frac{x}{4539033} + \frac{1686668}{1513011} \right) \left(-\frac{x}{3341334} + \frac{1287435}{1113778} \right) \left(-\frac{x}{2285027} \right. \\
& + \frac{2805998}{2285027} \left(-\frac{x}{1586209} + \frac{2107180}{1586209} \right) \left(-\frac{x}{1354786} + \frac{1875757}{1354786} \right) \left(-\frac{x}{810825} \right. \\
& + \frac{443932}{270275} \left(-\frac{x}{383729} + \frac{904700}{383729} \right) \left(-\frac{x}{122677} + \frac{643648}{122677} \right)
\end{aligned}$$

Gang ud, reducer udtryk, tilnærm og sortér efter graden:

$$\begin{aligned}
& \text{sort}(\text{evalf}(\text{simplify}(\text{expand}(\%))), x) = \\
& -4.992699730 \times 10^{-176} x^{26} + 2.208324201 \times 10^{-167} x^{25} - 4.375813657 \times 10^{-159} x^{24} + 5.166827289 \\
& \times 10^{-151} x^{23} - 4.083350825 \times 10^{-143} x^{22} + 2.302019817 \times 10^{-135} x^{21} - 9.640940814 \times 10^{-128} x^{20} \\
& + 3.085061437 \times 10^{-120} x^{19} - 7.698446427 \times 10^{-113} x^{18} + 1.520793050 \times 10^{-105} x^{17} - 2.404601358 \\
& \times 10^{-98} x^{16} + 3.066842135 \times 10^{-91} x^{15} - 3.170977386 \times 10^{-84} x^{14} + 2.664833342 \times 10^{-77} x^{13} \\
& - 1.820805548 \times 10^{-70} x^{12} + 1.009630038 \times 10^{-63} x^{11} - 4.524646357 \times 10^{-57} x^{10} + 1.628224423 \\
& \times 10^{-50} x^9 - 4.662041480 \times 10^{-44} x^8 + 1.049129845 \times 10^{-37} x^7 - 1.825673300 \times 10^{-31} x^6 \\
& + 2.404409570 \times 10^{-25} x^5 - 2.327364225 \times 10^{-19} x^4 + 1.587729211 \times 10^{-13} x^3 - 7.150242704 \\
& \times 10^{-8} x^2 + 0.01888780900 x - 2190.345173
\end{aligned}$$

Der er altså tale om et 26. grads polynomium!

Tjek

```
[seq(P_Lagrange(L_x[i]), i=1..27)] ==  
[96, 79, 76, 61, 52, 33, 31, 21, 21, 21, 21, 21, 21, 20, 17, 15, 15, 15, 15, 14, 12, 11, 9, 9, 7, 6, 6, 6]  
% - ~L_y = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
```

Tallene stemmer. mandat værdierne af Lagrange polynomiet i de 27 befolkningstal er identisk med de givne mandattal.

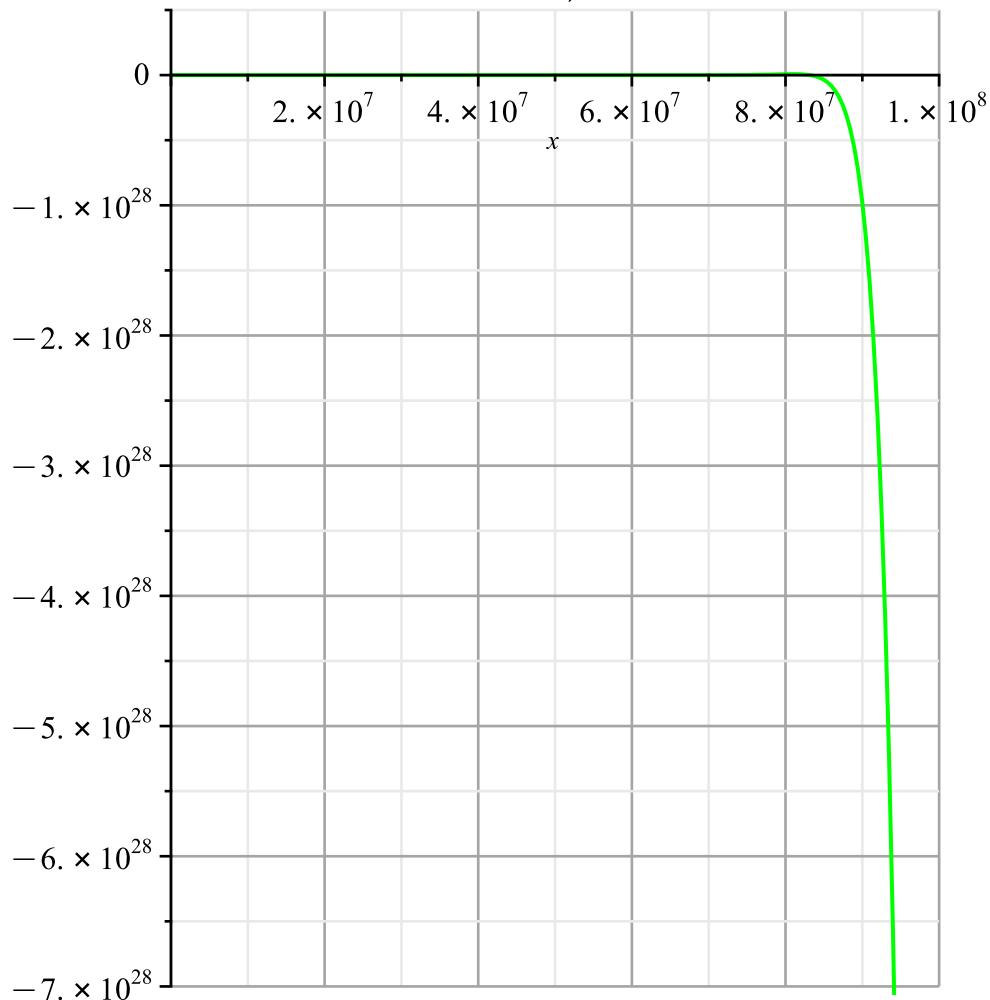
Fejlen er givet ved:

$$ERR_{\text{Lagrange}} := \left(\sum_{i=1}^{27} |P_{\text{Lagrange}}(L_x[i]) - L_y[i]|^2 \right)^{\frac{1}{2}} = 0$$

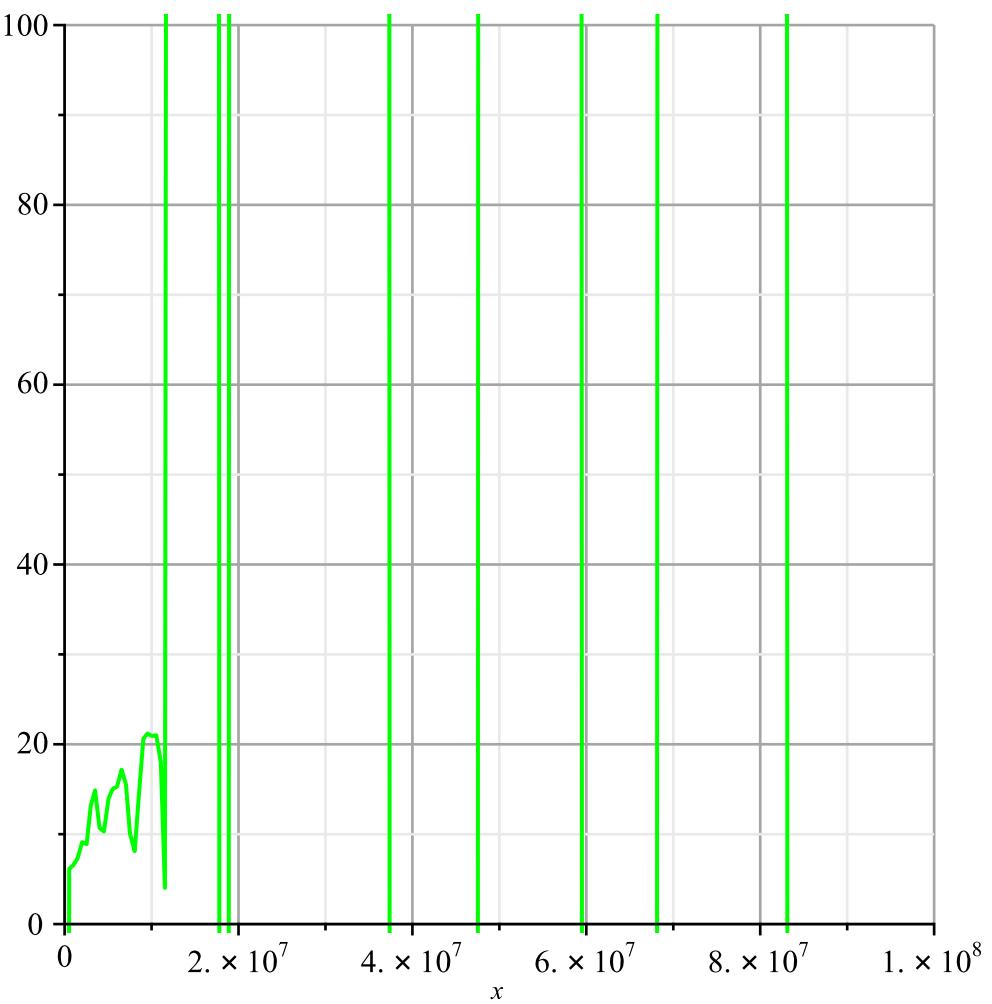
Den samlede fejl er 0, da Lagrange polynomiet går eksakt gennem alle punkter.

Plots

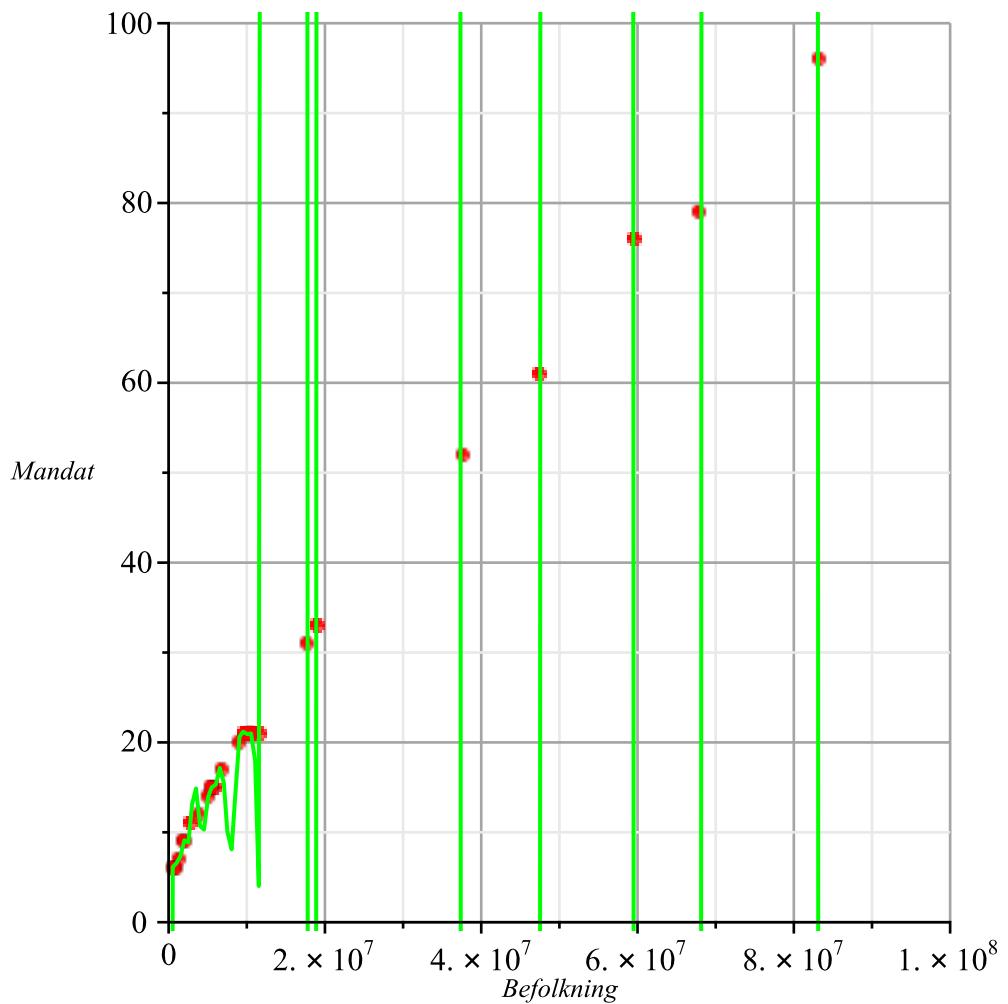
```
plot(P_Lagrange(x), x=0..100000000, gridlines, color=green)
```



```
LAG := plot(P_Lagrange(x), x=0..100000000, view=[0..100000000, 0..100], gridlines, color=green)
```



display(EUP, LAG)



Voldsomme udsving af Lagrange polynomiets graf!

Tilnærmet polynomium

I stedet for at bestemme et polynomium, som går eksakt gennem alle punkterne, kan man tilnærme punkterne med polynomier.

with(Statistics) :

Formler

F.eks. det bedste 8. grads polynomium som tilpasses dataene:

$\text{PolynomialFit}(8, L_x, L_y, x)$

Warning, model is not of full rank

$$3.74397881695117 \times 10^{-99} x + 2.20413132156157 \times 10^{-91} x^2 + 1.28086187015735 \times 10^{-83} x^3 \\ + 7.23751965778802 \times 10^{-76} x^4 + 3.87630264827828 \times 10^{-68} x^5 + 1.86323131836200 \\ \times 10^{-60} x^6 + 6.77469102413642 \times 10^{-53} x^7 - 7.73508678132896 \times 10^{-61} x^8 \quad (3.1.1)$$

Dette gøres for polynomier af alle grader fra 1 til 26:

for j **from** 1 **to** 26 **do**

$\text{Pol}[j] := \text{unapply}(\text{PolynomialFit}(j, L_x, L_y, x, \text{svdtolerance} = 10^{-20}), x)$:

end do:

Warning, model is not of full rank

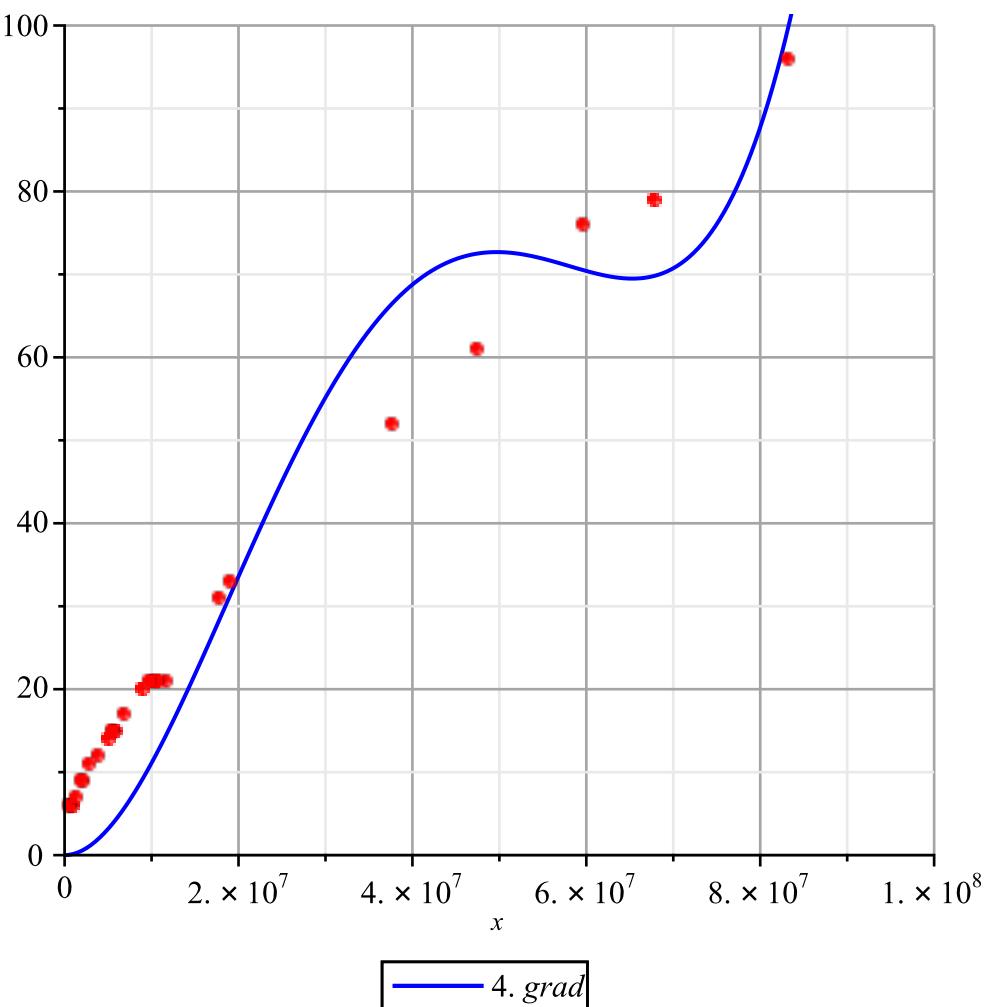
Fejlen beregnes:

$$\text{ERR} := \left[\text{seq}\left(\left(\sum_{i=1}^{26} |\text{Pol}[j](L_x[i]) - L_y[i]|^2 \right)^{\frac{1}{2}}, j = 1..26 \right) \right] =$$

[10.49125957, 6.396771385, 14.20620217, 46.30098247, 69.43042148, 78.66681992, 83.53648662, 87.50794303, 91.42652795, 95.24564061, 98.74070836, 101.7717357, 104.3097390, 106.3922685, 108.0826406, 109.4472513, 110.5460301, 111.4297047, 112.1399673, 112.7106331, 132.7878384, 133.4062804, 133.9492395, 134.4259167, 134.8443949, 135.2117767]

Grafer

```
POL[4] := plot(Pol[4](x), x=0..100000000, view=[0..100000000, 0..100], gridlines, color=blue, legend  
= [4. grad]) :  
display(POL[4], EUP)
```

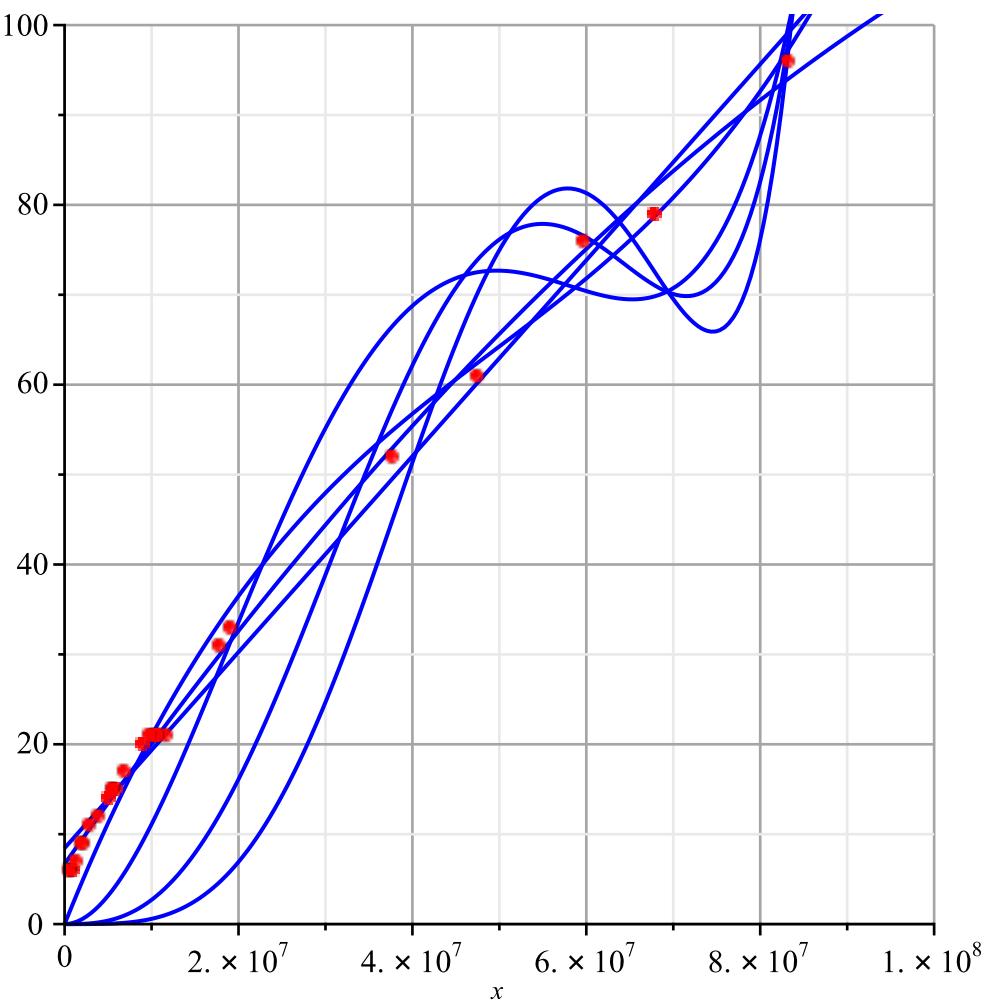


for j **from** 1 **to** 26 **do**

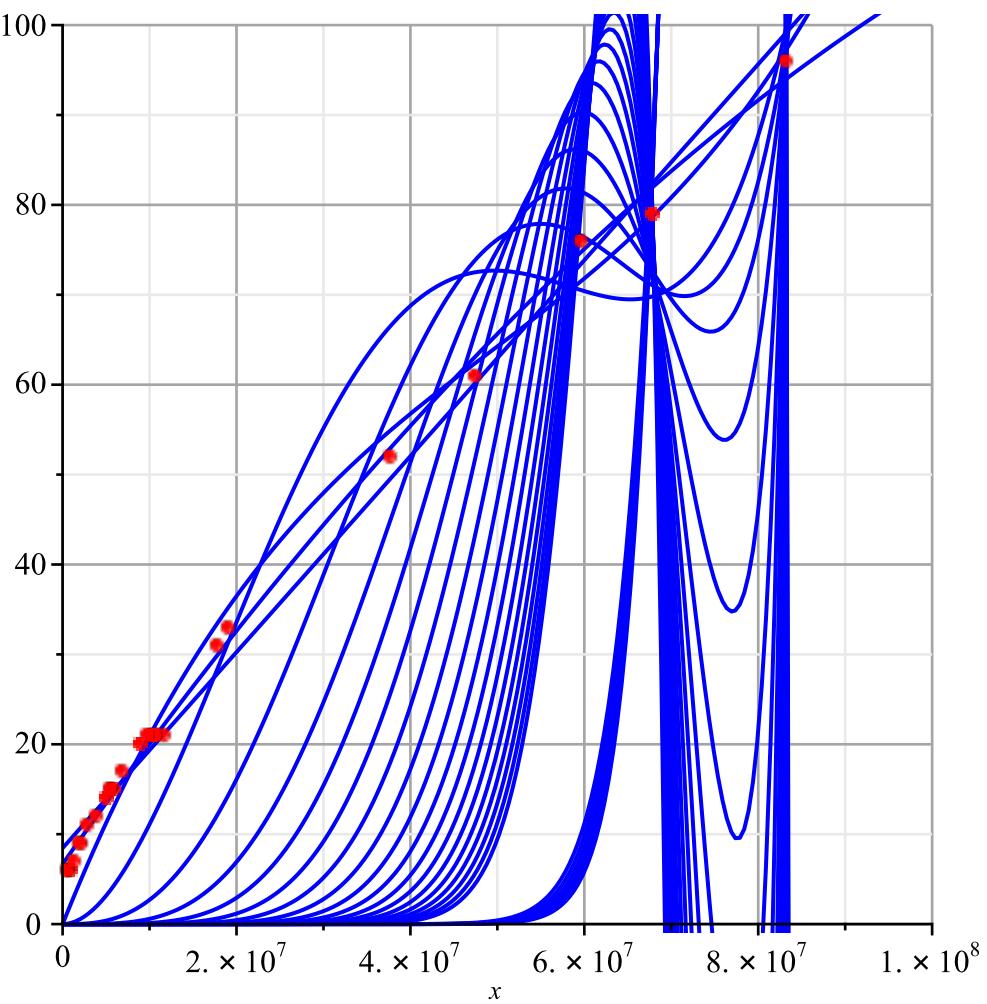
```
POL[j] := plot(Pol[j](x), x=0..100000000, view=[0..100000000, 0..100], gridlines, color=blue) :  
end do:
```

De 6 første polynomier plottet i samme koordinatsystem:

```
display(seq(POL[j], j=1..6), EUP)
```



Alle 26 grafer plottet i samme koordinatsystem:
display(seq(POL[j],j = 1 ..26), EUP)



Rammer altså langt ved siden af!

Algoritmen i *PolynomialFit* er ikke god!

Polynomiel tilnærmelse med Gym-pakken

with(Gym) :

for j **from** 1 **to** 26 **do**

$Pol[j] := unapply(PolyReg(L_x, L_y, j, x), x)$:

end do:

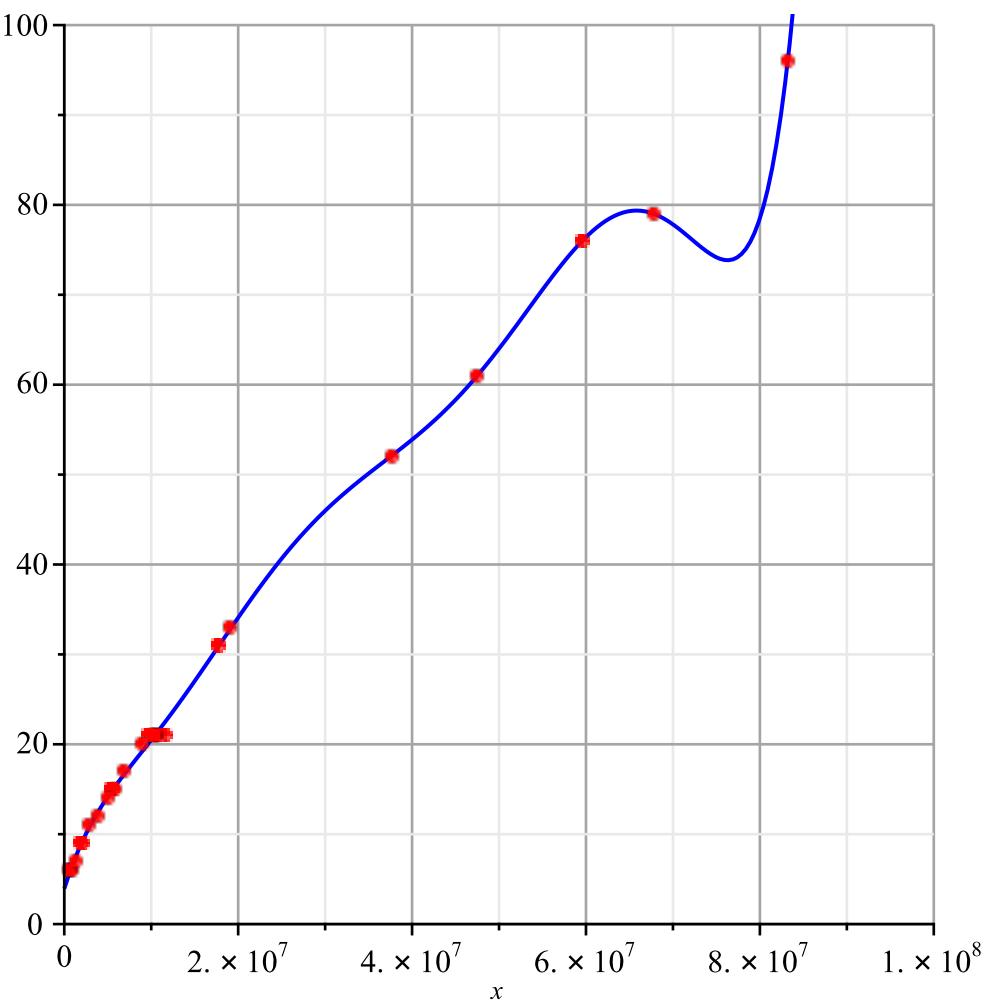
for j **from** 1 **to** 26 **do**

$POL[j] := plot(Pol[j](x), x = 0 .. 1000000000, view = [0 .. 1000000000, 0 .. 100], gridlines, color = blue)$:

end do:

8. grads polynomiet som tilnærmelse:

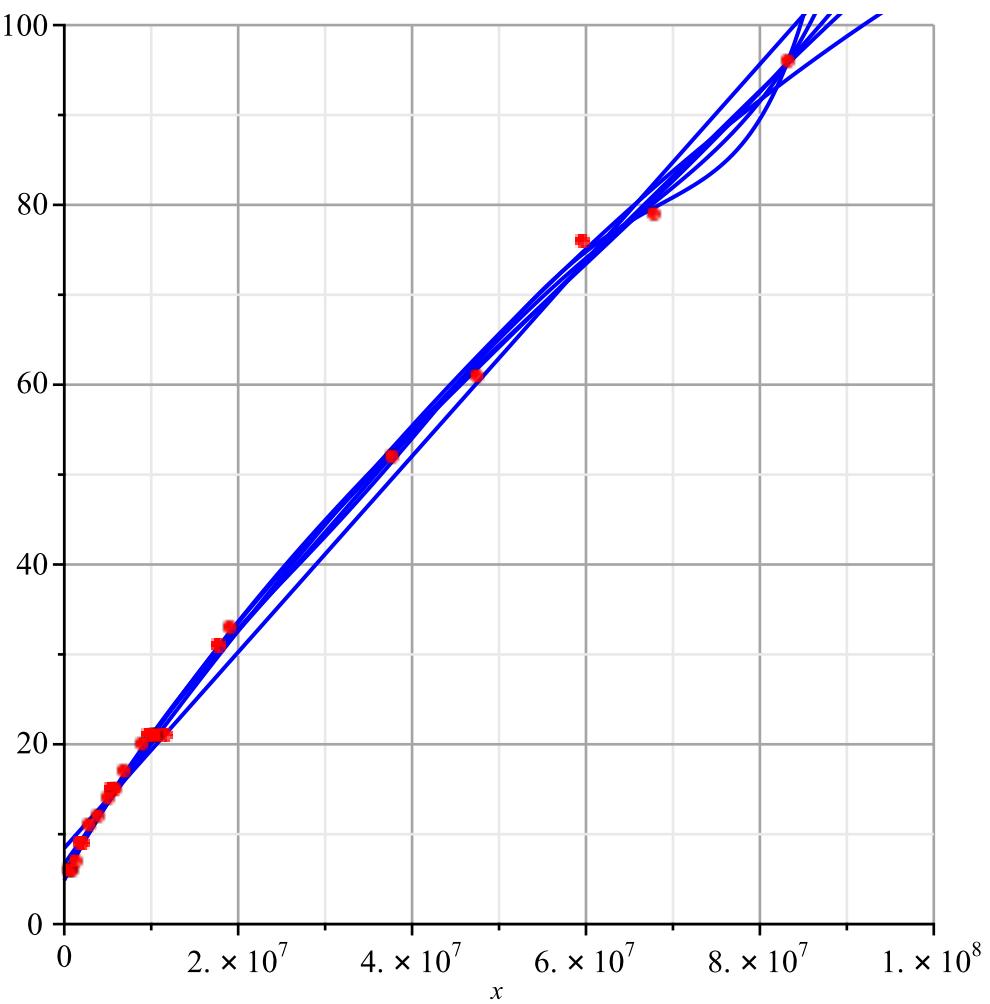
display(POL[8], EUP)



Det ses tydeligt, at de 2 Inde med højeste mandattal giver en grov afvigelse på polynomiet!

De 6 første polynomier plottet i samme koordinatsystem:

$$\text{display}(\text{seq}(POL[j], j = 1 .. 6), EUP)$$



Det ser meget bedre ud end før.

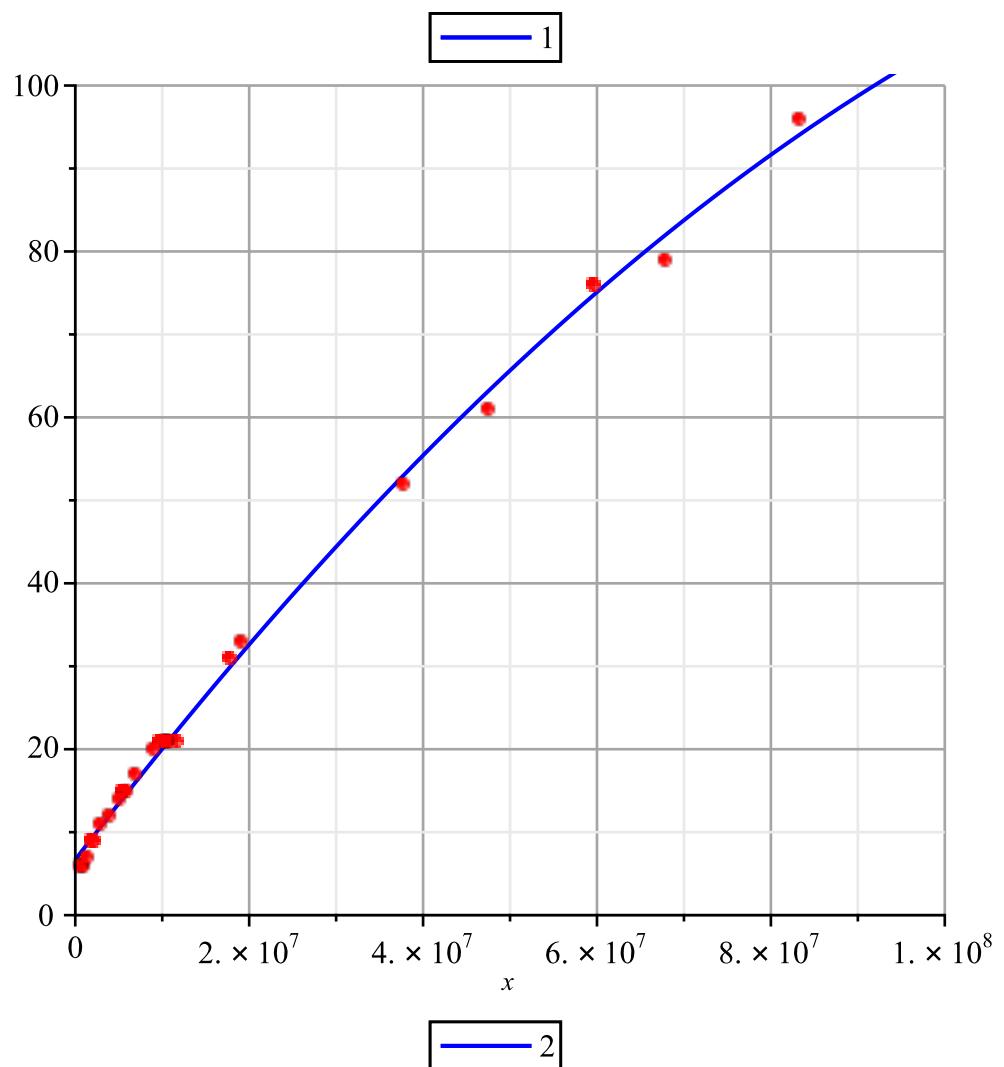
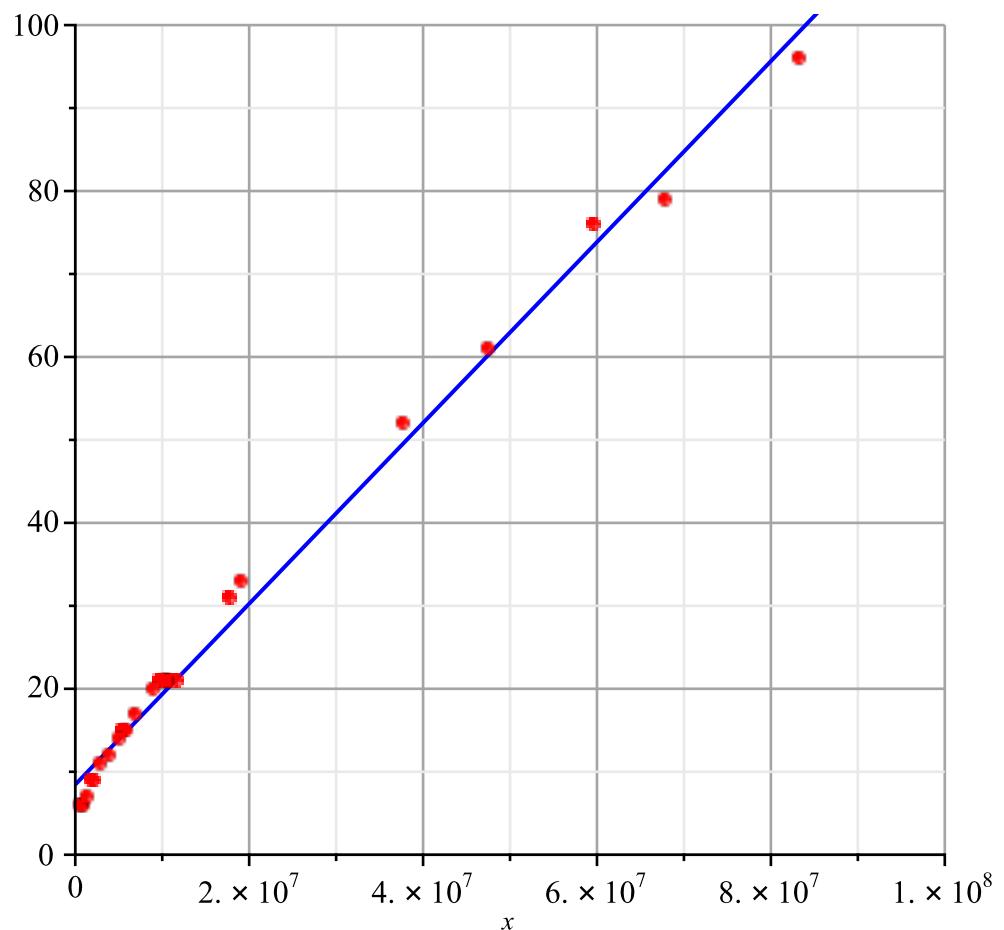
De første 8 grafer:

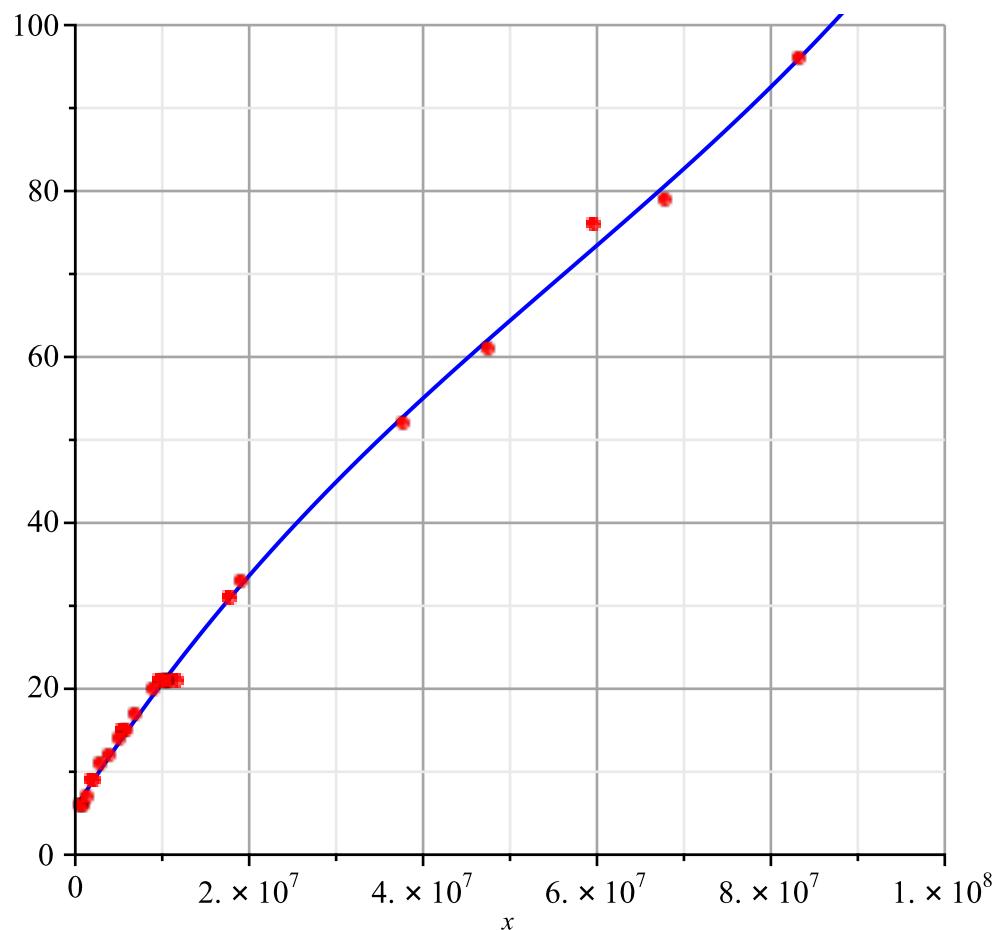
```

for j from 1 to 8 do
    POL2[j] := plot(Pol[j](x), x = 0 .. 100000000, view = [0 .. 100000000, 0 .. 100], gridlines, color = blue, legend
        = j):
end do:

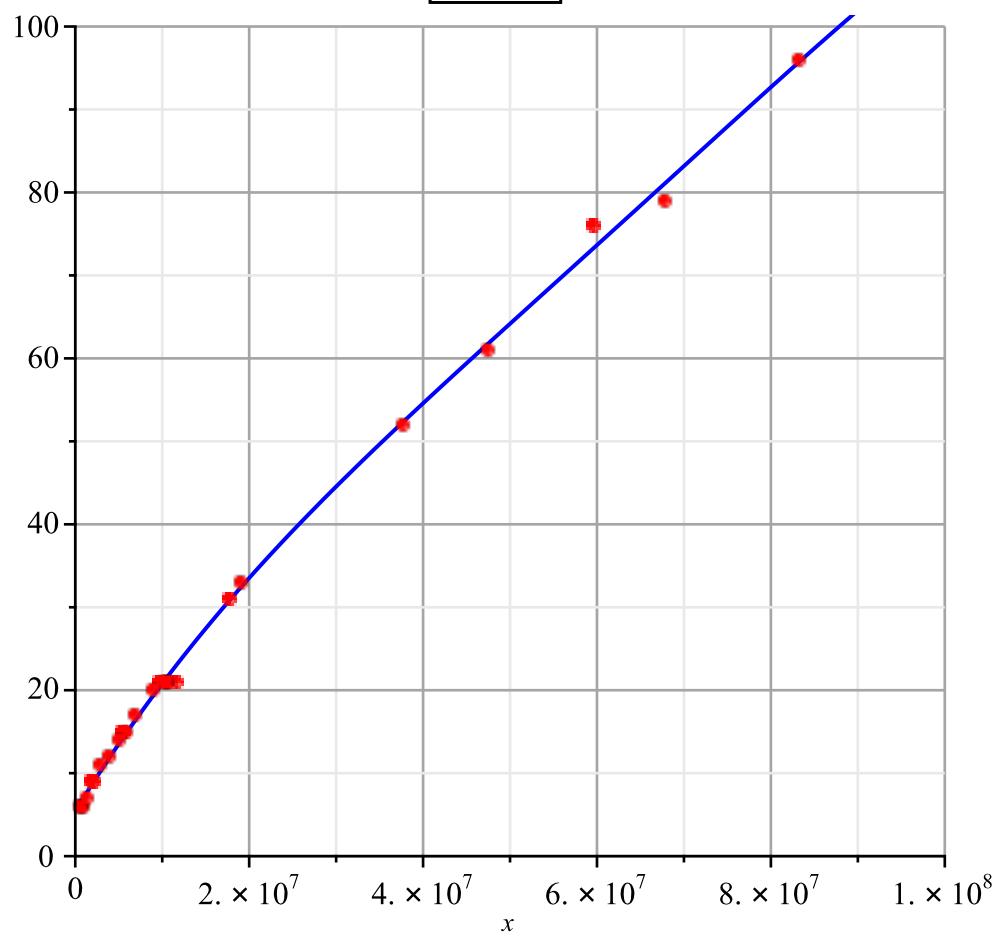
for j from 1 to 8 do
    display(POL2[j], EUP)
end do

```

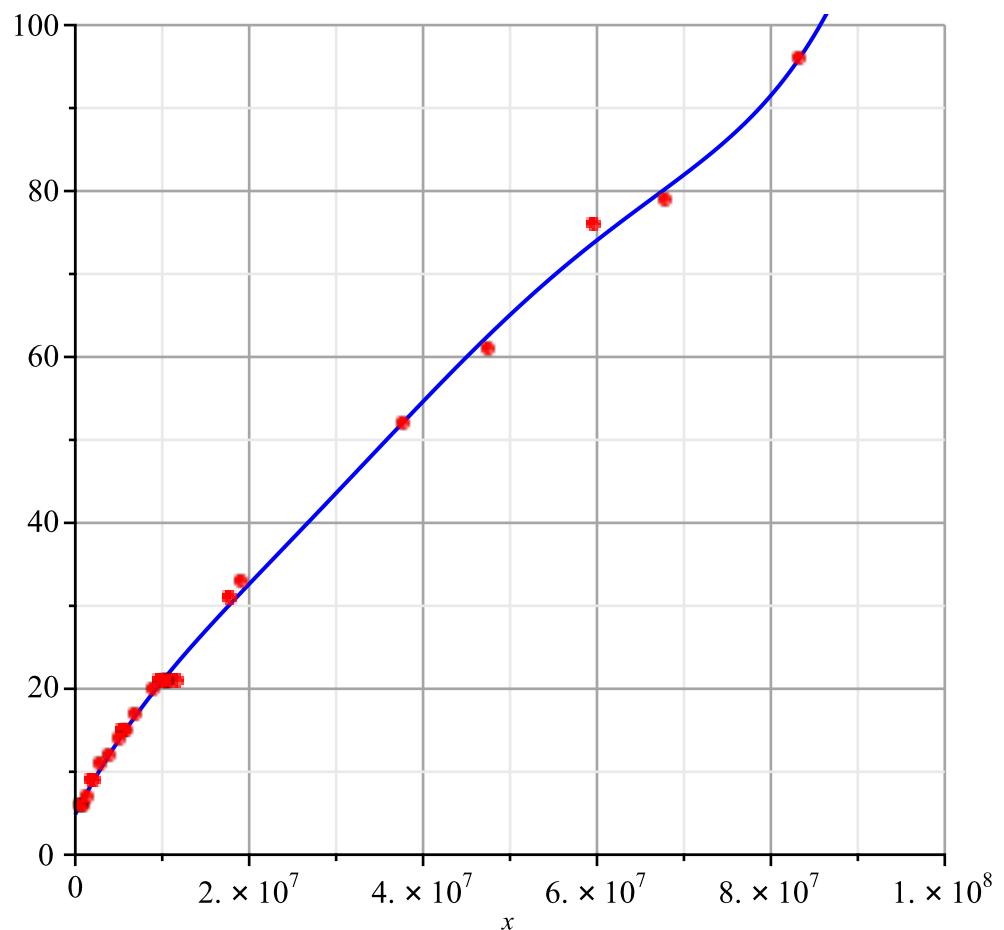




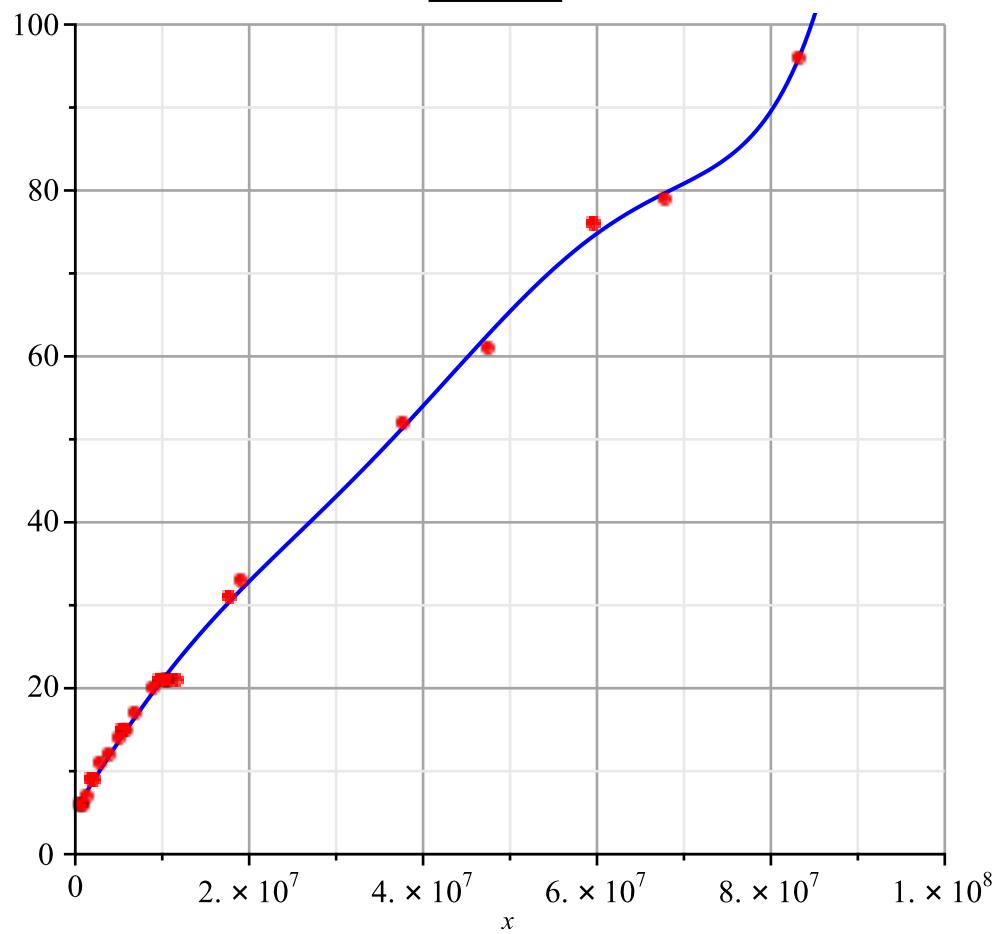
— 3



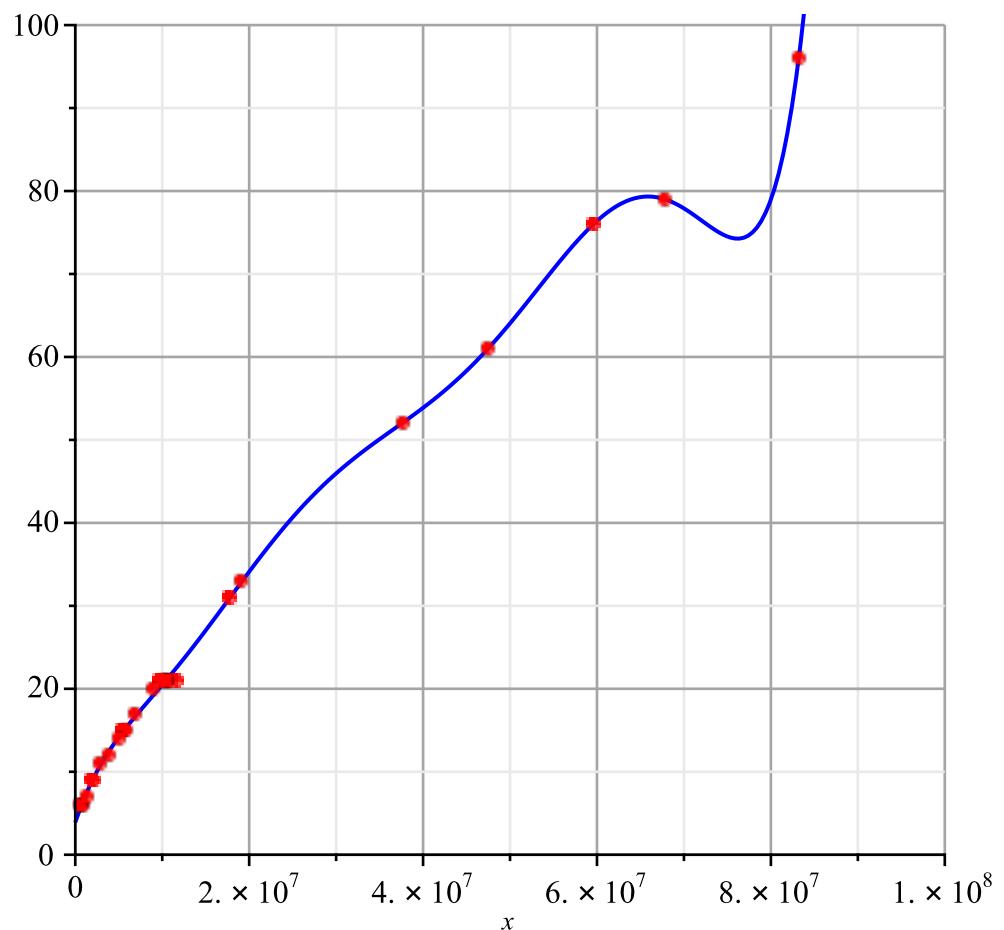
— 4



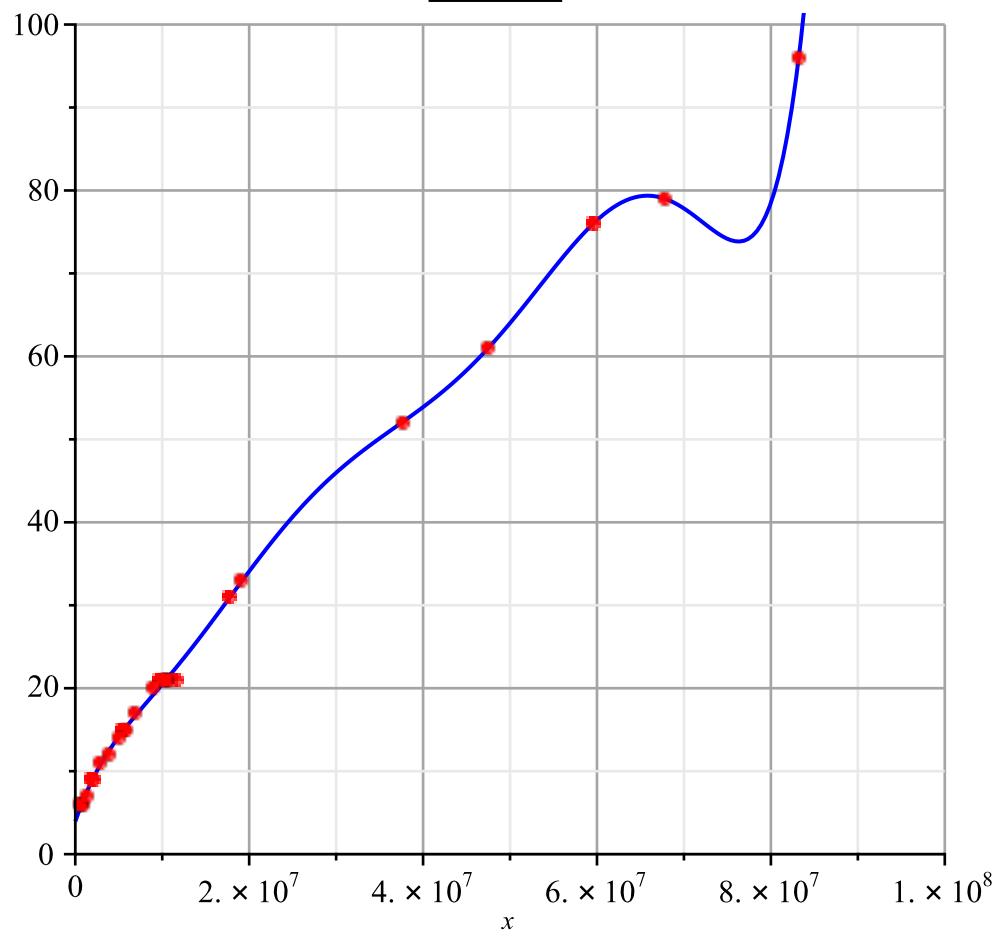
— 5



— 6



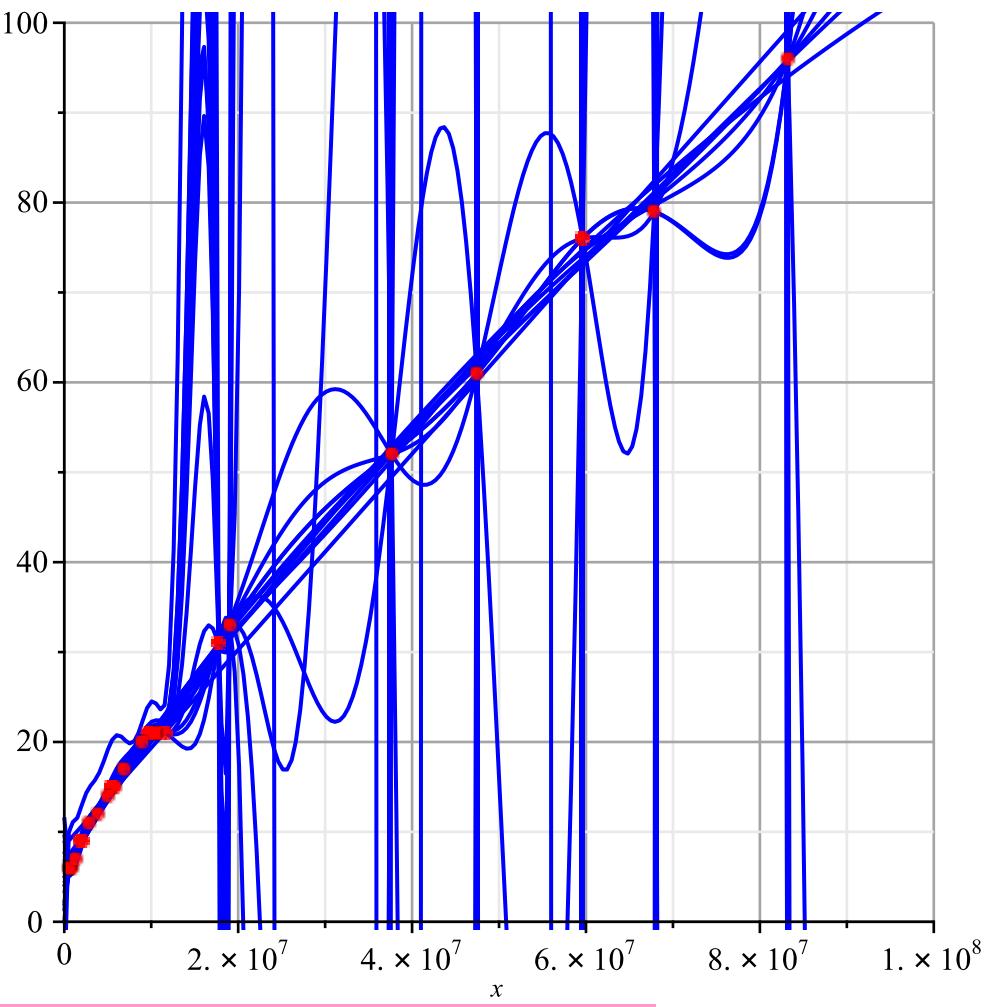
— 7



— 8

Alle 26 grafer plottet i samme koordinatsystem:

display(seq(POL[j], j = 1 .. 26), EUP)



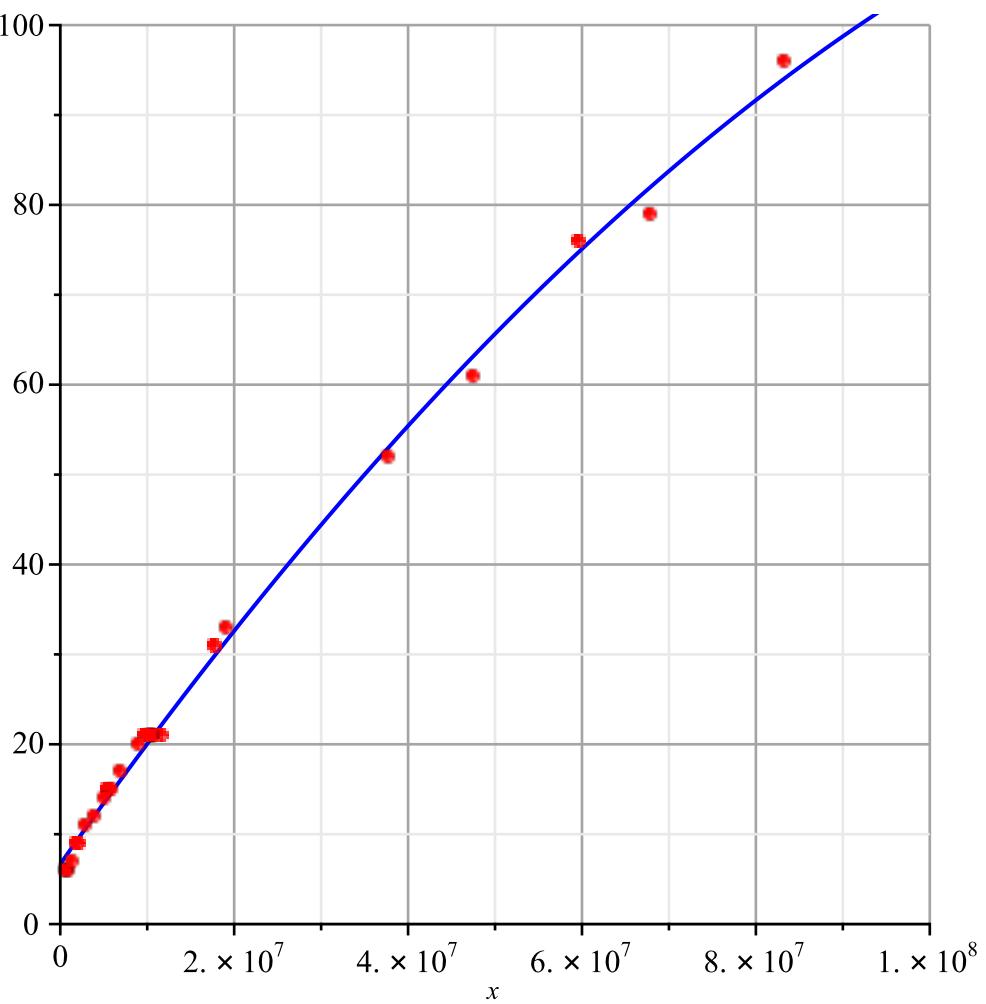
Rammer altså langt ved siden af for de høje grader af polynomiet.

Tilnærmelse med parabol, dvs. 2. grads polynomium

Inspireret af Victoriano Ramírez-González metode fra 2007.

$$Pol[2](x) = -3.91000142315624 \times 10^{-15} x^2 + 1.37481625967792 \times 10^{-6} x + 6.68091057995837$$

display(POL[2], EUP)



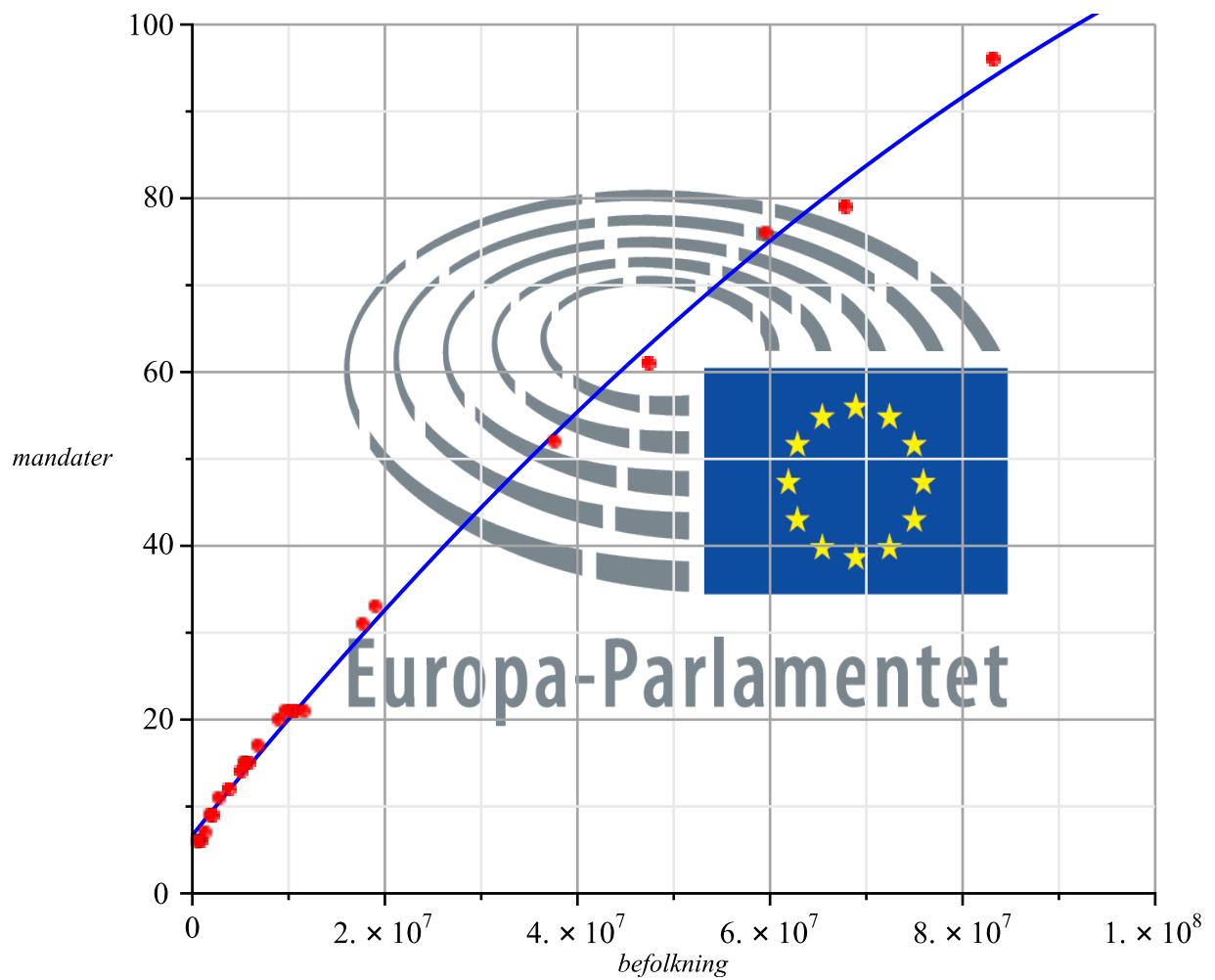
De 2 lande med flest mandater bør justeres!

Mere grafik

EP som baggrundsbillede

Dansk logo for Europaparlamentet kan downloades på: <https://download-centre.europarl.europa.eu/home.html>

```
display(POL[2], EUP, background = "C:/Users/Steen/Downloads/EP logo RGB_DA.png", size = [878·0.7,  
712·0.7], labels = [befolkning, mandater])
```



▼ Med halv højde

`display(POL[2], EUP, size = [800, 400], labels = [befolkning, mandater])`

